

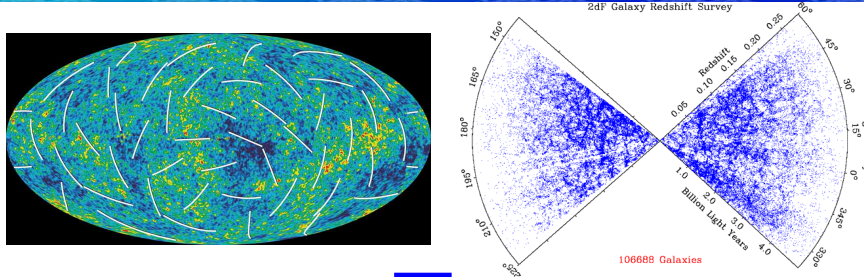
SuperBayeS.org

**A new statistical inference tool
for SUSY searches**

**Roberto Trotta
(collaborators: Roberto Ruiz de Austri,
Leszek Roszkowski & Joe Silk)**

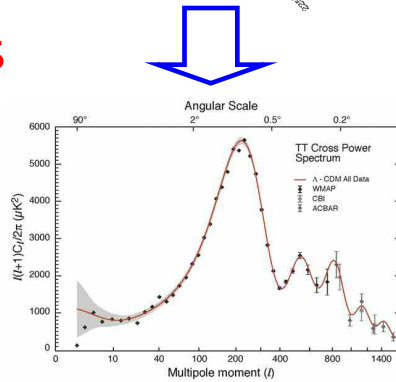
University of Oxford, Astrophysics
St Anne's College

Cosmological analysis pipeline



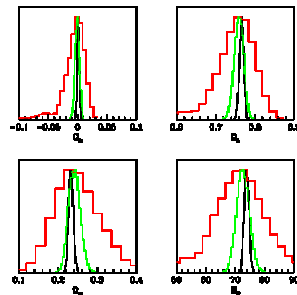
'raw' data » 10^6

Estimators



power spectra » 10^3

MCMC vs grid



cosmological params » 10

Purely Bayesian question

Λ CDM

model selection » 1

superbayes.org

The Constrained MSSM

- *General MSSM scenario: soft SUSY breaking*

105 free parameters in the Lagrangian

- *Assuming Universal boundary conditions at M_{GUT}*

Gaugino masses:

$$M_1 = M_2 = M_3 = m_{1/2}$$

Scalar masses:

$$m_{H_d}^2 = m_{H_u}^2 = M_L^2 = M_R^2 = M_Q^2 = M_D^2 = M_U^2 = m_0^2$$

Trilinear couplings

$$A_u = A_d = A_l = A_0$$

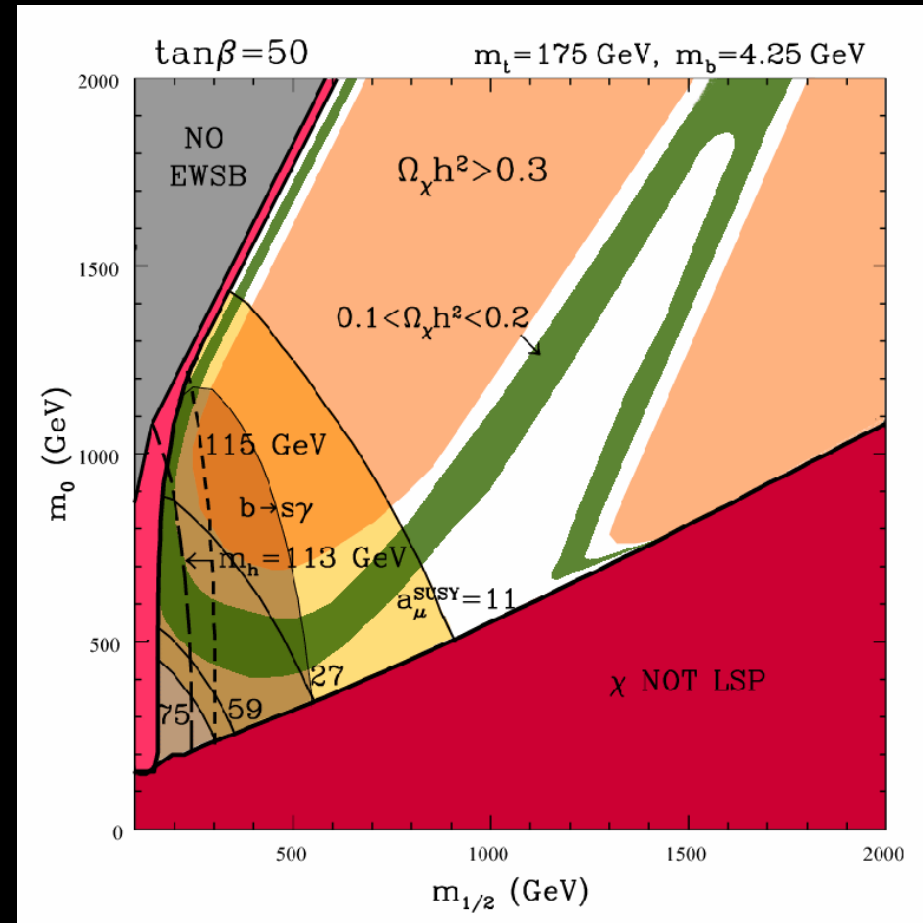
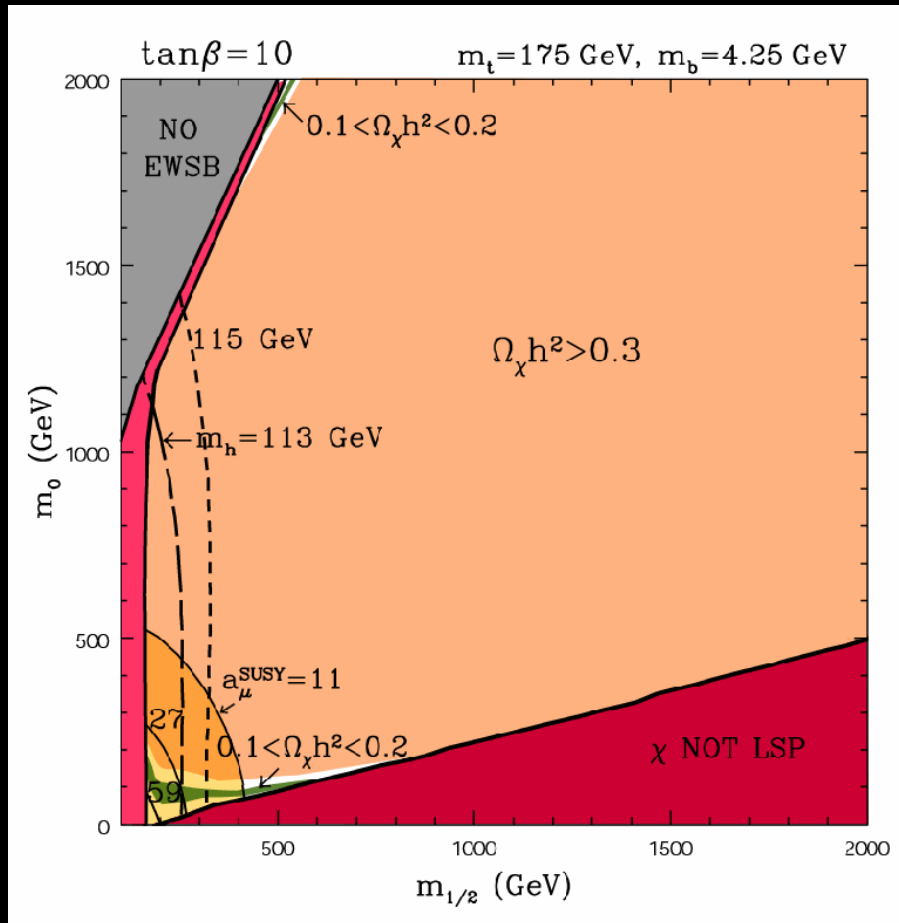
Higgs vev ratio

$$\tan\beta = v_u/v_d$$

μ^2 from EWSB

A 4 (5) parameters
benchmark scenario
 $m_{1/2}, m_0, A_0, \tan\beta$ (sign(μ))

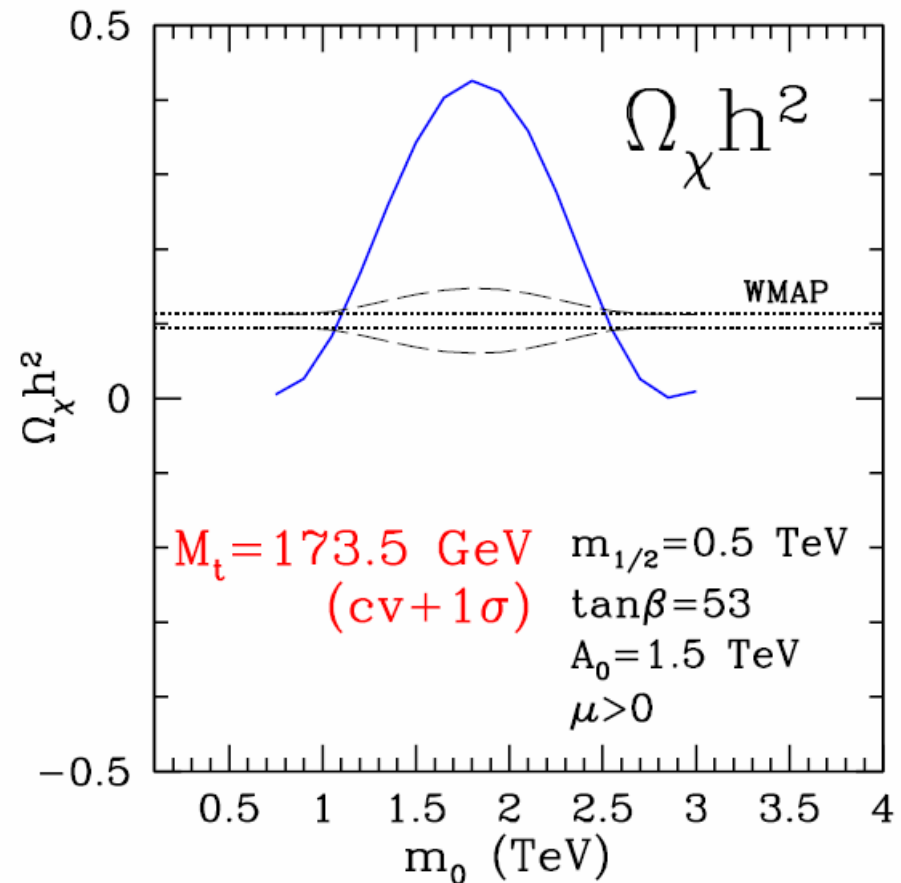
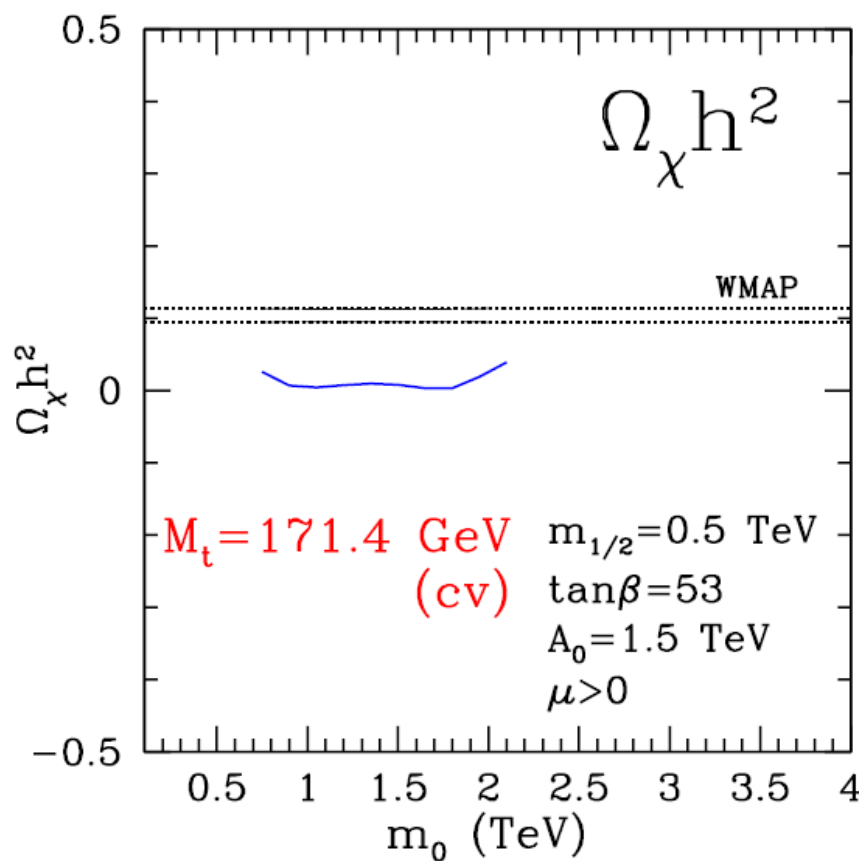
2D slices of CMSSM parameter space



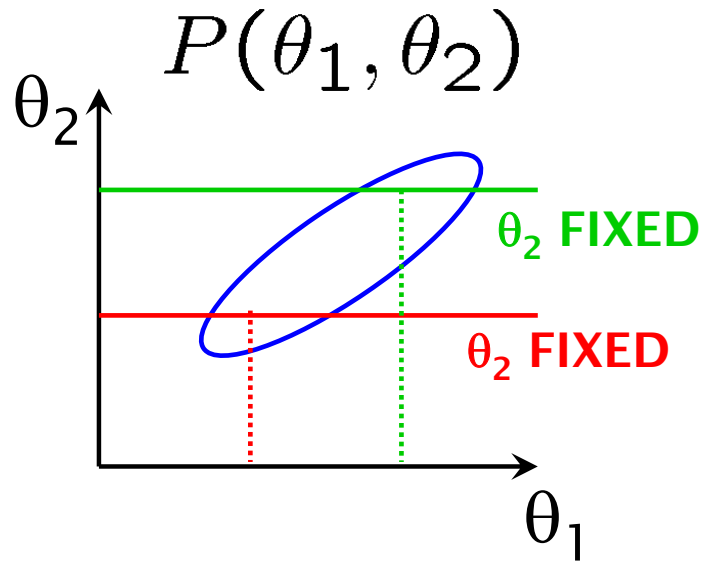
But this is only for fixed A_0 , $\tan\beta$

Fixing nuisance parameters is not enough

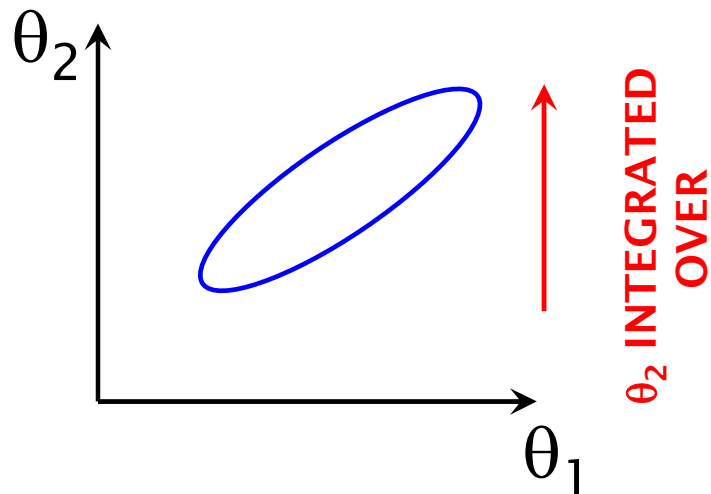
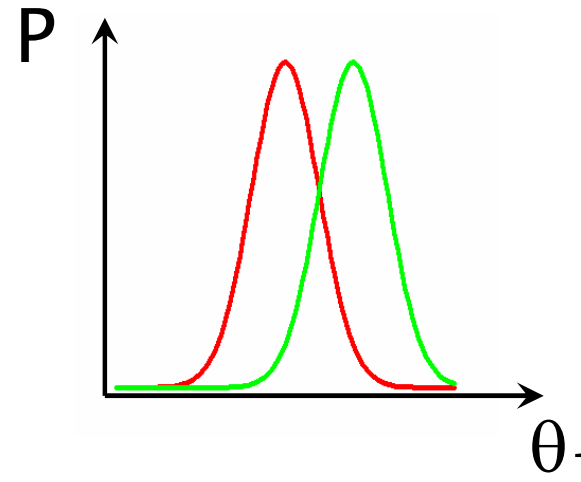
- Example: CDM relic abundance dependence on m_t



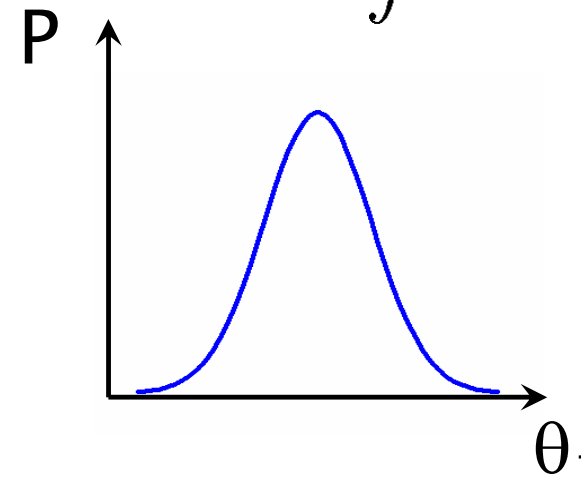
Marginalization



$$P(\theta_1) = P(\theta_1, \theta_2 = \theta_*)$$



$$P(\theta_1) = \int P(\theta_1, \theta_2) d\theta_2$$



A Bayesian analysis of the CMSSM

- *CMSSM parameters*

$$m_0, m_{1/2}, A_0, \tan \beta, \text{sgn}(\mu)$$

- *'Nuisance' parameters*

$$m_b(m_b)^{\overline{MS}} = 4.20 \pm 0.07 \text{ (GeV)}$$

$$m_t = 171.4 \pm 2.1 \text{ (GeV)}$$

$$1/\alpha_{\text{em}}(M_Z)^{\overline{MS}} = 127.955 \pm 0.018$$

$$\alpha_s(M_Z)^{\overline{MS}} = 0.1176 \pm 0.002$$

- *Observables*

(with full likelihood)

SUSY mass limits (LEP II),

Higgs limits, BR's, $g-2$, EW observables

cosmological CDM abundance

- *Output: probability distrib'ons for*

All observables and CMSSM parameters

Direct and indirect detection quantities (fluxes, cross sections...)

Collider cross sections and BR'os, sparticle masses, etc...

➤ Roszkowski, Ruiz de Austri & RT (2007)

➤ Roszkowski, Ruiz de Austri RT & Silk (2007)

➤ See also works by Baltz & Gondolo (2004), Allanach et al (2006)

superbayes.org

Bayesian parameter estimation

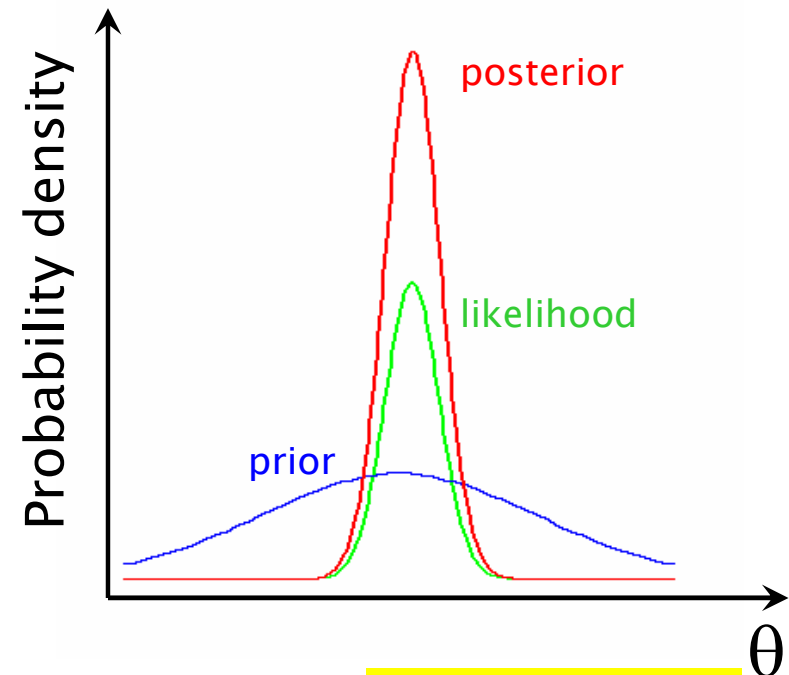
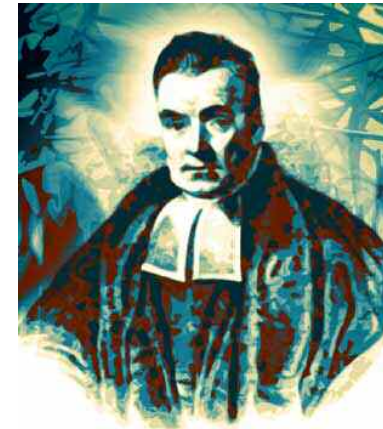
θ : parameters

d : data

Bayes' Theorem

$$\mathcal{P}(\theta|d) = \frac{\mathcal{L}(d|\theta)\pi(\theta)}{\mathcal{P}(d)}$$

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$



3 reasons to be Bayesian

- MCMC: a procedure to draw samples from the posterior pdf*

MCMC Bayesian Frequentist

<i>1) Efficiency</i>	/ N	/ k^N
<i>2) Marginalization</i>	trivial	close to impossible
<i>3) Predictivity for derived parameters</i>	YES	need estimator

Prior information

BE CAREFUL
THIS MACHINE HAS NO BRAIN USE YOUR OWN

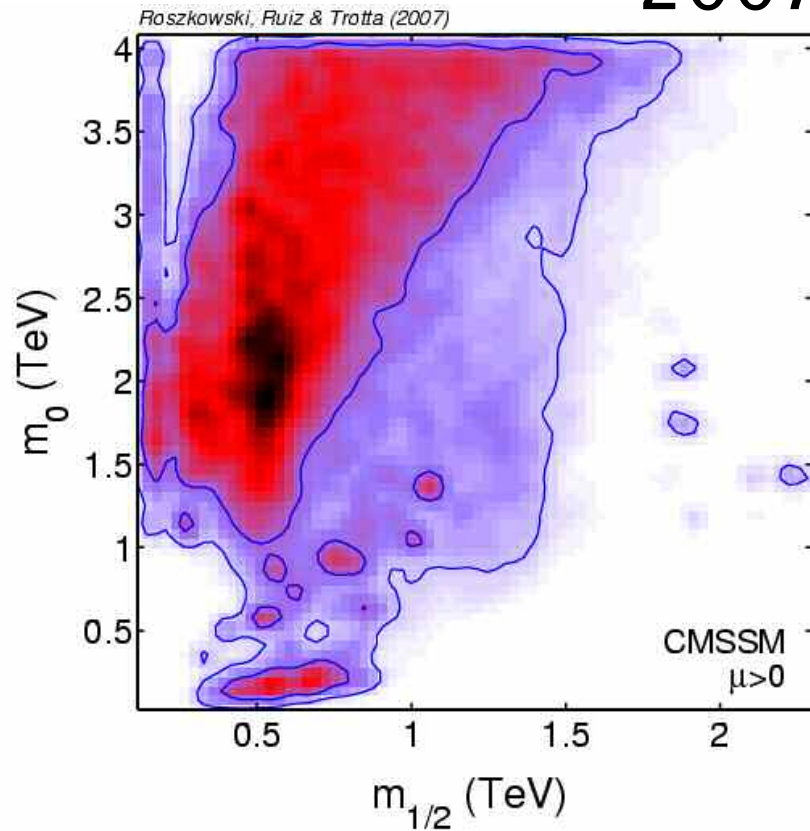
An 8-dimensional Bayesian scan

Experimental $b \rightarrow s\gamma$: $(3.55 \pm 0.26) \times 10^{-4}$

Theory SM: $(3.15 \pm 0.23) \times 10^{-4}$
(Misiak et al 07)

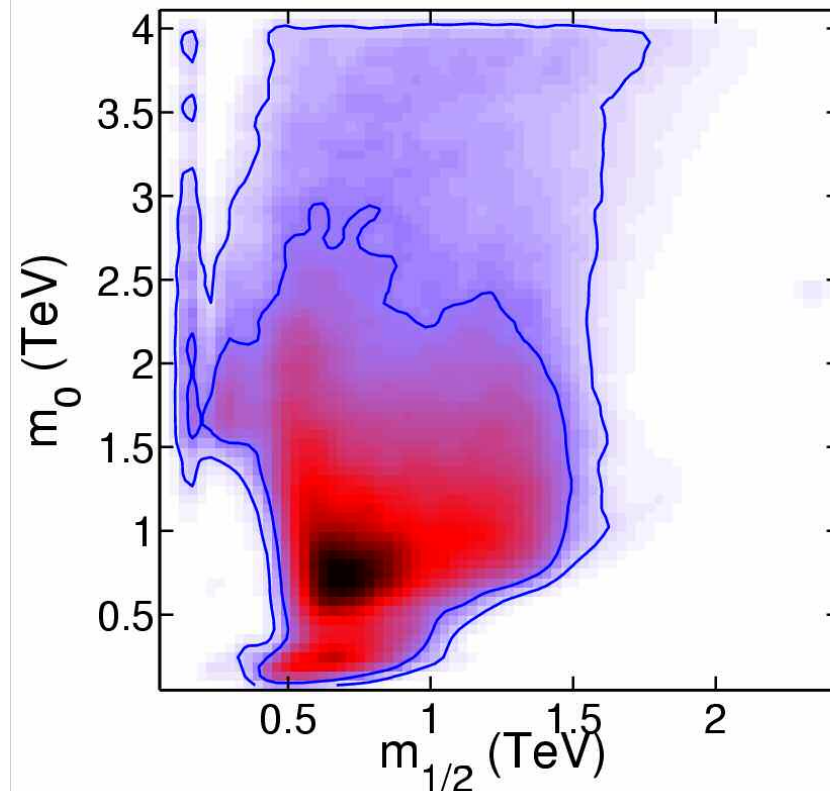
Old value: $(3.60 \pm 0.30) \times 10^{-4}$

2007



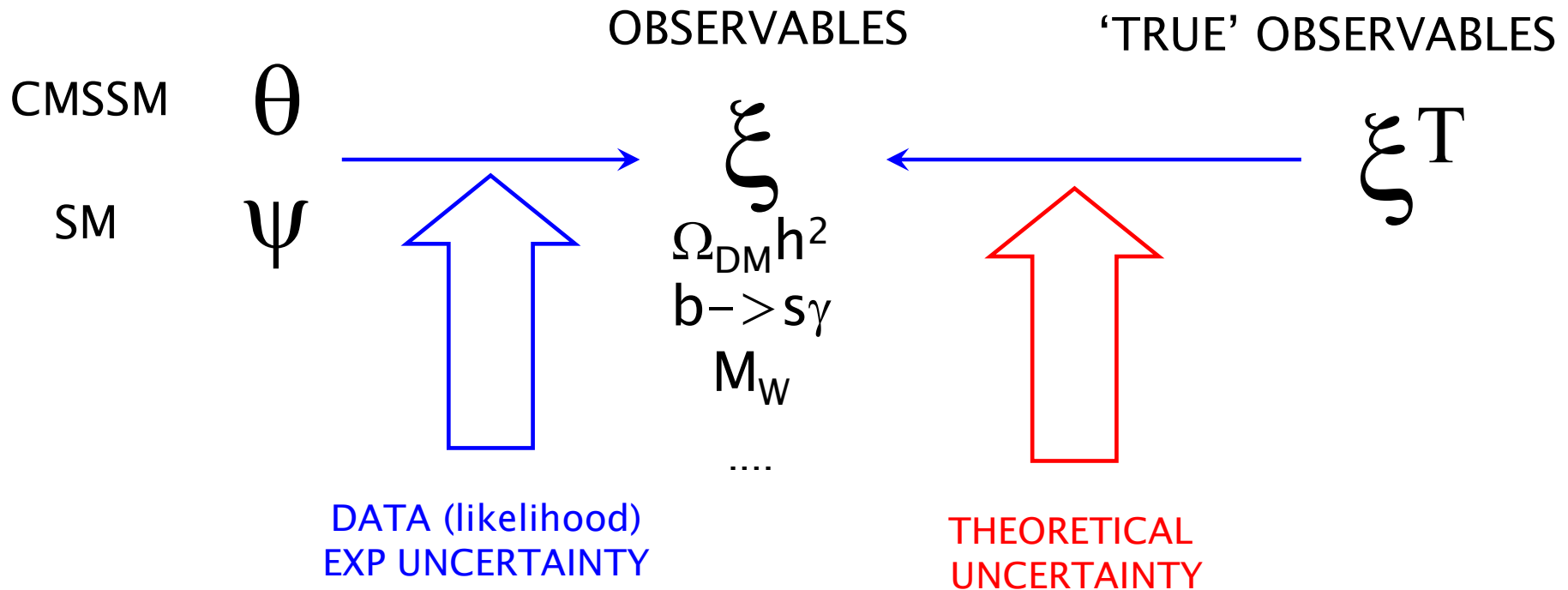
2006

Ruiz, Trotta & Roszkowski (2006)



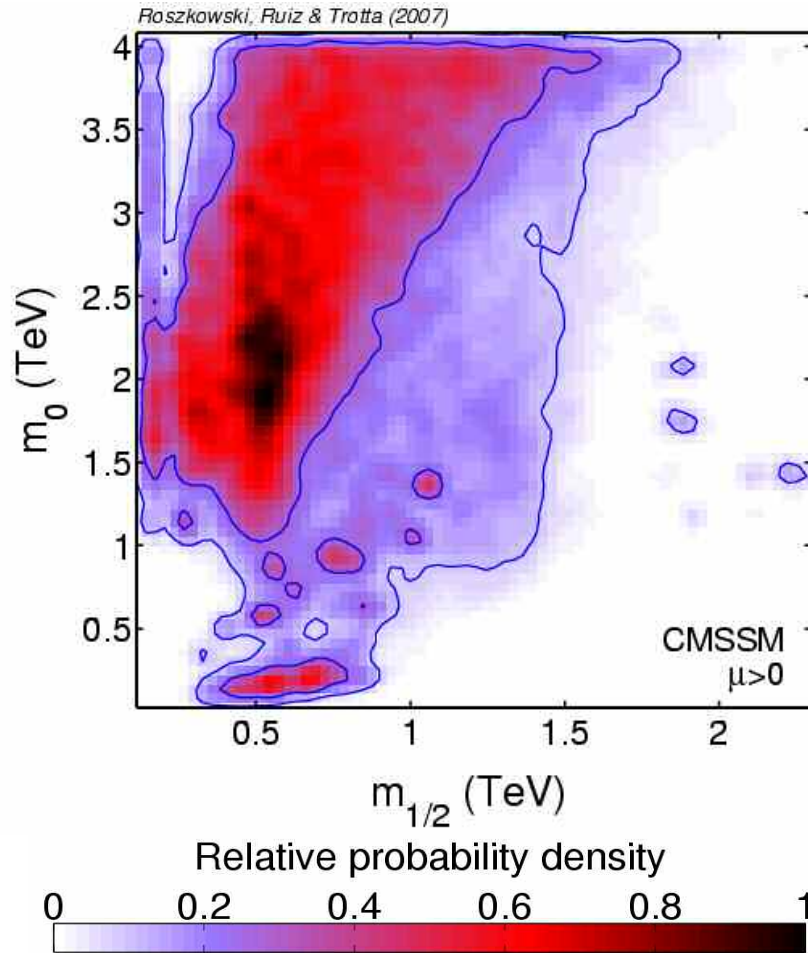
Theoretical uncertainties

- The Bayesian framework allows effortless incorporation of theoretical uncertainties:

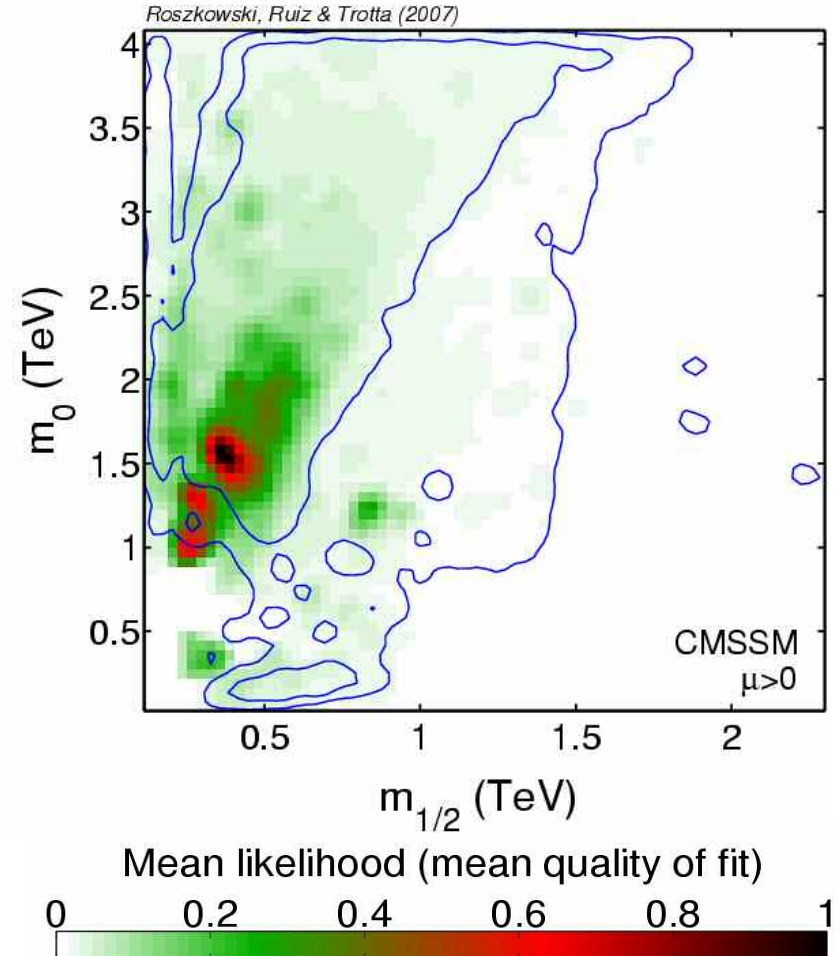


likelihood: $p(d | \theta, \psi) = \int p(d | \xi^T) p(\xi^T | \xi) d\xi$

Bayesian vs “quality-of-fit”



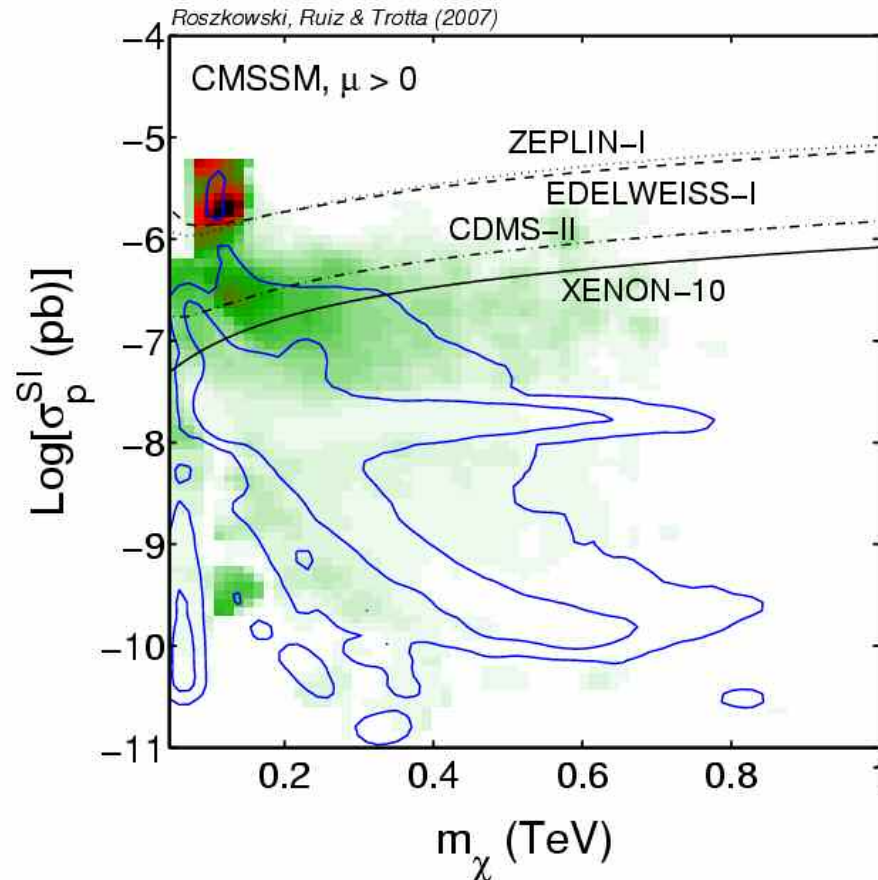
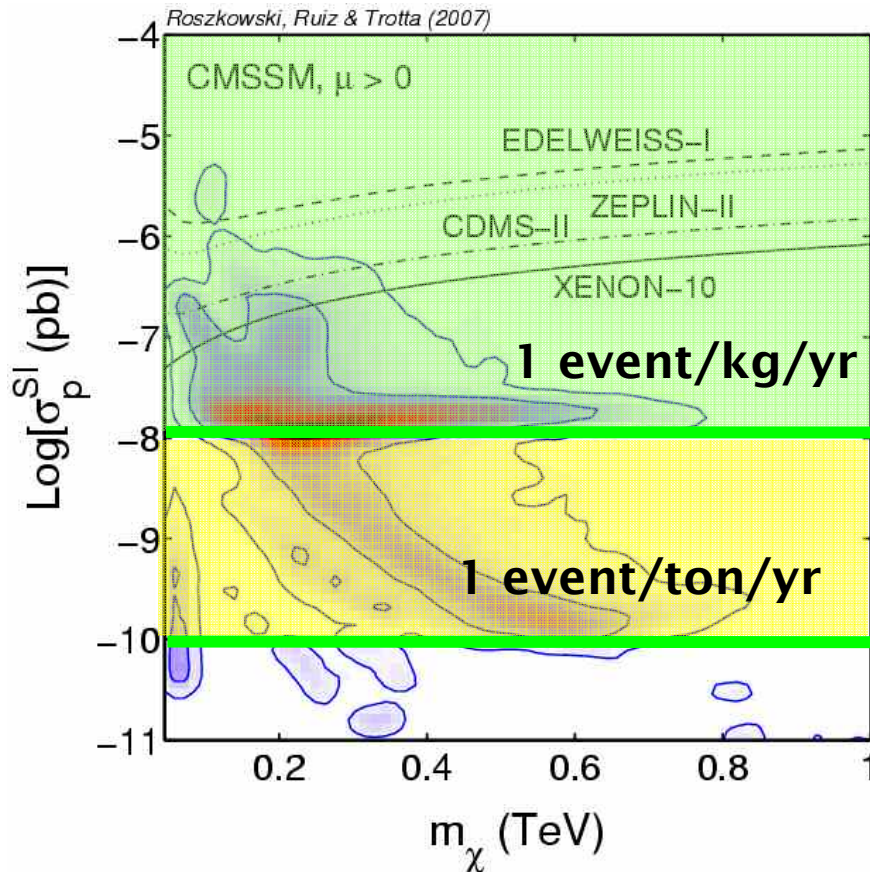
Posterior pdf
Represents “state of knowledge”
Volume effect of parameter space



Akin to “chi-square” statistics
Goodness of fit test

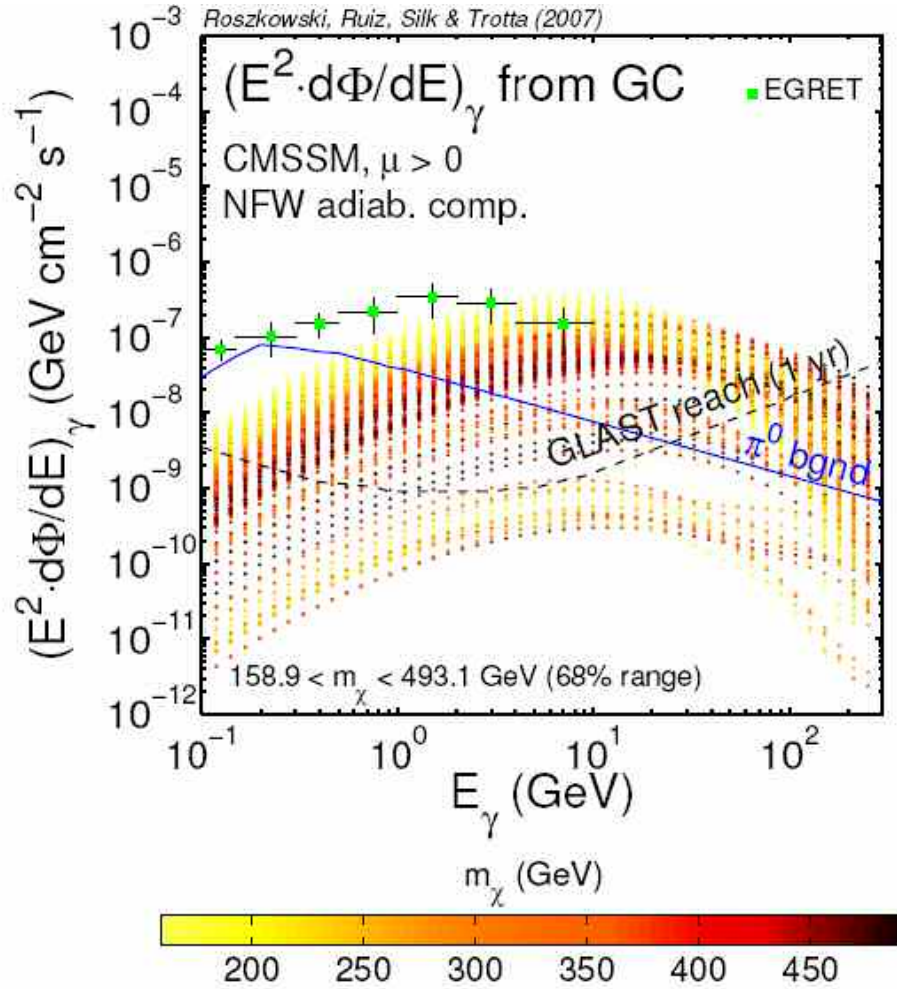
DM direct detection in the CMSSM

New $b \rightarrow s\gamma$ value (2007)
 $BR(B_s \rightarrow s\gamma) = 3.11 \pm 0.21$ (TH)

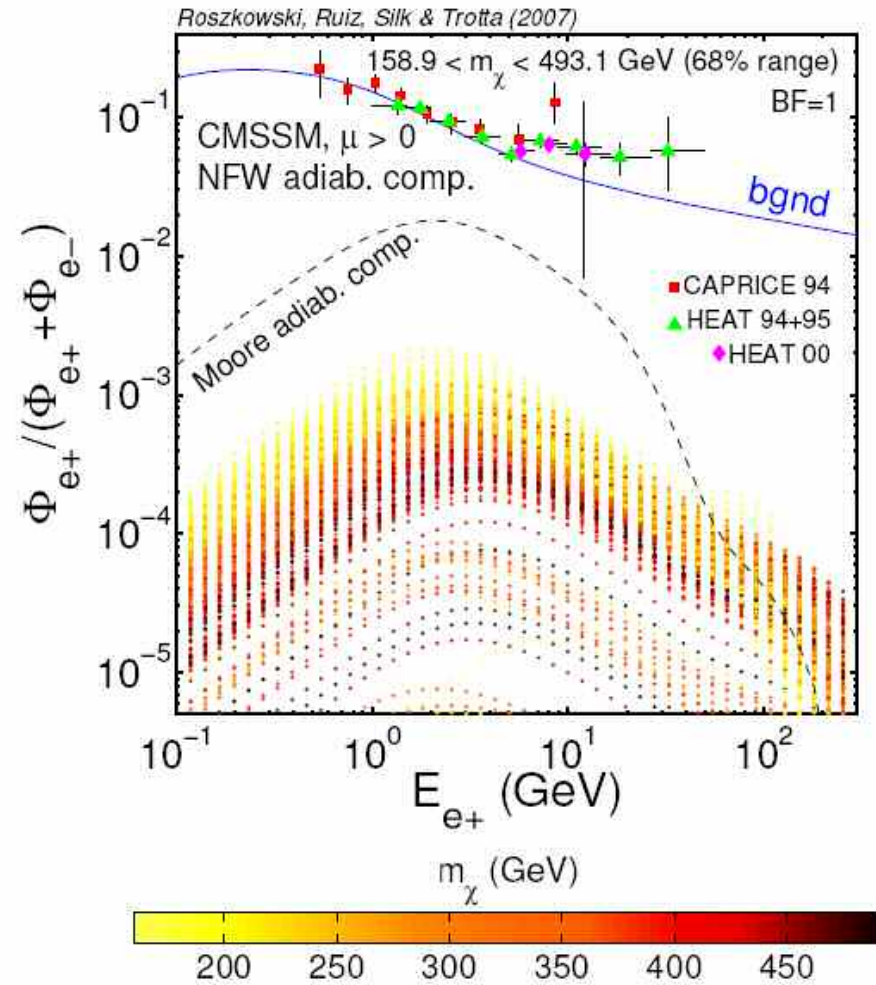


The new generation of detectors will probe most of the favoured region in the CMSSM

Predicted γ rays flux



Predicted positron flux



Code released in July 2007, v 1.0:

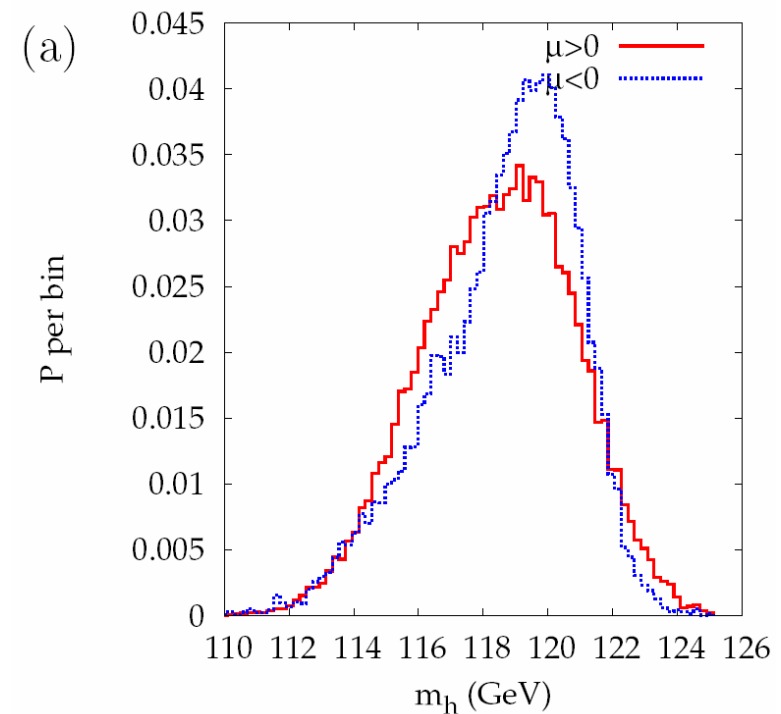
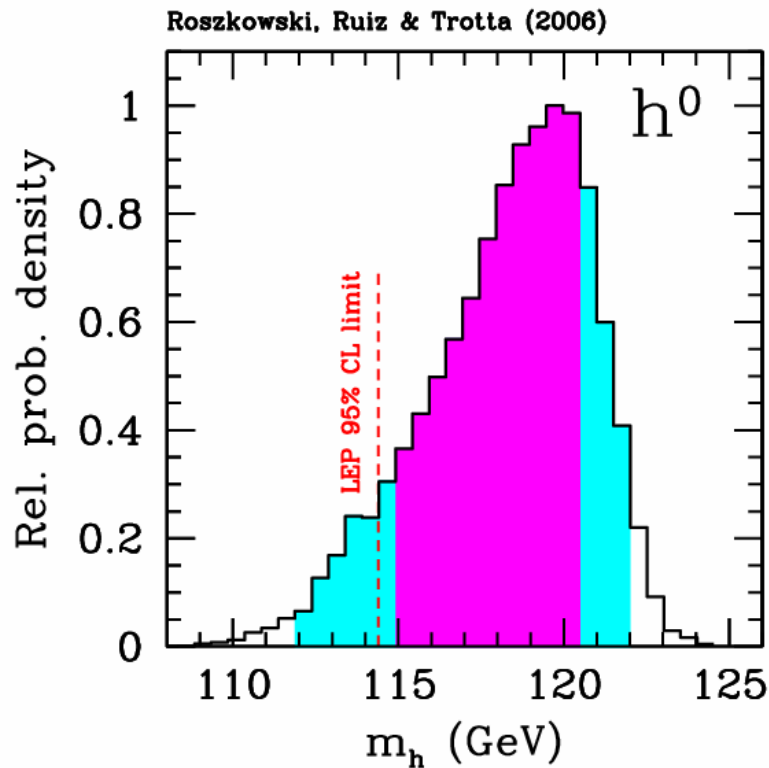
- *Implements the CMSSM, but can be easily extended to the general MSSM*
- *Includes up-to-date constraints from all observables*
- *Fully parallelized, MPI-ready, user-friendly interface*
- *Bayesian MCMC or grid scan mode, plotting routines*
- *Produces probability and quality of fit plots for all observables, CMSSM parameters, derived quantities, ...*



Thanks!

Light Higgs mass distribution

- *Detailed analysis in:* Roszkowski, Ruiz de Austri & RT (2006), *hep-ph/0611173*, $m_0 < 4$ TeV prior.
Recently updated with *new value*
 $BR(B_s \rightarrow s\gamma) \times 10^4 = 3.55 \pm 0.26$ (EXP), 3.11 ± 0.21 (TH) (Misiak et al 2006)



m_h range will be covered by Tevatron

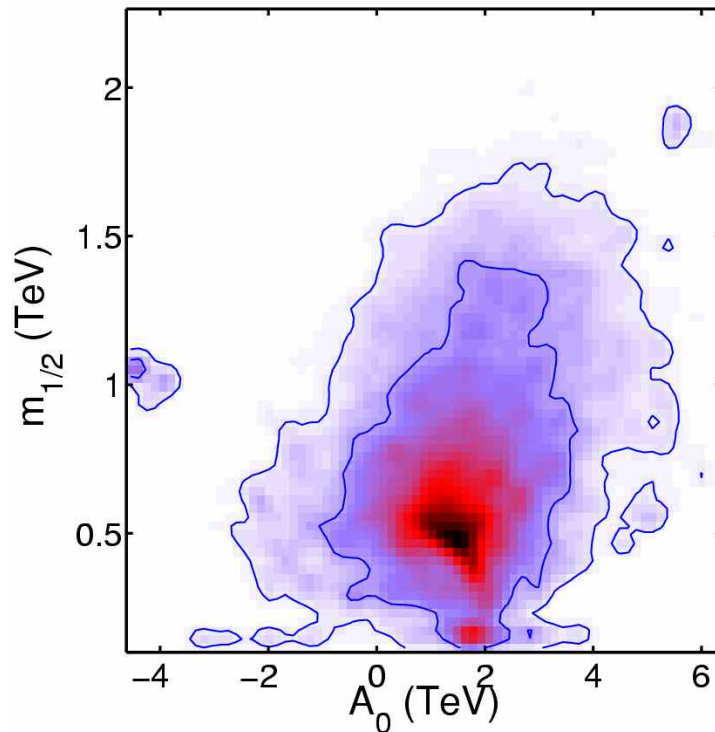
Allanach (2006); Allanach superbayes.org (2006)

Telling the truth with statistics

- Fully marginalised constraints vs chi-sq fits

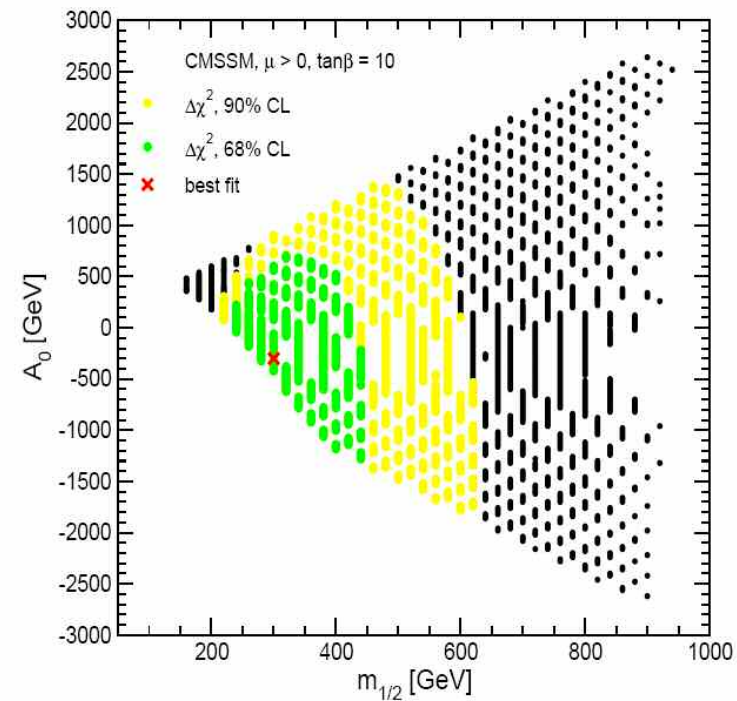
$m_t, m_b, \alpha_S, \alpha_{EM},$
 $\tan\beta, m_0$ constrained through data
and integrated over

Roszkowski, Ruiz & Trotta (2006)



Ruiz de Austri et al (2006)
Roszkowski et al (2007, in prep)

m_t fixed
 $\tan\beta = 10$ fixed
 m_0 fitted to WMAP

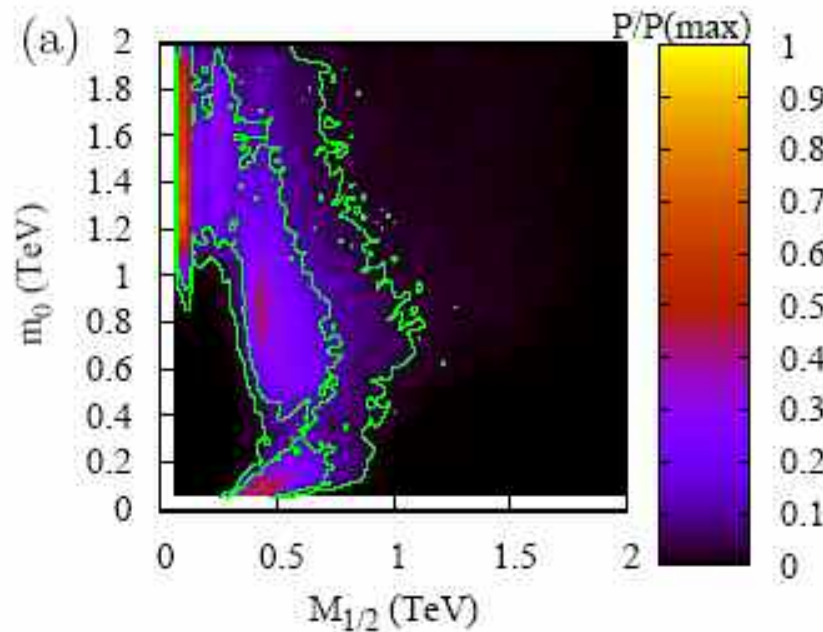


Ellis et al (2005), hep-ph/0508169

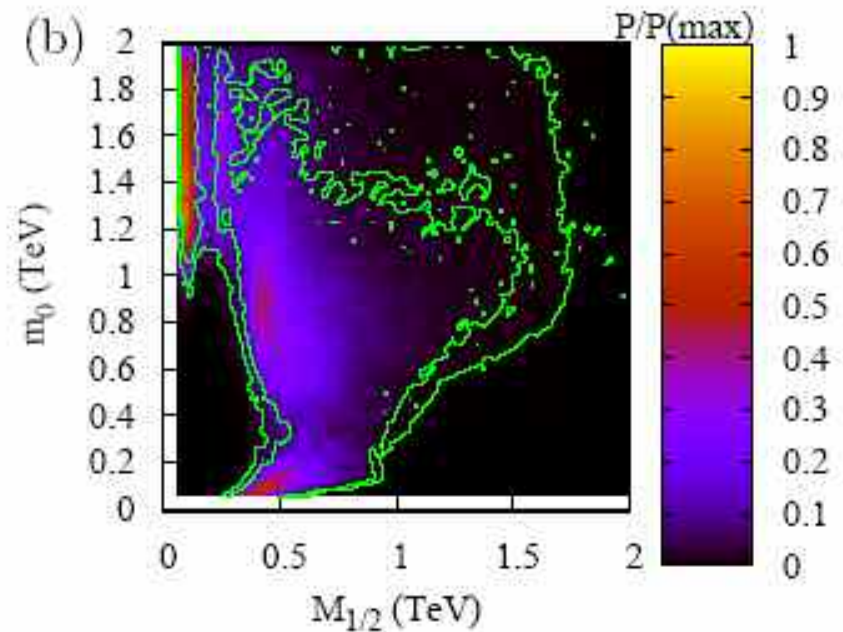
Change in priors I

- The fine tuning problem:
$$\frac{M_Z^2}{2} = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

- Amount of fine tuning:
$$c_i \equiv \left| \frac{\partial \ln M_Z}{\partial \ln p_i} \right|, \quad c \equiv \max\{c_i\}.$$



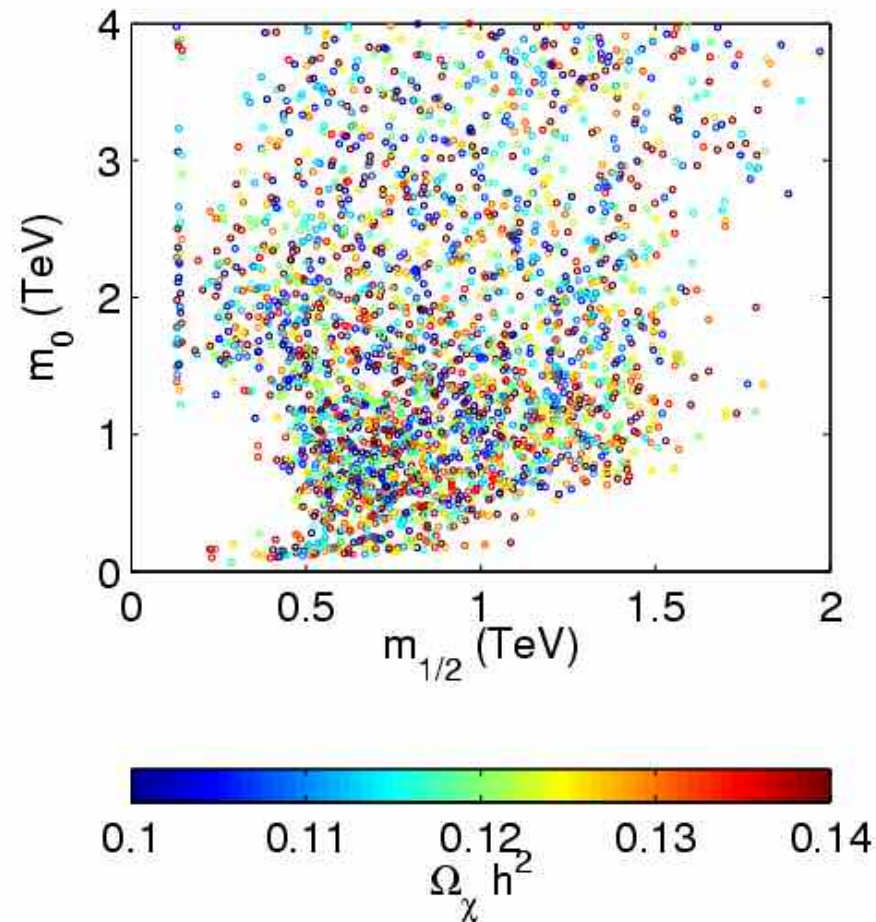
Naturalness prior



Flat prior

Just how constraining is $\Omega_m h^2$?

*Not very much
apart from setting an upper limit to $m_{1/2}$!*



(WMAP1 data)

MCMC Metropolis–Hastings algorithm

MCMC = Markov Chain Monte Carlo

- (1) *Select a random point in parameter space, θ_0
Compute $P(\theta_0) = \text{Like} * \text{Prior}$*
- (2) *Propose a new point, θ_1
with transition probability T , satisfying*
$$T(\theta_0, \theta_1) = T(\theta_1, \theta_0)$$
- (3) *Evaluate $P(\theta_1) = \text{Like} * \text{Prior}$*
- (4) *If $P(\theta_1) > P(\theta_0)$ move to θ_1
else
move to θ_1 with probability = $P(\theta_1)/P(\theta_0)$*

Obtain a Markov Chain: $\theta_i, i = 1, \dots, N$

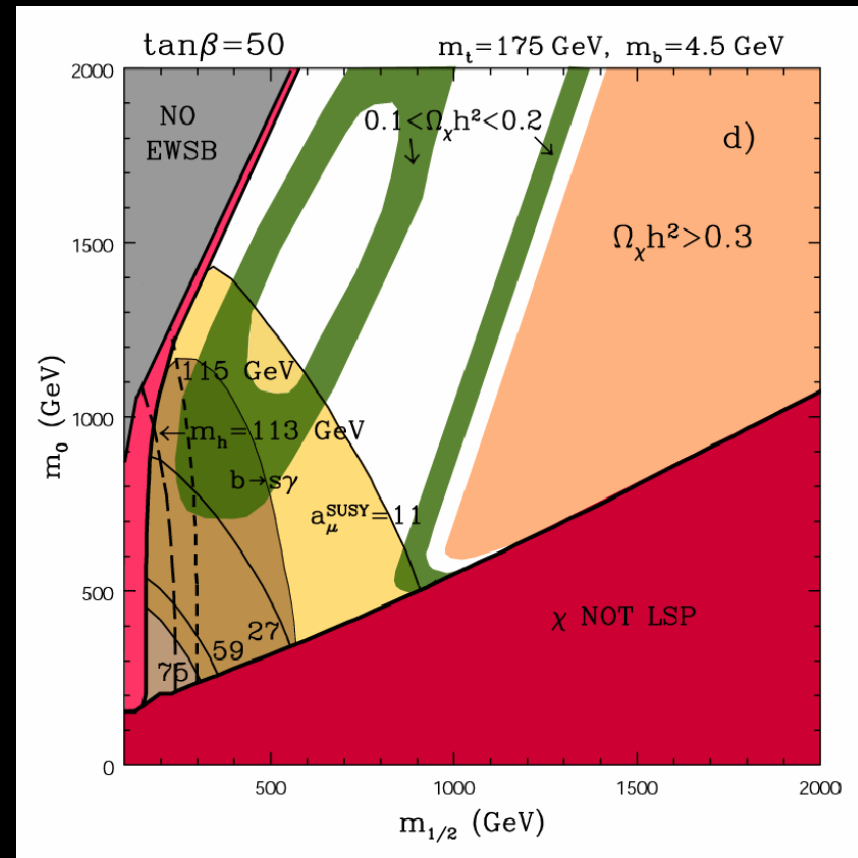
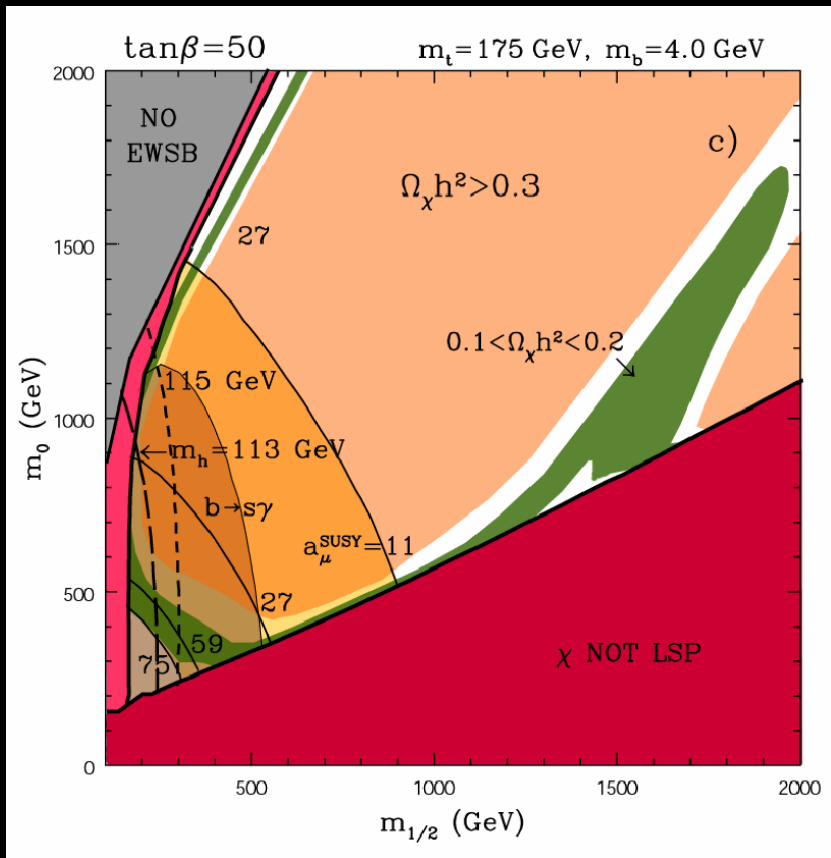
The density of points is proportional to the target distribution, $P(\theta)$

Statistical inference eg:

$$\langle f(\theta) \rangle = 1/N \sum_j f(\theta_j)$$

$m_b = 4.0 \text{ GeV}$

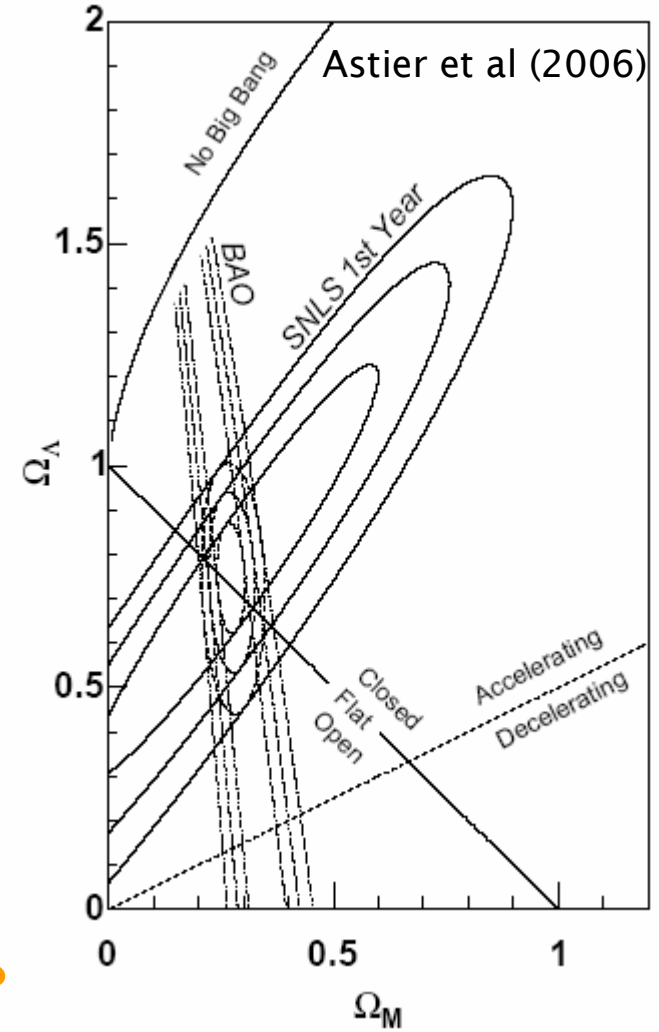
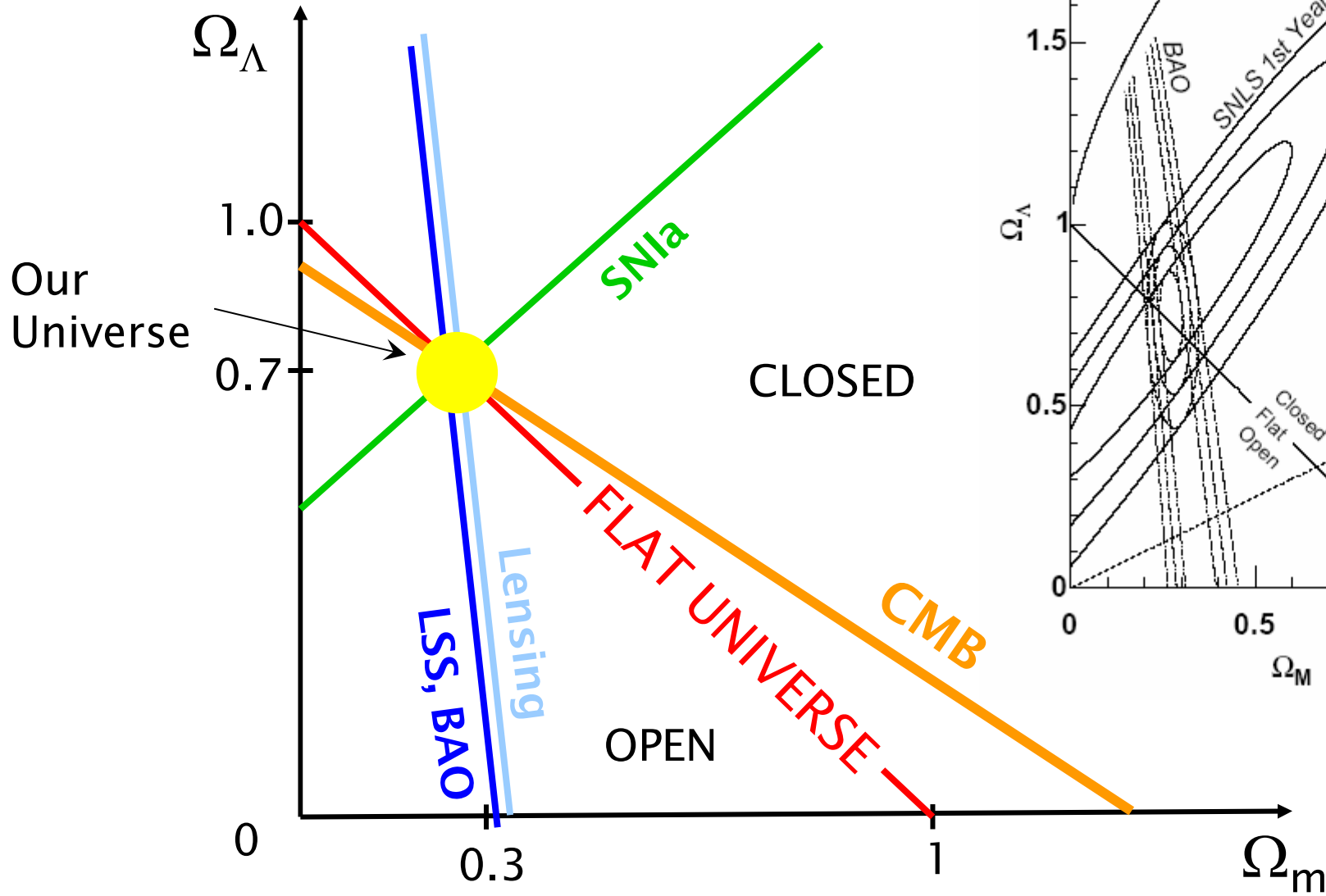
$m_b = 4.5 \text{ GeV}$



Uncertainty in SM parameters
cannot be neglected

(Roszkowski, Ruiz de Austri, Nihei 2001)

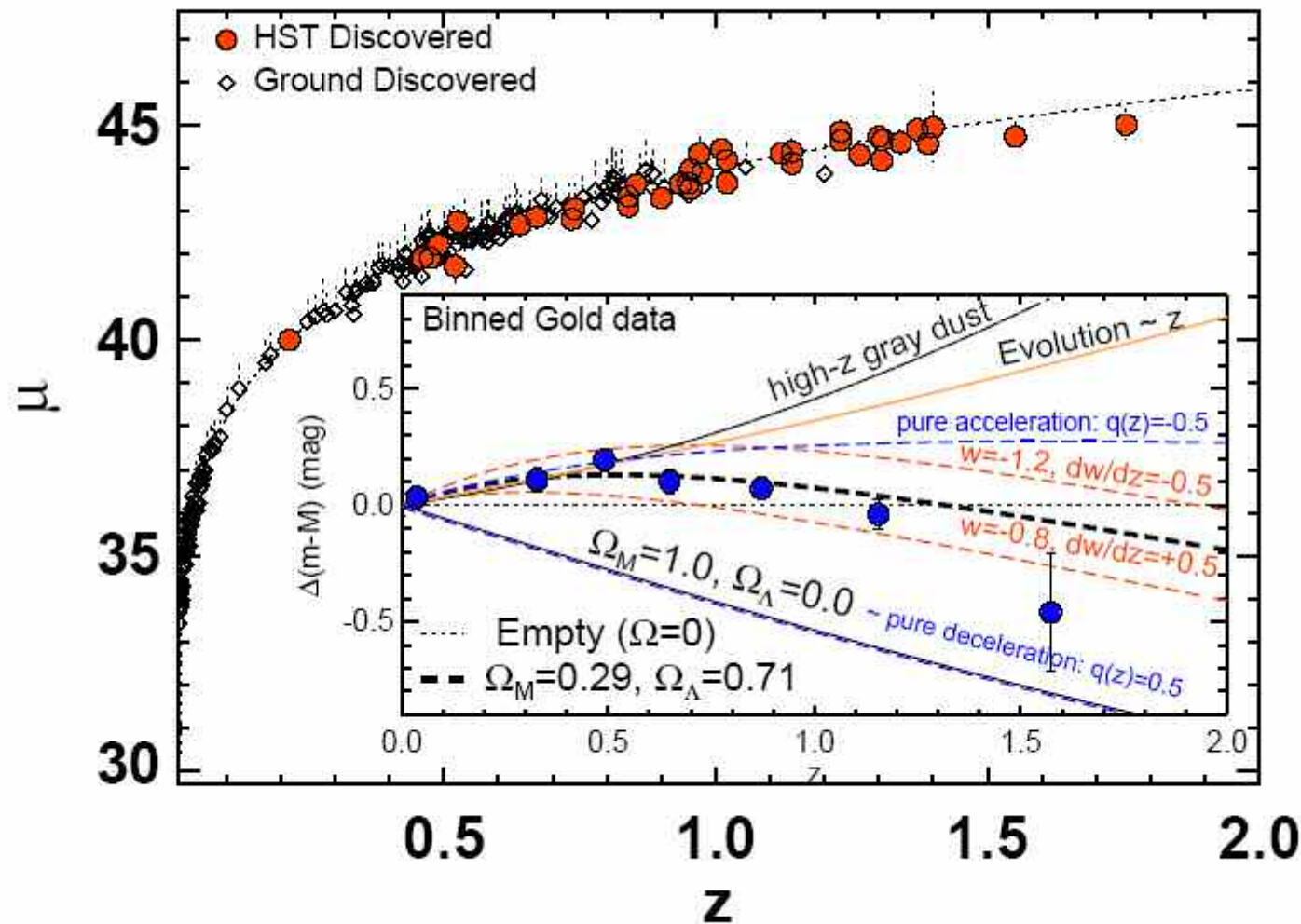
Accumulating evidence



$\Omega_m = 0.271 \pm 0.020, \Omega_\Lambda = 0.751 \pm 0.082$ suprbayes.org

Luminosity distance measurements

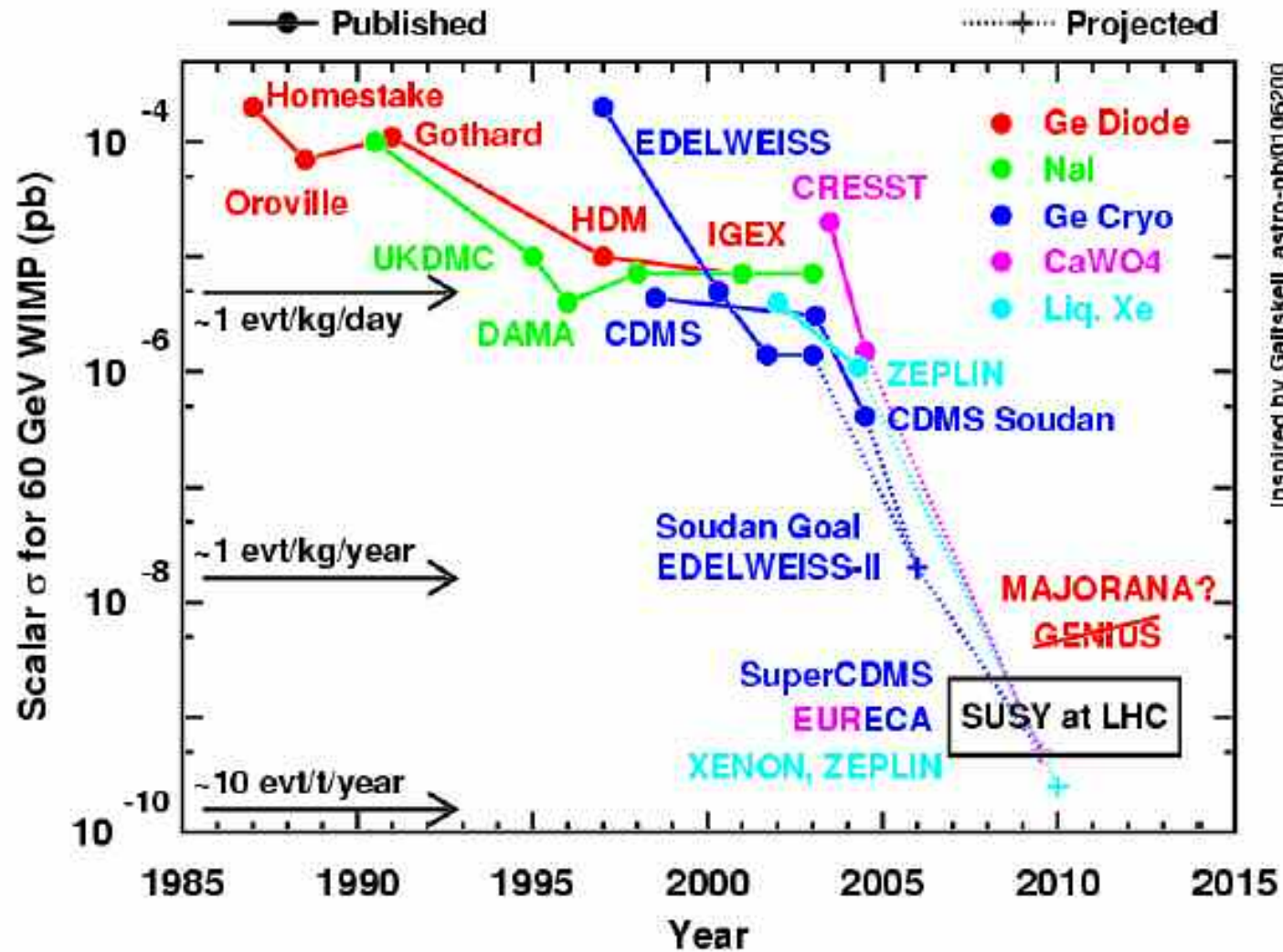
- *Supernovae type Ia as (almost) standard candles*



Riess et al (2006)

$D_L(z)$ (a function of $\Omega_m, \Omega_\Lambda, \Omega_k, H_0$)

Direct searches: present & future

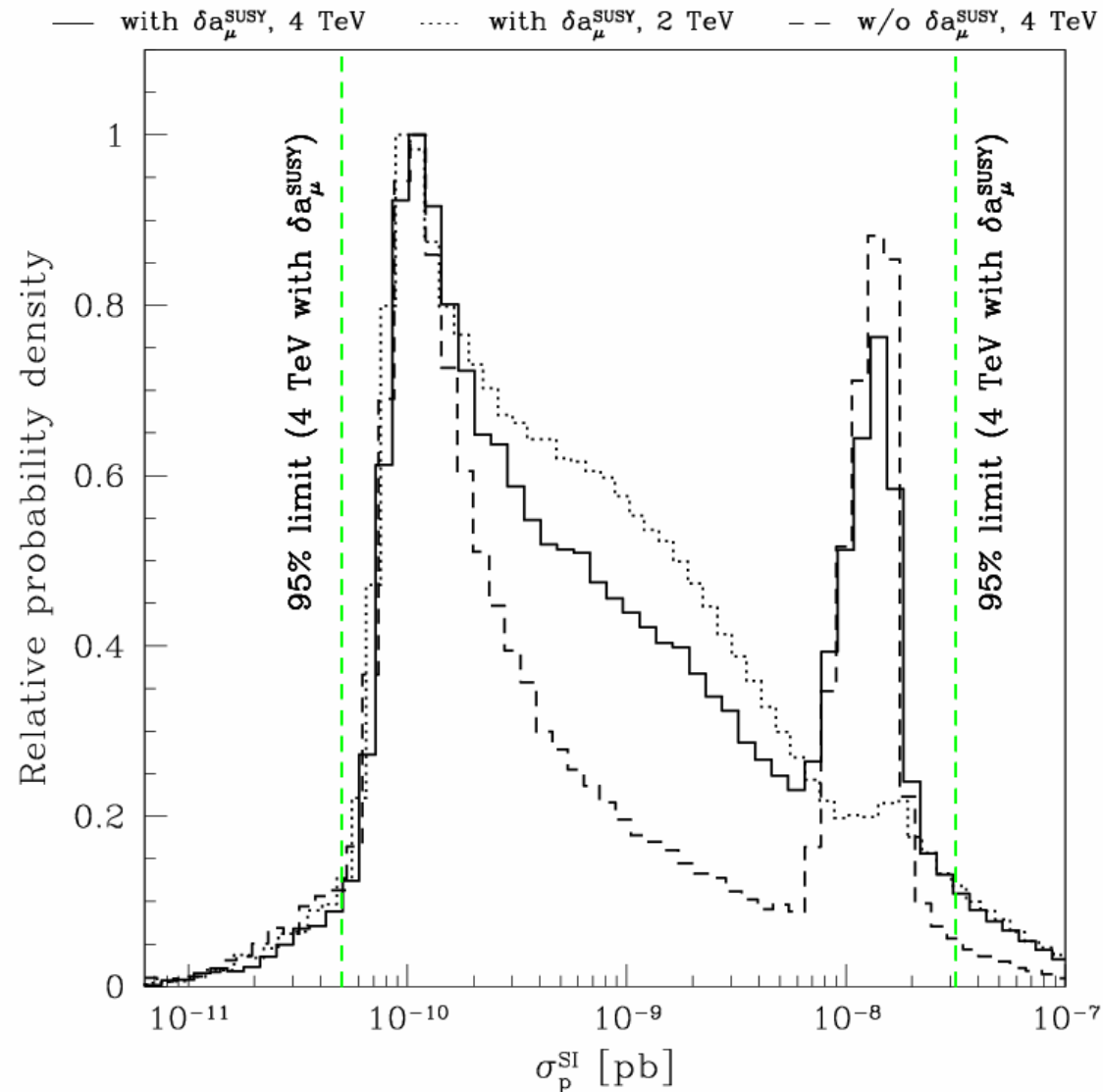


Courtesy Hans Kraus

superbayes.org

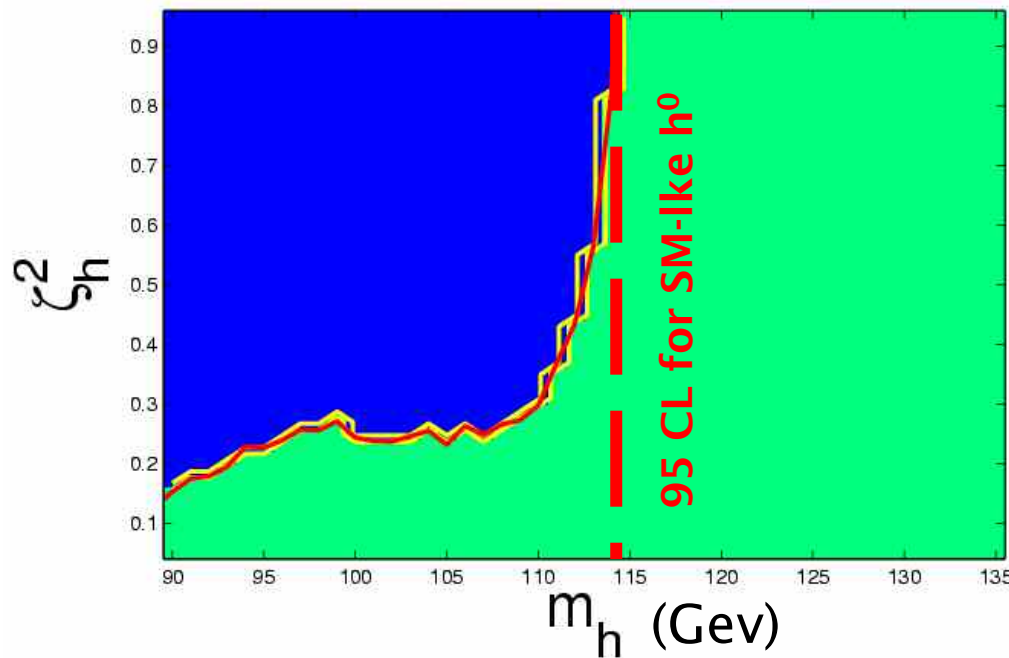
Sensitivity to assumptions

1D probability distribution
fairly robust
with respect to
a change in
prior ranges or
inclusion of g-2
data

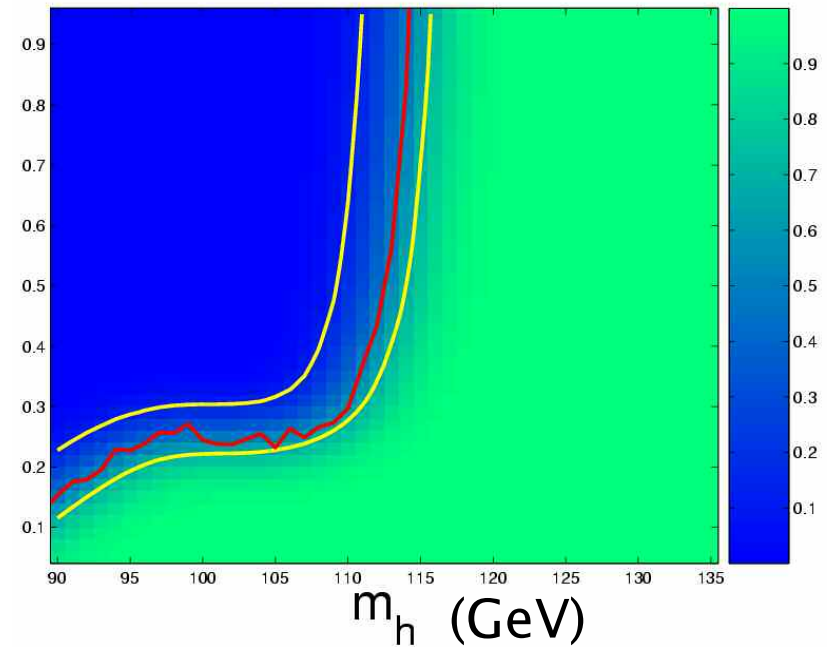


An example: Higgs mass LEP bounds

- *Need to consider likelihood in the (m_h, ξ_h^2) plane. Cannot simply assume that h^0 is SM-like*



NO THEORETICAL ERRORS



THEORETICAL ERRORS
in m_h (3 GeV) and ξ_h^2 (10%)