Electroweak two-loop results for Δr in the SM

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based on collaboration with

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1. Introduction

Theoretical prediction for M_W in terms of M_Z, α, G_μ from muon decay:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1 + \Delta r),$$

$\Updownarrow$

loop corrections

Current experimental values:

$$\begin{array}{lcll} M_W[\,\text{GeV}] & = & 80.419 \pm 0.038 & 0.05\% \\ M_Z[\,\text{GeV}] & = & 91.1871 \pm 0.0021 & 0.002\% \\ G_\mu[\,\text{GeV}^{-2}] & = & 1.16637(1)\,10^{-5} & 0.001\% \end{array}$$

SM prediction for Δr, one-loop:

[*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\begin{array}{lccccccc} \Delta r_{1-\text{loop}} = & \Delta\alpha & - & \frac{c_W^2}{s_W^2}\Delta\rho & + & \Delta r_{\text{rem}}(M_H) \\ & \sim \log\frac{M_Z}{m_f} & & \sim m_t^2 & & \\ & \sim 6\% & & \sim 3.3\% & & \sim 1\% \end{array}$$

$\Delta\alpha$, $\Delta\rho$: fermion-loop contributions from charge and mixing-angle renormalization

Higher-order results for M_W in the SM:

Resummation of leading 1-loop contributions:

$$1 + \Delta r \to \frac{1}{(1 - \Delta\alpha)\left(1 + \frac{c_W^2}{s_W^2}\Delta\rho\right) - \Delta r_{rem}}$$

$\Rightarrow$ $(\Delta\alpha)^2$, $(\Delta\rho)^2$, $(\Delta\alpha\Delta\rho)$, $(\Delta\alpha\Delta r_{rem})$ terms correct at 2-loop order

[*W. Marciano '79*], [*A. Sirlin '84*]

[*M. Consoli, W. Hollik, F. Jegerlehner '89*]

QCD corrections:

Results in $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha\alpha_s^2)$ known

[*A. Djouadi, C. Verzegnassi '87*], [*A. Djouadi '89*]

[*B.A. Kniehl '90*], [*F. Halzen, B.A. Kniehl '91*]

[*L. Avdeev, J. Fleischer, S.M. Mikhailov, O. Tarasov '94*]

[*K. Chetyrkin, J. Kühn, M. Steinhauser '95*]

EW 2-loop corrections:

Expansion for asymptotically large m_{t}:

Leading $m_{\text{t}}^4/M_{\text{W}}^4$ terms:

[*J. van der Bij, F. Hoogeveen '87*]

[*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*]

[*J. Fleischer, O.V. Tarasov, F. Jegerlehner '93*]

Next-to-leading $m_{\text{t}}^2/M_{\text{W}}^2$ terms:

expansions:

$$M_{\text{W}}, M_{\text{Z}}, M_{\text{H}} \ll m_{\text{t}}$$

$$M_{\text{W}}, M_{\text{Z}} \ll m_{\text{t}}, M_{\text{H}}$$

+ interpolation

[*G. Degrassi, P. Gambino, A. Vicini '96*]

[*G. Degrassi, P. Gambino, A. Sirlin '97*]

→ Same order of magnitude as $m_{\text{t}}^4/M_{\text{W}}^4$ terms, same sign

⇒ Exact evaluation of EW 2-loop contributions desirable (no expansion in masses)

Expansion for asymptotically large M_{H}:

Leading $M_{\mathrm{H}}^2/M_{\mathrm{W}}^2$ terms:

expansion: $M_{\mathrm{W}}, M_{\mathrm{Z}}, m_{\mathrm{t}} \ll M_{\mathrm{H}}$

[*J. van der Bij, M. Veltman '84*], [*J. van der Bij '84*]

Exact result for Higgs-mass dependence of fermionic contributions:

$M_{\mathrm{W,subtr}}(M_{\mathrm{H}}) = M_{\mathrm{W}}(M_{\mathrm{H}}) - M_{\mathrm{W}}(M_{\mathrm{H}} = 65\,\mathrm{GeV})$

[*S. Bauberger, G.W. '98*]

⇒ good agreement with expansion in m_{t} up to next-to-leading order:

[*P. Gambino, A. Sirlin, G.W. 99*]

Leading EW corrections beyond 2-loop:

Pure fermion-loop contributions up to 4-loop order

[*A. Stremplat '98*], [*G.W. '98*]

2. Exact fermionic two-loop contributions to M_W

Muon decay in Fermi Model:

$$\Gamma_\mu = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1+\Delta q)$$

$\Rightarrow$ definition of G_μ

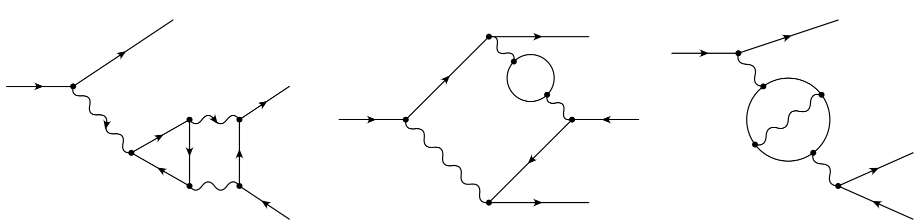
Δq: QED corr. [*R. Behrends, R. Finkelstein, A. Sirlin '56*]
[*T. van Ritbergen, R. Stuart '99*]
[*M. Steinhauser, T. Seidensticker '99*]

Muon decay in SM:

$$\Rightarrow\ G_\mu = \frac{\pi\alpha}{\sqrt{2}\left(1-\frac{M_W^2}{M_Z^2}\right)M_W^2}(1+\Delta r)$$

QED corrections to Fermi Model have to be split off

Two-loop fermionic contributions to muon decay:



$\Rightarrow$ full dependence on m_t, complete light fermion contributions at 2-loop order
[*A. Freitas, S. Heinemeyer, W. Hollik, W. Walter, G.W. '00*]

External momenta and masses negligible

⇒ Two-loop vacuum diagrams

On-shell renormalization (physical parameters)

⇒ Two-loop 2-point functions, $p \neq 0$

Method of calculation:

– Algebraic evaluation:

FeynArts [*J. Küblbeck, A. Denner, M. Böhm '90*]
[*H. Eck '95*], [*T. Hahn '99*]

⇒ Feynman amplitudes and counterterms

TwoCalc [*G. W. '92*], [*G. W., R. Scharf, M. Böhm '94*]

⇒ tensor integral reduction, etc.
standard scalar integrals

– General R_ξ gauge

– Regularisation:
DREG [*G. 't Hooft, M. Veltman '72*]
Careful treatment of γ_5 necessary
[*P. Breitenlohner, D. Maison '77*]

– Two-loop renormalization:
  On-shell scheme (physical parameters)

– Numerical evaluation:
  One-dimensional integral representations with elementary functions
  [*S. Bauberger, F. Berends, M. Böhm, M. Buza '95*]
  [*S. Bauberger, M. Böhm '95*]

  $\Rightarrow$ Exact eval. of 2-loop 2-point functions

Treatment of the γ_5 problem

Problem:

Dirac-algebra for γ_5

$\left\{\gamma_5, \gamma_\alpha\right\} = 0 \quad \text{for} \quad \alpha = 1, \ldots, 4$

$\mathrm{Tr}\left\{\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\right\} = 4 i \epsilon_{\mu\nu\rho\sigma}$

cannot be translated to D dimensions

Naively anti-commuting γ_5:

$\left\{\gamma_5, \gamma_\alpha\right\} = 0 \quad \text{for} \quad \alpha = 1, \ldots, D$

$\Rightarrow \mathrm{Tr}\left\{\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\right\} = 0$

$\Rightarrow$ inconsistent

Consistent scheme:

treat first 4 and remaining dimensions separately

$$\gamma_\mu = \bar{\gamma}_\mu + \widehat{\gamma}_\mu$$

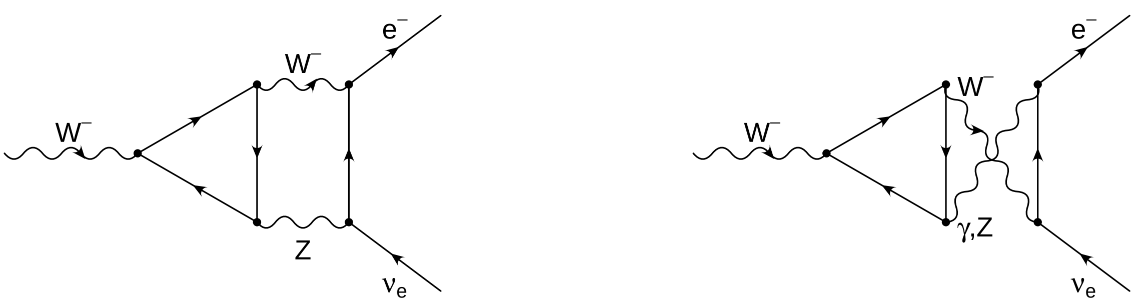
[*G. 't Hooft, M. Veltman '72*]

[*P. Breitenlohner, D. Maison '77*]

non-invariant regulator $\Rightarrow$ violates Ward identities

have to be restored with additional counterterms

Graphs sensitive to γ_5 problem:



Triangle subgraph:

$\left\{\gamma_5, \gamma_\alpha\right\} = 0$ in D dim. $+$ $\mathrm{Tr}\left\{\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\right\} = 4i\epsilon_{\mu\nu\rho\sigma}$

comparison with HVBM scheme

$\Rightarrow$ terms of $\mathcal{O}\left((D-4)^0\right)$ correct

Analysis of divergency structure of second loop

$\Rightarrow$ only finite contributions

$\Rightarrow$ consistent evaluation

Renormalization in gauge-fixing and ghost sector:

Unrenormalized gauge-fixing term:

$$\mathcal{L}_{\mathsf{gf}} = -\frac{1}{2}\Big((F^\gamma)^2 + (F^Z)^2 + F^+F^- + F^-F^+\Big),$$

$$F^\gamma = \left(\xi_1^\gamma\right)^{-\frac{1}{2}}\partial_\mu A_0^\mu + \frac{\xi^{\gamma Z}}{2}\partial_\mu Z_0^\mu,$$

$$F^Z = \left(\xi_1^Z\right)^{-\frac{1}{2}}\partial_\mu Z_0^\mu + \frac{\xi^{Z\gamma}}{2}\partial_\mu A_0^\mu - \left(\xi_2^Z\right)^{\frac{1}{2}} M_{Z0}\,\chi_0,$$

$$F^\pm = \left(\xi_1^W\right)^{-\frac{1}{2}}\partial_\mu W_0^{\pm\mu} \mp i\left(\xi_2^W\right)^{\frac{1}{2}} M_{W0}\,\phi_0^\pm.$$

Keep gauge-fixing term invariant under renormalization → no counterterm contributions

⇒ gauge parameter renormalization

FP-ghost Lagrangian:

$$\mathcal{L}_{\mathsf{FP}} = \sum_{a,b=\gamma,Z,\pm} \bar{u}^a \frac{\delta F^a}{\delta\theta^b} u^b = \int d^4y \sum_{a,b} \bar{u}^a(x)\frac{\delta F^a(x)}{\delta\theta^b(y)} u^b(y)$$

⇒ Counterterm contributions in ghost sector

need to be taken into account from 2-loop order on [*S. Bauberger, G.W. '96*], [*A. Freitas '99*]

⇒ finite building blocks in general R_ξ gauge

Mass renormalization of unstable particles:

For propagator from 2-loop order on:

real part of complex pole $\neq$ pole of real part

$$\delta M^2_{(2)} = \delta \widetilde{M}^2_{(2)} + \underbrace{\mathrm{Im}\left\{\Sigma'_{\mathrm{T},(1)}(M^2)\right\} \mathrm{Im}\left\{\Sigma_{\mathrm{T},(1)}(M^2)\right\}}_{\text{gauge-parameter dependent!}}$$

S-matrix theory $\Rightarrow$ complex pole gauge-invariant
[*A. Sirlin '91*], [*R.G. Stuart '91*], [*H. Veltman '94*]
[*P. Gambino, P.A. Grassi '99*]

Explicit 2-loop calculation in general R_ξ-gauge:

$\Rightarrow$ $\delta s_{\mathrm{W},(2)}$ gauge-parameter independent only with mass definition according to complex pole, $\delta M^2_{(2)}$
[*A. Freitas, G.W. '00*]

$\Rightarrow$ complex pole definition adopted

Experimental determinations of M_W, M_Z use Breit-Wigner parameterization with running width:
corresponds to real pole definition

$\Rightarrow M_\mathrm{W} \approx \widetilde{M}_\mathrm{W}^{\mathrm{exp}} - 26.4$ MeV, $\quad M_\mathrm{Z} \approx \widetilde{M}_\mathrm{Z}^{\mathrm{exp}} - 34.0$ MeV

3. Results

Checks of the calculation:

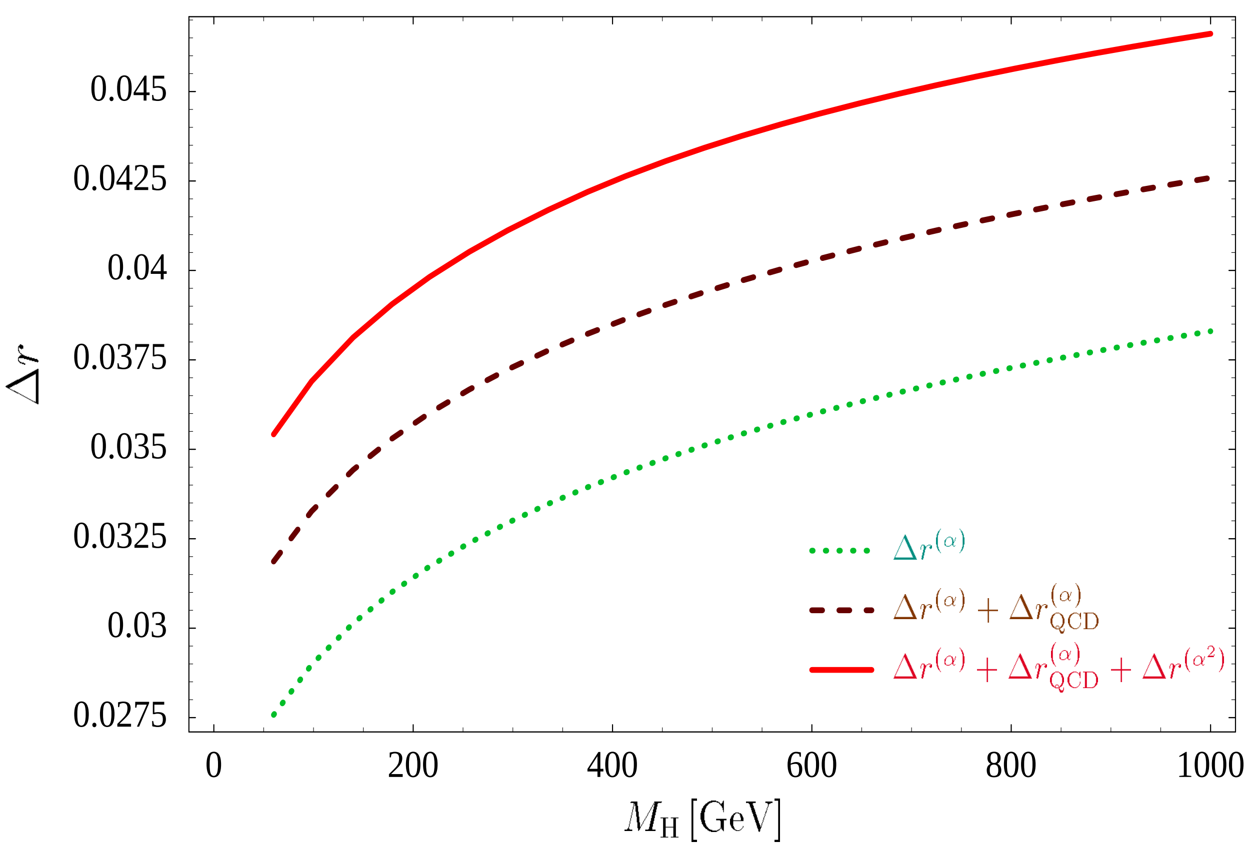
UV-, IR-finiteness,
general R_ξ-gauge $\Rightarrow$ gauge-parameter independence
at algebraic level

Higgs-mass dependence agrees with existing result
[*S. Bauberger, G.W. '98*]

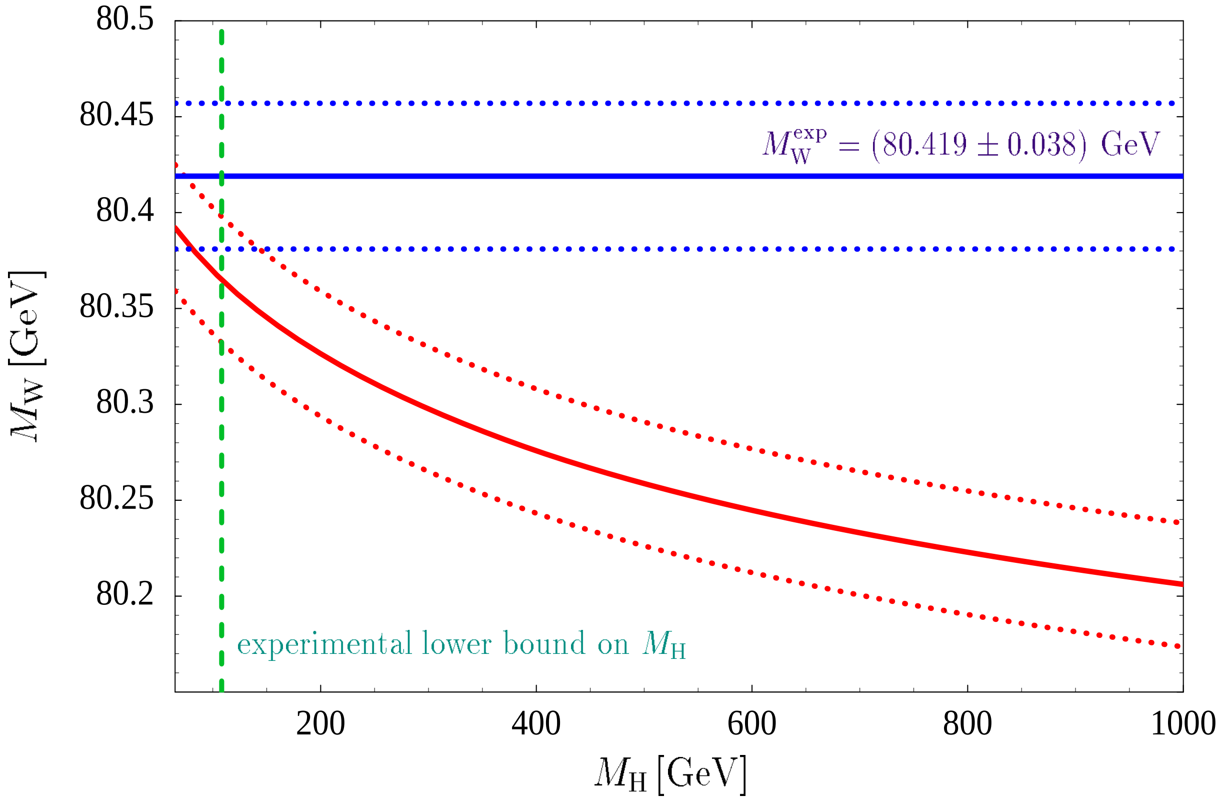
1-loop, QCD and ew 2-loop contributions to Δr:

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)}$$

fully expanded, no resummation needed



SM prediction for M_W vs. experimental result:



Comparison with expansion in m_t:

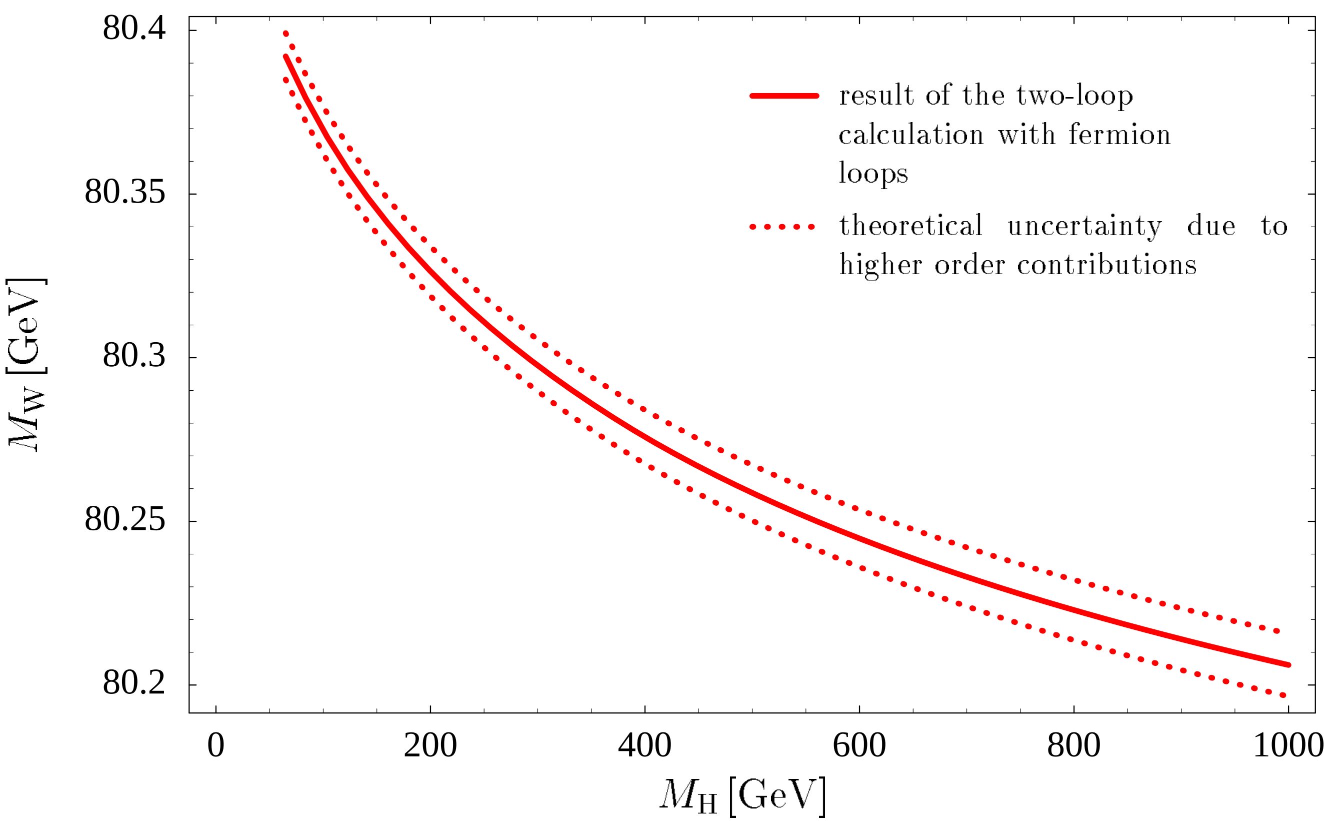
[*G. Degrassi, P. Gambino, A. Vicini '96*]
[*G. Degrassi, P. Gambino, A. Sirlin '97*]

M_H/ GeV	$\widetilde{M}_W$/ GeV	M_W^{expa}/ GeV	ΔM_W/ MeV
65	80.4009	80.4039	−3.0
100	80.3785	80.3805	−2.0
300	80.3068	80.3061	0.7
600	80.2540	80.2521	1.9
1000	80.2155	80.2129	2.6

$\Rightarrow$ good agreement

Estimate of theoretical uncertainty from unknown higher-order corrections:

- Purely bosonic $\mathcal{O}(\alpha^2)$ electroweak corrections
- Higher-order electroweak corrections: $\mathcal{O}(\alpha^3)$
- Higher-order QCD corrections: $\mathcal{O}(\alpha\alpha_s^3)$, $\mathcal{O}(\alpha^2\alpha_s)$



For light Higgs masses:

$$\Delta M_W^{\text{theo,ew}} \lesssim 6\,\text{MeV}$$

4. Future prospects for electroweak precision tests from M_W

High-lumi. LC (TESLA) in low-energy mode: **GigaZ**

$$\delta M_W^{exp} \approx \pm 6 \text{ MeV}$$

Future theoretical uncertainties:

Parametric uncertainties from experimental errors of input parameters:

$$\begin{aligned} \delta m_t &= \pm 130 \text{ MeV} && \Rightarrow \delta M_W \approx \pm 1 \text{ MeV} \\ \delta M_H &= \pm 50 \text{ MeV} && \\ \delta \alpha_s &= \pm 1 \times 10^{-3} && \\ \delta M_Z &= \pm 2.1 \text{ MeV} && \Rightarrow \delta M_W \approx \pm 2.5 \text{ MeV} \\ \delta(\Delta\alpha) &= \pm 5 \times 10^{-5} && \Rightarrow \delta M_W \approx \pm 1 \text{ MeV} \end{aligned}$$

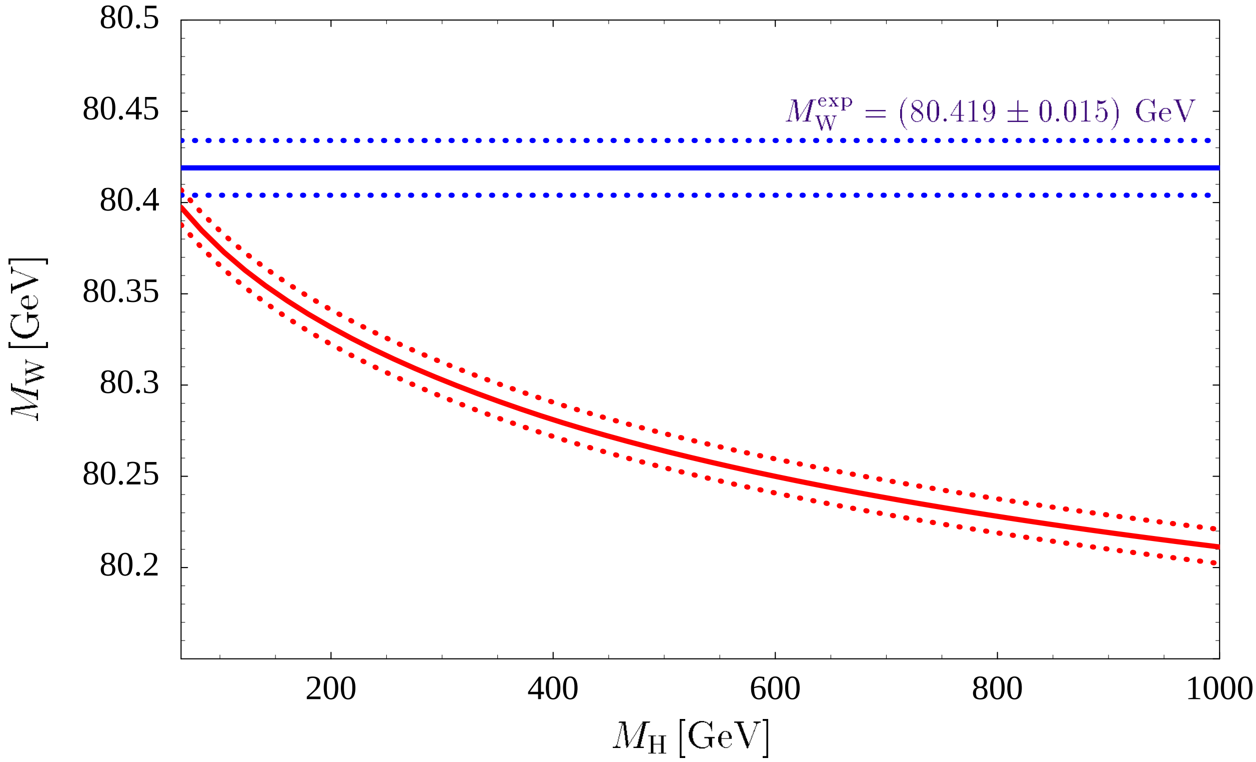
$\Rightarrow$ Error of M_Z non-negligible!

Estimate for future theoretical uncertainties from unknown higher-order corrections (including $\delta(\Delta\alpha)$ uncertainty):

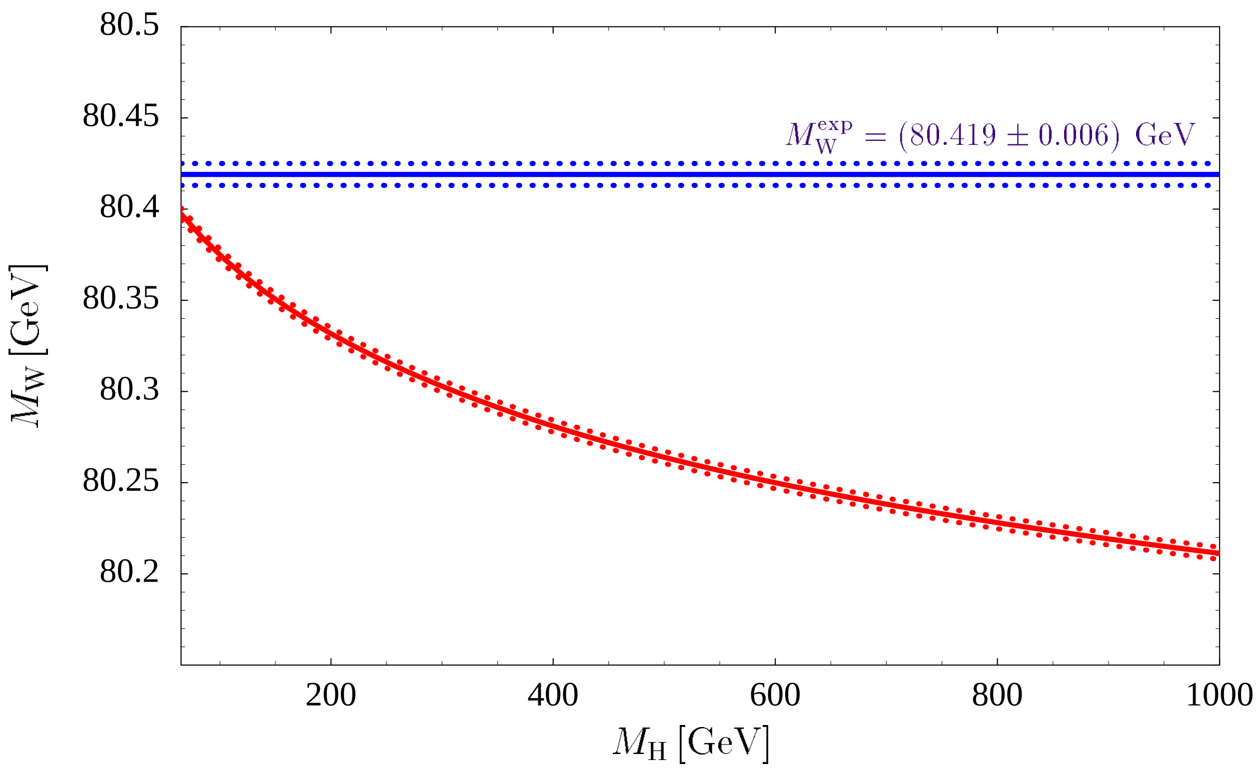
$$\delta M_W(\text{theory}) \approx \pm 3 \text{ MeV}$$

$M_W(M_H)$ in the SM, prospective future accuracies:

LHC precision:



GigaZ precision:



Expected precisions of indirect Higgs-mass determinations:

[*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]

	M_W	$\sin^2\theta_{eff}$	all
now	229 %	62 %	61 %
Tevatron Run IIA	77 %	46 %	41 %
Tevatron Run IIB	39 %	28 %	26 %
LHC	28 %	24 %	21 %
LC	18 %	20 %	15 %
GigaZ	12 %	7 %	7 %

5. Conclusions

- Exact result for fermionic two-loop corrections to M_W in the electroweak SM

- Comparison with result of expansion in m_t:
  good agreement, deviations up to $\Delta M_W \approx 3$ MeV

- Estimate of theoretical uncertainties from unknown higher-order corrections: $\delta M_W \lesssim 6$ MeV

- At LC:
  improved accuracy of precision observables M_W, $\sin^2\theta_{eff}$, ... and input parameters m_t, ...
  $\Rightarrow$ Highly sensitive test of the electroweak theory

- GigaZ:
  Indirect determination of M_H in SM:

$$\delta M_H / M_H \approx 7\%$$