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ECFA/DESY Linear Collider Workshop

Padova, 5-8 May 2000

Prospects of Measuring General Higgs Couplings at e^+e^- Linear Colliders*)

Contents

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DESY 99-190, hep-ph/0002043, Eur. Phys. J. C (to appear).

1) General HZZ and HZγ couplings

Consider effective HZV interaction Lagrangian (Hagiwara, Stong '94)

$$\mathcal{L}_{eff} = (1+a_Z)\frac{g_Z m_Z}{2} H Z_\mu Z^\mu + \frac{g_Z}{m_Z}\sum_{V=Z,\gamma}\Big[b_V H Z_{\mu\nu} V^{\mu\nu}$$
$$+ c_V (\partial_\mu H Z_\nu - \partial_\nu H Z_\mu) V^{\mu\nu} + \tilde{b}_V H Z_{\mu\nu} \tilde{V}^{\mu\nu}\Big]$$

- $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\tilde{V}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} V^{\alpha\beta}$
- scalar components neglected, i.e. $\partial_\mu Z^\mu = \partial_\mu V^\mu = 0$
- operators w/ $d > 5$ neglected
- 7 couplings : $\underbrace{a_Z, b_Z, c_Z, b_\gamma, c_\gamma}_{\text{CP even}}, \underbrace{\tilde{b}_Z, \tilde{b}_\gamma}_{\text{CP odd}}$

Derive $H Z_\alpha V_\beta$ vertex

$$\Gamma^V_{\alpha\beta}(q, p_Z) = g_Z m_Z \left[h_1^V(s)\, g_{\alpha\beta} + \frac{h_2^V(s)}{m_Z^2} q_\alpha p_{Z\beta} + \frac{h_3^V(s)}{m_Z^2} \varepsilon_{\alpha\beta\mu\nu} q^\mu p_Z^\nu \right]$$

- only 6 form factors ↝ 1 comb. of coupl. cannot be meas.

$$h_1^Z(s) = (1+a_Z) + 2c_Z \frac{s+m_Z^2}{m_Z^2} + 2(b_Z - c_Z)\frac{s+m_Z^2-m_H^2}{m_Z^2}$$

$$h_2^Z(s) = -4(b_Z - c_Z)$$

$$h_3^Z(s) = -4\tilde{b}_Z$$

$$h_1^\gamma(s) = 2c_\gamma \frac{s}{m_Z^2} + (b_\gamma - c_\gamma)\frac{s+m_Z^2-m_H^2}{m_Z^2}$$

$$h_2^\gamma(s) = -2(b_\gamma - c_\gamma)$$

$$h_3^\gamma(s) = -2\tilde{b}_\gamma$$

2) Optimal-observable method

Atwood, Soni '92
Davier et al. '93
Diehl, Nachtmann '94
Gunion, Grzadkowski, He '96

Consider $\frac{d\sigma}{d\Omega} = \Sigma(\Omega) = \sum_i c_i f_i(\Omega)$

f_i: known

Choose any set of weighting functions $\{w_i\}$ with

$$\int d\Omega \, w_i(\Omega) \, f_j(\Omega) = \delta_{ij} .$$

Then $c_i = \int d\Omega \, w_i(\Omega) \, \Sigma(\Omega)$.

$\{w_i\}$ are not unique.

Optimal choice minimizes stat. error in c_i:

$$w_i(\Omega) = \sum_j M^{-1}_{ij} f_j(\Omega) / \Sigma(\Omega) , \text{ where}$$

$$M_{ij} = \int d\Omega \, f_i(\Omega) f_j(\Omega) / \Sigma(\Omega) .$$

Then

$$c_i = \sum_j M^{-1}_{ij} \int d\Omega \, f_j(\Omega) , \text{ and}$$

$$\chi^2 = \sum_{i,j} (c_i - c_i^0)(c_j - c_j^0) V^{-1}_{ij} , \text{ where}$$

$$V^{-1}_{ij} = \varepsilon \cdot L \cdot M_{ij} .$$

ε: efficiency; L: luminosity

statistical error on c_i is $\sqrt{V_{ii}}$

correlation between errors on c_i, c_j is $V_{ij} / \sqrt{V_{ii} V_{jj}}$

3) Angular distributions of $e^+e^- \to Hf\bar{f}$

- Calculate $e^+e^- \to HZ$ from $\mathcal{L}_{eff}$
- Narrow-width approximation for Z propagator
- Use SM amplitude for $Z \to f\bar{f}$

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$$\frac{d\sigma}{d\cos\Theta d\cos\theta d\varphi} = \sum_{i=1}^{9}\left[c_i^{(V)}F_i^{(V)}(\Theta,\theta,\varphi) + c_i^{(A)}F_i^{(A)}(\Theta,\theta,\varphi)\right];$$

$$F_1^{(V)} = \frac{r}{4}\sin^2\Theta\sin^2\theta,$$
$$F_2^{(V)} = \frac{r}{16}(1+\cos^2\Theta)(1+\cos^2\theta) - \frac{rPA_f}{4}\cos\Theta\cos\theta,$$
$$F_3^{(V)} = -\frac{r}{16}\sin 2\Theta\sin 2\theta\cos\varphi + \frac{rPA_f}{4}\sin\Theta\sin\theta\cos\varphi,$$
$$F_4^{(V)} = \frac{r}{8}\sin^2\Theta\sin^2\theta\cos 2\varphi,$$
$$F_5^{(V)} = -\frac{r}{16}\sin 2\Theta\sin 2\theta\sin\varphi + \frac{rPA_f}{4}\sin\Theta\sin\theta\sin\varphi,$$
$$F_6^{(V)} = \frac{r}{8}\sin^2\Theta\sin^2\theta\sin 2\varphi,$$
$$F_7^{(V)} = -\frac{rA_f}{8}(1+\cos^2\Theta)\cos\theta + \frac{rP}{8}\cos\Theta(1+\cos^2\theta),$$
$$F_8^{(V)} = \frac{rA_f}{8}\sin 2\Theta\sin\theta\cos\varphi - \frac{rP}{8}\sin\Theta\sin 2\theta\cos\varphi,$$
$$F_9^{(V)} = \frac{rA_f}{8}\sin 2\Theta\sin\theta\sin\varphi - \frac{rP}{8}\sin\Theta\sin 2\theta\sin\varphi.$$

$$F_i^{(V)} = \alpha_i + \beta_i P \quad \curvearrowright \quad F_i^{(A)} = \alpha_i P + \beta_i$$

P = e^- polarization

$$r = \frac{1}{4}\frac{\beta_{HZ}}{32\pi s}\frac{3}{4\pi}\mathrm{Br}(Z\to f\bar{f}), \qquad \beta_{HZ} = \sqrt{1 - 2\frac{m_Z^2+m_H^2}{s} + \left(\frac{m_Z^2-m_H^2}{s}\right)^2}.$$

$$A_f = \frac{\left(g_L^f\right)^2 - \left(g_R^f\right)^2}{\left(g_L^f\right)^2 + \left(g_R^f\right)^2}.$$

- $P = 0 \curvearrowright C^{(A)}_{1,4,6}$ unmeasurable

Coefficients $c_i^{(V,A)}$ are model dependent ↷ a_Z, ...

$$M_\sigma^\lambda(e^+e^- \to HZ) = \hat{M}_\sigma^\lambda d^1_{\sigma,\lambda}(\Theta),$$

where

$$d^1_{\sigma,\lambda=0}(\Theta) = -\frac{1}{\sqrt{2}}\sigma\sin\Theta, \qquad d^1_{\sigma,\lambda=\pm}(\Theta) = \frac{1}{2}(1+\sigma\lambda\cos\Theta).$$

SM:

$$c_1^{(V,A)} = |\hat{M}_R^0|^2 \pm |\hat{M}_L^0|^2,$$
$$c_2^{(V,A)} = |\hat{M}_R^+|^2 + |\hat{M}_R^-|^2 \pm \left(|\hat{M}_L^+|^2 + |\hat{M}_L^-|^2\right),$$
$$c_3^{(V,A)} = \mathrm{Re}\left[\hat{M}_R^0(\hat{M}_R^+)^* + \hat{M}_R^-(\hat{M}_R^0)^*\right] \pm \mathrm{Re}\left[\hat{M}_L^0(\hat{M}_L^+)^* + \hat{M}_L^-(\hat{M}_L^0)^*\right],$$
$$c_4^{(V,A)} = \mathrm{Re}\left[\hat{M}_R^-(\hat{M}_R^+)^*\right] \pm \mathrm{Re}\left[\hat{M}_L^-(\hat{M}_L^+)^*\right],$$
$$c_5^{(V,A)} = \mathrm{Im}\left[\hat{M}_R^0(\hat{M}_R^+)^* + \hat{M}_R^-(\hat{M}_R^0)^*\right] \pm \mathrm{Im}\left[\hat{M}_L^0(\hat{M}_L^+)^* + \hat{M}_L^-(\hat{M}_L^0)^*\right],$$
$$c_6^{(V,A)} = \mathrm{Im}\left[\hat{M}_R^-(\hat{M}_R^+)^*\right] \pm \mathrm{Im}\left[\hat{M}_L^-(\hat{M}_L^+)^*\right],$$
$$c_7^{(V,A)} = |\hat{M}_R^+|^2 - |\hat{M}_R^-|^2 \pm \left(|\hat{M}_L^+|^2 - |\hat{M}_L^-|^2\right),$$
$$c_8^{(V,A)} = \mathrm{Re}\left[\hat{M}_R^0(\hat{M}_R^+)^* - \hat{M}_R^-(\hat{M}_R^0)^*\right] \pm \mathrm{Re}\left[\hat{M}_L^0(\hat{M}_L^+)^* - \hat{M}_L^-(\hat{M}_L^0)^*\right],$$
$$c_9^{(V,A)} = \mathrm{Im}\left[\hat{M}_R^0(\hat{M}_R^+)^* - \hat{M}_R^-(\hat{M}_R^0)^*\right] \pm \mathrm{Im}\left[\hat{M}_L^0(\hat{M}_L^+)^* - \hat{M}_L^-(\hat{M}_L^0)^*\right].$$

- light quarks: cannot distinguish q, $\bar{q}$

  ↷ average over $(\Theta, \vartheta, \varphi)$, $(\Theta, \pi-\vartheta, \varphi\pm\pi)$:
  (corresponds to $A_f = 0$)

$$\bar{F}_i^{(V,A)}(\Theta,\theta,\varphi) = \frac{1}{2}\left[F_i^{(V,A)}(\Theta,\theta,\varphi) + F_i^{(V,A)}(\Theta,\pi-\theta,\varphi\pm\pi)\right].$$

- neutrinos: can only measure Θ

  ↷ integrate over ϑ, φ:

$$\frac{d\sigma}{d\cos\Theta} = \sum_{i=1,2,7}\left[c_i^{(V)}\tilde{F}_i^{(V)}(\Theta) + c_i^{(A)}\tilde{F}_i^{(A)}(\Theta)\right],$$

$$\tilde{F}_1^{(V)} = \frac{2\pi r}{3}\sin^2\Theta,$$
$$\tilde{F}_2^{(V)} = \frac{\pi r}{3}(1+\cos^2\Theta),$$
$$\tilde{F}_7^{(V)} = \frac{2\pi r P}{3}\cos\Theta,$$
$$\tilde{F}_1^{(A)} = \frac{2\pi r P}{3}\sin^2\Theta,$$
$$\tilde{F}_2^{(A)} = \frac{\pi r P}{3}(1+\cos^2\Theta),$$
$$\tilde{F}_7^{(A)} = \frac{2\pi r}{3}\cos\Theta,$$

Consider discrete symmetries CP, $CP\tilde{T}$.

$\tilde{T}$ = naive, i.e. flips momentum and spin, but does not reverse time flow from initial to final state.

$$(P, A_f; \Theta, \vartheta, \varphi) \xrightarrow{CP} (P, A_f; \pi-\Theta, \pi-\vartheta, 2\pi-\varphi)$$

$$(P, A_f; \Theta, \vartheta, \varphi) \xrightarrow{\tilde{T}} (P, A_f; \Theta, \vartheta, 2\pi-\varphi)$$

↷ CP and $CP\tilde{T}$ properties of $F_i^{(V,A)}$:

i	1	2	3	4	5	6	7	8	9
CP	+	+	+	+	−	−	−	−	+
$CP\tilde{T}$	+	+	+	+	+	+	−	−	−

(for i = 7, 8, 9:) via absorptive parts of light-particle loops

↷ Correlation matrix

$$V_{ij}^{-1} = L \int d\cos\Theta \, d\cos\vartheta \, d\varphi \; F_i(\Omega) F_j(\Omega) / \Sigma_{SM}(\Omega)$$

becomes block diagonal.

4) Constraints on HZV couplings

Consider 3 options:

(i) Measure tau helicity, via $\tau \to \nu_\tau + h$.

Assume efficiency $\epsilon_\tau = 40\%$ for τ^+ or τ^-

(ii) Identify charge of B hadron, via $B \to \ell\nu + X$.

Assume efficiency $\epsilon_b = 20\%$ for b or $\bar{b}$.

(iii) Use e^- beam polarization.

Assume $|P| = 90\%$.

Furthermore, consider $\sqrt{s} = 250, 500$ GeV to gain sensitivity to combination

$$a_Z - (b_Z + c_Z)\frac{m_Z^2}{2(s+m_Z^2)}$$

← indep. of f, P

which is unmeasurable at fixed energy.

Procedure:

- Determine errors and correlations for $c_i^{(V,A)}$
- Translate results to $a_Z, \ldots$

$C_{1,2,3,4}^{(V,A)} \longrightarrow \mathrm{Re}\,(a_Z, b_Z, c_Z, b_\gamma, c_\gamma)$

$C_{5,6}^{(V,A)} \longrightarrow \mathrm{Re}\,(\tilde{b}_Z, \tilde{b}_\gamma)$

$C_{7,8}^{(V,A)} \longrightarrow \mathrm{Im}\,(\tilde{b}_Z, \tilde{b}_\gamma)$

$C_9^{(V,A)} \longrightarrow \mathrm{Im}\,(a_Z, b_Z, c_Z, b_\gamma, c_\gamma)$

$\sqrt{s} = 250$ GeV
$m_H = 120$ GeV
$L = 10\ fb^{-1}$

Table 2: Optimal errors on the real parts of the general HZV couplings at $\sqrt{s} =$ 250 GeV.

ϵ_τ	—	0.4	—	—	0.4
ϵ_b	—	—	0.2	—	0.2
$\lvert P \rvert$	—	—	—	0.9	0.9
$\mathrm{Re}\,(b_Z + .059 a_Z)$	.0061	.0036	.0033	.0030	.0029
$\mathrm{Re}\,(c_Z + .059 a_Z)$	.013	.0076	.0070	.0061	.0061
$\mathrm{Re}\, b_\gamma$	.19	.072	.053	.0085	.0084
$\mathrm{Re}\, c_\gamma$	.12	.047	.035	.0053	.0052
$\mathrm{Re}\, \tilde{b}_Z$	.012	.011	.010	.010	.0091
$\mathrm{Re}\, \tilde{b}_\gamma$	.094	.036	.026	.016	.013

orthogonal to unmeas. comb. (arrows to the $\mathrm{Re}\,(b_Z + .059 a_Z)$ and $\mathrm{Re}\,(c_Z + .059 a_Z)$ rows)

a_Z fixed!

For $\epsilon_\tau = \epsilon_b = P = 0$

$$\begin{array}{ll} \mathrm{Re}\,(b_Z + .059 a_Z) = 0 \pm .0061 \\ \mathrm{Re}\,(c_Z + .059 a_Z) = 0 \pm .013 \\ \mathrm{Re}\, b_\gamma = 0 \pm .19 \\ \mathrm{Re}\, c_\gamma = 0 \pm .12 \\ \mathrm{Re}\, \tilde{b}_Z = 0 \pm .012 \\ \mathrm{Re}\, \tilde{b}_\gamma = 0 \pm .094 \end{array} , \quad \begin{pmatrix} 1 & & & & & \\ -.95 & 1 & & & & \\ -.86 & .88 & 1 & & & \\ .83 & -.89 & -.99 & 1 & & \\ 0 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & -.46 & 1 \end{pmatrix} .$$

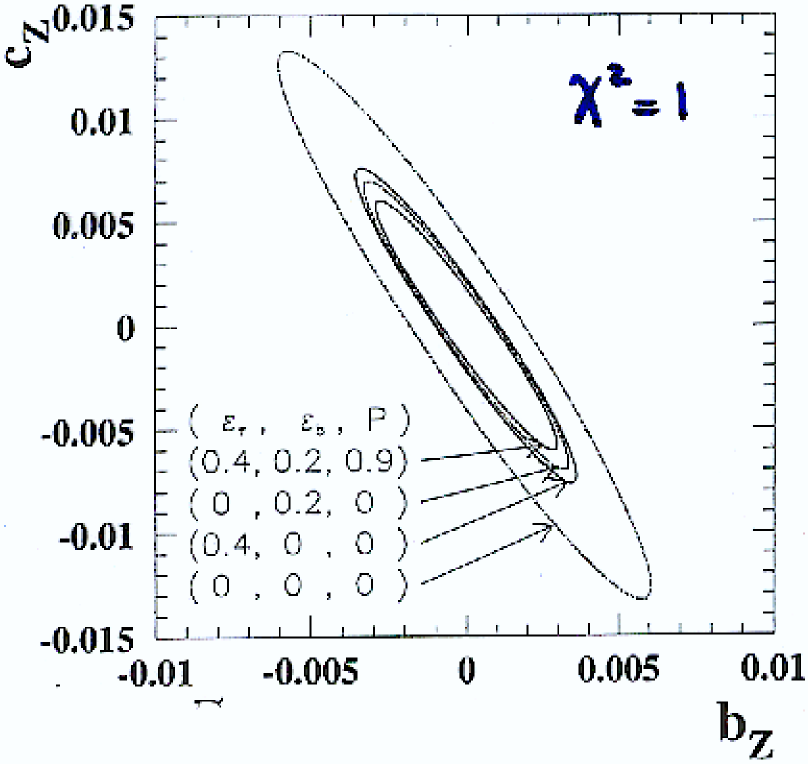
In multi-Higgs-doublet models, e.g. MSSM, have $b_V = c_V = \tilde{b}_V = 0$ at tree level. $\rightarrow$ $a_Z = 0 \pm 0.010$

For $\epsilon_\tau = 40\%$, $\epsilon_b = 20\%$ and $|P| = 90\%$

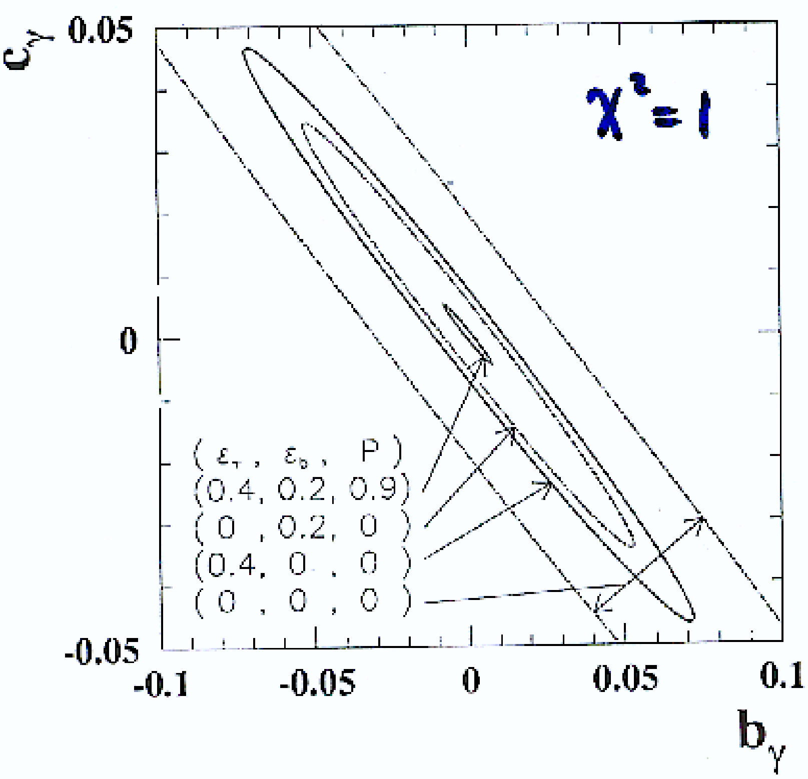
$$\begin{array}{ll} \mathrm{Re}\,(b_Z + .059 a_Z) = 0 \pm .0029 \\ \mathrm{Re}\,(c_Z + .059 a_Z) = 0 \pm .0061 \\ \mathrm{Re}\, b_\gamma = 0 \pm .0084 \\ \mathrm{Re}\, c_\gamma = 0 \pm .0052 \\ \mathrm{Re}\, \tilde{b}_Z = 0 \pm .0091 \\ \mathrm{Re}\, \tilde{b}_\gamma = 0 \pm .013 \end{array} , \quad \begin{pmatrix} 1 & & & & & \\ -.96 & 1 & & & & \\ -.08 & .08 & 1 & & & \\ .08 & -.09 & -.99 & 1 & & \\ 0 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & -.09 & 1 \end{pmatrix}$$

In MHDM : $a_Z = 0 \pm 0.010$

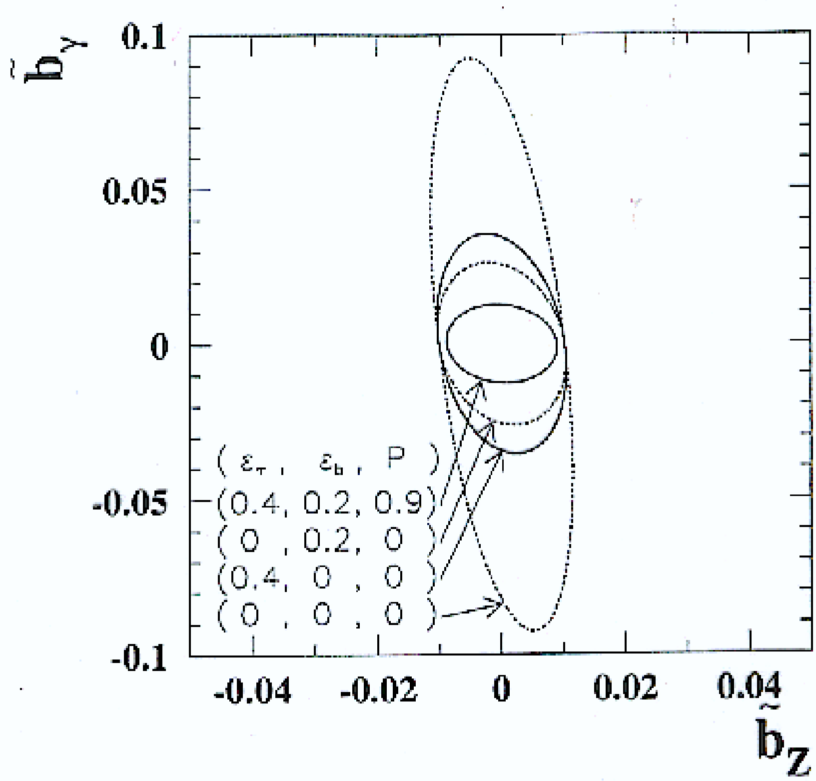
√s = 250 GeV



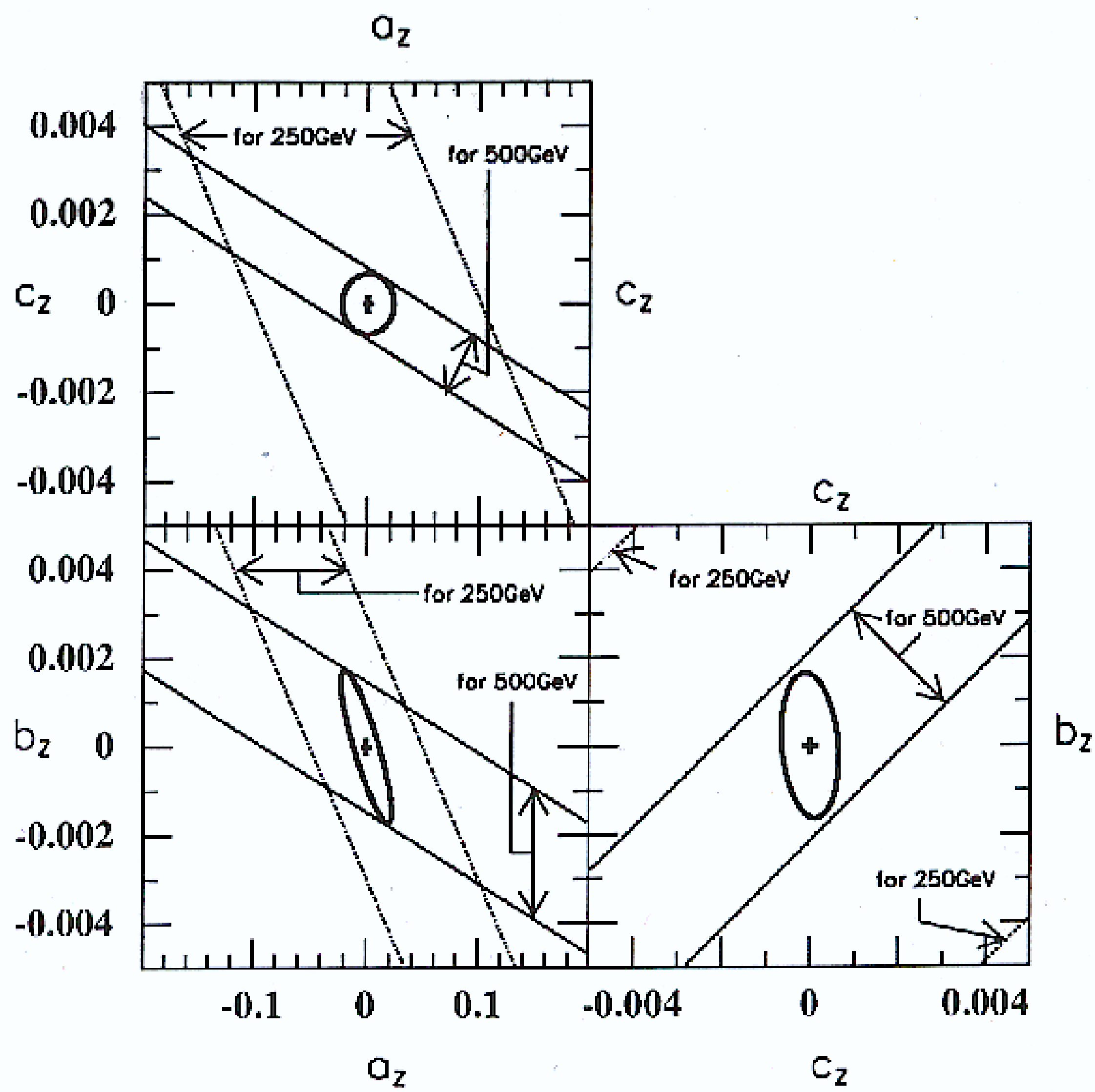
c_Z
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.01
-0.005
0
0.005
0.01
b_Z
$\chi^2 = 1$
(ε_τ , ε_b , P)
(0.4, 0.2, 0.9)
(0 , 0.2, 0)
(0.4, 0 , 0)
(0 , 0 , 0)



c_γ
0.05
0
-0.05
-0.1
-0.05
0
0.05
0.1
b_γ
$\chi^2 = 1$
(ε_τ , ε_b , P)
(0.4, 0.2, 0.9)
(0 , 0.2, 0)
(0.4, 0 , 0)
(0 , 0 , 0)



$\tilde{b}_\gamma$
0.1
0.05
0
-0.05
-0.1
(ε_τ , ε_b , P)
(0.4, 0.2, 0.9)
(0 , 0.2, 0)
(0.4, 0 , 0)
(0 , 0 , 0)
-0.04
-0.02
0
0.02
0.04
$\tilde{b}_Z$



a_Z
0.004
0.002
c_Z 0
-0.002
-0.004
for 250GeV
for 500GeV
c_Z
c_Z
0.004
0.002
b_Z 0
-0.002
-0.004
for 250GeV
for 500GeV
for 250GeV
for 500GeV
for 250GeV
b_Z
-0.1
0
0.1
-0.004
0
0.004
a_Z
c_Z

$\sqrt{s} = 500$ GeV, $\epsilon_c = 40\%$, $\epsilon_b = 20\%$, $|P| = 90\%$

$$\begin{array}{ll} \mathrm{Re}\,(b_Z + .016 a_Z) = 0 \pm .0015 \\ \mathrm{Re}\,(c_Z + .016 a_Z) = 0 \pm .0007 \\ \mathrm{Re}\,b_\gamma = 0 \pm .0024 \\ \mathrm{Re}\,c_\gamma = 0 \pm .0005 \\ \mathrm{Re}\,\tilde{b}_Z = 0 \pm .0042 \\ \mathrm{Re}\,\tilde{b}_\gamma = 0 \pm .0052 \end{array}, \quad \begin{pmatrix} 1 & & & & & \\ -.77 & 1 & & & & \\ -.09 & .07 & 1 & & & \\ .07 & -.09 & -.84 & 1 & & \\ 0 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & -.09 & 1 \end{pmatrix}$$

In MHDM: $a_Z = 0 \pm 0.021$

Combine analyses for $\sqrt{s} = 250, 500$ GeV ($L_{250} = L_{500}$)

$$\begin{array}{ll} \mathrm{Re}\,(b_Z + .066 a_Z) = 0 \pm .0009 \\ \mathrm{Re}\,c_Z = 0 \pm .0006 \\ \mathrm{Re}\,b_\gamma = 0 \pm .0015 \\ \mathrm{Re}\,c_\gamma = 0 \pm .0004 \\ \mathrm{Re}\,\tilde{b}_Z = 0 \pm .0038 \\ \mathrm{Re}\,\tilde{b}_\gamma = 0 \pm .0049 \end{array}, \quad \begin{pmatrix} 1 & & & & & \\ -.68 & 1 & & & & \\ -.08 & .07 & 1 & & & \\ .06 & -.08 & -.79 & 1 & & \\ 0 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & -.09 & 1 \end{pmatrix}.$$

Get handle on a_Z

$$\chi^2_{\min} = \left(\frac{a_Z}{0.024}\right)^2$$

Combined analysis with a_Z not fixed.

$$\begin{array}{l} \mathrm{Re}\,a_Z = 0 \pm .024 \\ \mathrm{Re}\,b_Z = 0 \pm .0018 \\ \mathrm{Re}\,c_Z = 0 \pm .0006 \\ \mathrm{Re}\,b_\gamma = 0 \pm .0015 \\ \mathrm{Re}\,c_\gamma = 0 \pm .0004 \\ \mathrm{Re}\,\tilde{b}_Z = 0 \pm .0038 \\ \mathrm{Re}\,\tilde{b}_\gamma = 0 \pm .0049 \end{array}, \quad \begin{pmatrix} 1 & & & & & & \\ -.87 & 1 & & & & & \\ -.02 & -.31 & 1 & & & & \\ .00 & -.04 & .07 & 1 & & & \\ .00 & .03 & -.08 & -.79 & 1 & & \\ 0 & 0 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 0 & -.09 & 1 \end{pmatrix}.$$

5) Conclusions

- For $m_H = 120$ GeV, $\sqrt{s} = 250$ GeV, $L = 10\,fb^{-1}$, gain optimal errors $\sim 10^{-2}$, 10^{-1} on HZZ, HZγ couplings.

- As for HZγ couplings, gain

  $\sim 2/5$ with $\varepsilon_\tau = 40\%$

  $\sim 2/7$ with $\varepsilon_b = 20\%$

  $\sim 1/20$ $(1/6)$ for CP-even (-odd) with $|P| = 90\%$

  At most $1/2$ for HZZ couplings.

- For HZZ and HZγ couplings, gain $\sim 1/2 - 1/10$ with $\sqrt{s} = 500$ GeV.

- Need 2 different energies to measure all 7 HZV couplings.