# Subleading Electroweak Sudakov Logarithms to all orders

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Introduction

Subleading Corrections in Non-Abelian theories

Subleading EW Sudakov Logarithms

Comparison with existing calculations

Conclusions

* work based on hep-ph/0004056.

# Introduction

Precision NLC physics ($<1\%$) to disentangle new physics.

At 1 Loop, EW DL's 10-20 %, at 2 Loops few %. Subleading could be significant too.

$\longrightarrow$ Need a complete leading & subl. log analysis through 2 Loops!

In addition:

Theoretical understanding of IR-structure of broken gauge theories.

Subleading Corrections in Non-Abelian theories

Logarithms at fixed angle, $s \sim |t| \sim |u| \gg \mu^2 \geq m^2$

Double Logarithms from IR evolution equation

Non-Abelian Gribov Theorem ($\boldsymbol{k}'^2_\perp \gg \boldsymbol{k}^2_\perp \gg \mu^2$):

$$\mathcal{M}(p_1, ..., p_n; \mu^2) = \mathcal{M}_{\text{Born}}(p_1, ..., p_n)$$
$$-\frac{i}{2}\frac{g_s^2}{(2\pi)^4} \sum_{j,l=1,j\neq l}^{n} \int_{s\gg \boldsymbol{k}^2_\perp \gg \mu^2} \frac{d^4k}{k^2+i\epsilon} \frac{p_j p_l}{(kp_j)(kp_l)}$$
$$\times T^a(j) T^a(l) \mathcal{M}(p_1, ..., p_n; \boldsymbol{k}^2_\perp)$$

on the rhs is to be taken on the mass shell, but with the substituted infrared cutoff: $\mu^2 \longrightarrow \boldsymbol{k}^2_\perp$.

The IR-evolution equation is gauge invariant ($k^\mu, k^\nu$ terms give vanishing contribution do to conservation of total color charge $\sum_a T^a = 0$).

For $j \neq l$ we have $\frac{p_j p_l}{k p_j} \sim \frac{E_l}{w}$:

$$\mathcal{M}(p_1, ..., p_n; \mu^2) = \mathcal{M}_{\text{Born}}(p_1, ..., p_n) - \frac{2g^2}{(4\pi)^2} \sum_{l=1}^{n} \int_{\mu^2}^{s} \frac{dk_\perp^2}{k_\perp^2} \int_{|k_\perp|/\sqrt{s}}^{1} \frac{dv}{v} \times C_l \mathcal{M}(p_1, ..., p_n; k_\perp^2)$$

($C_l$ is the eig.val. of the Casimir operator $T^a(l)T^a(l)$ ($C_l = C_A$ for gauge bosons in the adjoint representation of $SU(N)$ and $C_l = C_F$ for fermions in the fundamental representation).

The differential form is thus:

$$\frac{\partial \mathcal{M}(p_1, ..., p_n; \mu^2)}{\partial \log(\mu^2)} = K(\mu^2)\mathcal{M}(p_1, ..., p_n; \mu^2)$$

$$K(\mu^2) \equiv -\frac{1}{2} \sum_{l=1}^{n} \frac{\partial W_l(s, \mu^2)}{\partial \log(\mu^2)}$$

$$W_l(s, \mu^2) = \frac{g^2}{8\pi^2} C_l \log \frac{s}{\mu^2}$$

$W_l$ is the probability to emit a soft and almost collinear gauge boson from the particle $l$, subject to the IR cut-off $\mu$ on the transverse momentum.

To logarithmic accuracy, we obtain:

$$\frac{\partial W_l(s,\mu^2)}{\partial \log(\mu^2)} = -\frac{g^2}{8\pi^2} C_l \log\frac{s}{\mu^2}$$

The initial condition is given by:

$$\mathcal{M}(p_1,...,p_n;s) = \mathcal{M}_{\text{Born}}(p_1,...,p_n)$$

The solution is thus given by the product of the Born amplitude and the Sudakov form factors:

$$\mathcal{M}(p_1,...,p_n;\mu^2) = \mathcal{M}_{\text{Born}}(p_1,...,p_n) \times \exp\left(-\frac{1}{2}\sum_{l=1}^{n} W_l(s,\mu^2)\right)$$

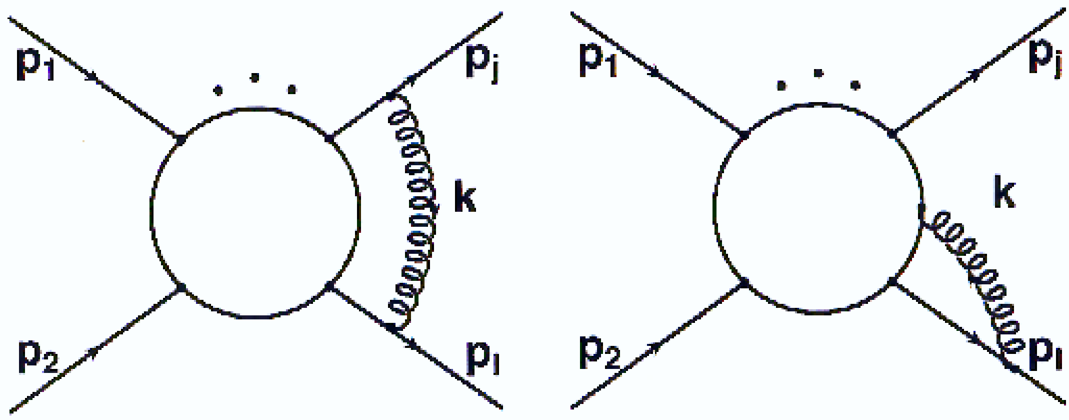
$\longrightarrow$ exactly analogous Sudakov exponentiation for the gauge group $SU(N)$ to the Abelian case.

In massless case, no subleading *soft* log.:

$$C_0(s/\mu^2) \equiv$$

$$\int \frac{d^4k_{[k_\perp^2>\mu^2]}}{(2\pi)^4(k^2+i\varepsilon)(k^2+2p_jk+i\varepsilon)(k^2-2p_lk+i\varepsilon)}$$

$$= \frac{i}{2(4\pi)^2 s}\log^2\frac{s}{\mu^2} \; + \; const.$$




$\longrightarrow$ Subleading log.: Collinear or RG

$\longrightarrow$ Use virtual contr. to A.P. splitting functions:

$P_{BA}$ describes probability of $B$ inside $A$ with fraction $z$ of energy of $A$ with probability $\mathcal{P}_{BA}$:

$$d\mathcal{P}_{BA}(z) = \frac{\alpha_s}{2\pi} P_{BA} dt$$

where $t = \log \frac{s}{\mu^2}$

$$d\mathcal{P}_{BA}(z) = \frac{\alpha_s}{2\pi} \frac{z(1-z)}{2} \overline{\sum_{spins}} \frac{|V_{A \longrightarrow B+C}|^2}{\boldsymbol{k}_\perp^2} d \log \boldsymbol{k}_\perp^2$$

$V_{A \longrightarrow B+C}$: elementary vertices

$$P_{BA}(z) = \frac{z(1-z)}{2} \overline{\sum_{spins}} \frac{|V_{A \longrightarrow B+C}|^2}{\boldsymbol{k}_\perp^2}$$

Calculate *virtual* contributions:

$$P^V_{qq}(z) = C_F\left(-2\log\frac{s}{\mu^2}+3\right)\delta(1-z)$$
$$P^V_{gg}(z) = C_A\left(-2\log\frac{s}{\mu^2}+\frac{4}{C_A}\beta_0^{\text{QCD}}\right)\delta(1-z)$$

These fulfill Altarelli-Parisi Equations:

$$\frac{\partial q(z,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_z^1\frac{dy}{y}q(z/y,t)P^V_{qq}(y)$$
$$\frac{\partial g(z,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_z^1\frac{dy}{y}g(z/y,t)P^V_{gg}(y)$$

$$q(1,t)=q_0\exp\left[-\frac{\alpha_s(s)C_F}{2\pi}\left(\log^2\frac{s}{\mu^2}-3\log\frac{s}{\mu^2}\right)\right]$$
$$g(1,t)=g_0\exp\left[-\frac{\alpha_s(s)C_A}{2\pi}\left(\log^2\frac{s}{\mu^2}-\frac{4}{C_A}\beta_0^{\text{QCD}}\log\frac{s}{\mu^2}\right)\right]$$

where $\beta_0^{\text{QCD}} = \frac{11}{12}C_A-\frac{1}{3}T_F n_f$ with $C_A=3$, $T_F=\frac{1}{2}$

Universality leads to general scattering amplitude:

$$\mathcal{M}(p_1,\ldots,p_n,g_s,\mu) = \mathcal{M}(p_1,\ldots,p_n,g_s(s))\times$$
$$\exp\left(-\frac{1}{2}\sum_{j=1}^{n_q}W_j^q(s,\mu^2)-\frac{1}{2}\sum_{l=1}^{n_g}W_l^g(s,\mu^2)\right)$$

with $n_q + n_g = n$, and

$$W^q(s,\mu^2) = \frac{\alpha_s(s)C_F}{4\pi}\left(\log^2\frac{s}{\mu^2} - 3\log\frac{s}{\mu^2}\right)$$

$$W^g(s,\mu^2) = \frac{\alpha_s(s)C_A}{4\pi}\left(\log^2\frac{s}{\mu^2} - \frac{4}{C_A}\beta_0^{\mathrm{QCD}}\log\frac{s}{\mu^2}\right)$$

Solution solves generalized IR evolution (or RG) equation with IR singular anomalous dimensions:

$$\left(\frac{\partial}{\partial t} + \beta^{\mathrm{QCD}}\frac{\partial}{\partial g_s} + n_g\left(\Gamma_g(t) - \frac{1}{2}\frac{\alpha_s}{\pi}\beta_0^{\mathrm{QCD}}\right)\right.$$
$$\left. + n_q\left(\Gamma_q(t) + \frac{1}{2}\gamma_{q\bar{q}}\right)\right)\mathcal{M}(p_1,\ldots,p_n,g_s,\mu) = 0$$

with

$$\Gamma_q(t) \equiv \frac{C_F\alpha_s}{4\pi}t \;\; ; \;\; \Gamma_g(t) \equiv \frac{C_A\alpha_s}{4\pi}t$$

## Subleading EW Sudakov Logarithms

Consider massive(M) boson at rest: $k^\nu = (M, 0, 0, 0)$
Polarization vector is lin.comb. of:

$e_1 \equiv (0, 1, 0, 0)$ , $e_1 \equiv (0, 0, 1, 0)$ , $e_3 \equiv (0, 0, 0, 1)$

Boost along 3-axis, pol. vect. still satisfy:

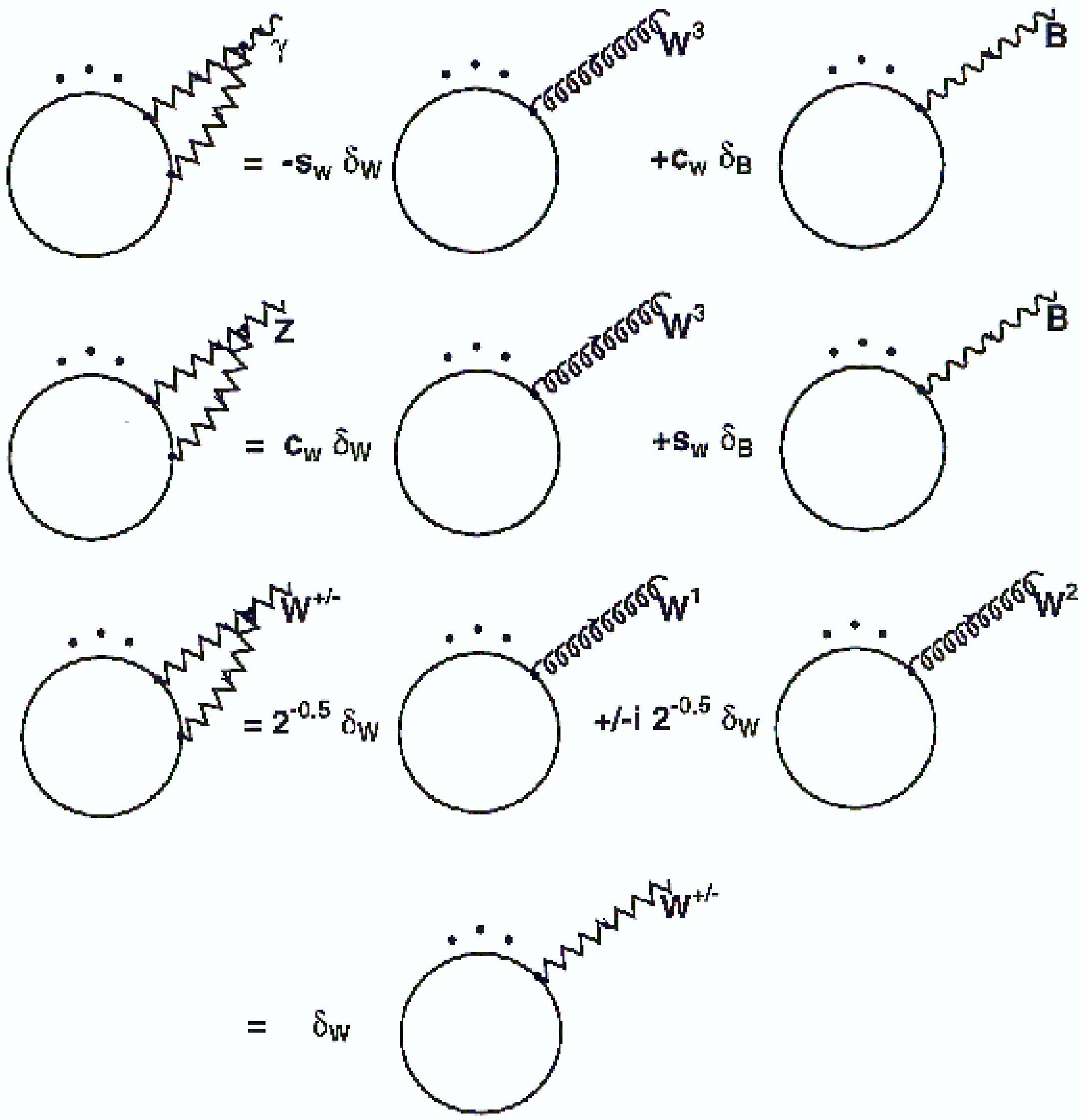
$$e_\nu k^\nu = 0 \quad , \quad e^2 = -1$$

$\longrightarrow$ 2 transverse $(e_1, e_2)$ and 1 longitudinal:

$$e_L^\nu(k) = (k/M, 0, 0, E_k/M) = k^\nu/M + \mathcal{O}(M/E_k)$$

Transverse d.o.f. correspond to massless theory!
Use massless virtual QCD results at high energies:
Additional complications: mixing & broken sym.

$$\begin{aligned} W_\nu^\pm &= \frac{1}{\sqrt{2}}\left(W_\nu^1 \pm i W_\nu^2\right) \\ Z_\nu &= \cos\theta_W W_\nu^3 + \sin\theta_W B_\nu \\ A_\nu &= -\sin\theta_W W_\nu^3 + \cos\theta_W B_\nu \end{aligned}$$




Only corrections to external $W^{\pm}$ factorize with respect to the physical amplitude!

Strategy:

Calculate in terms of massless fields for $\mu \geq M$ (where $M_Z \sim M_W \sim M_{\text{Higgs}} \sim M$)

Then solve IR evolution equation:

$$\left(\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial g} + \beta'\frac{\partial}{\partial g'} + \sum_{i=1}^{n_g} \Gamma_g^i(t) - n_{\text{W}}\frac{1}{2}\frac{\alpha}{\pi}\beta_0 - n_{\text{B}}\frac{1}{2}\frac{\alpha'}{\pi}\beta_0' + \sum_{k=1}^{n_f}\left(\Gamma_f^k(t) + \frac{1}{2}\gamma_{q\overline{q}}^k\right)\right)\mathcal{M}^{\perp}(p_1, ..., p_n, g, g', \mu) = 0$$

$$\text{with } \beta(g(\overline{\mu}^2)) = \frac{\partial g(\overline{\mu}^2)}{\partial \log \overline{\mu}^2} \approx -\beta_0\frac{g^3(\overline{\mu}^2)}{8\pi^2}$$

$$\beta'(g'(\overline{\mu}^2)) = \frac{\partial g'(\overline{\mu}^2)}{\partial \log \overline{\mu}^2} \approx -\beta_0'\frac{g'^3(\overline{\mu}^2)}{8\pi^2}$$

and

$$\beta_0 = \frac{11}{12}C_A - \frac{1}{3}n_{gen} - \frac{1}{24}n_h$$

$$\beta_0' = -\frac{5}{9}n_{gen} - \frac{1}{24}n_h$$

$$\Gamma_{f,g}^i(t) = \left(\frac{\alpha}{4\pi}T_i(T_i+1) + \frac{\alpha'}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)t$$

$$\gamma_{q\overline{q}}^i = -3\left(\frac{\alpha}{4\pi}T_i(T_i+1) + \frac{\alpha'}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)$$

Solution:

$$\mathcal{M}^{\perp}(p_1,\dots,p_n,g,g',\mu) = \mathcal{M}^{\perp}_{\text{Born}}(p_1,\dots,p_n,g(s),g'(s))$$
$$\times \exp\Bigg\{-\frac{1}{2}\sum_{i=1}^{n_g}\left(\frac{\alpha(s)}{4\pi}T_i(T_i+1)+\frac{\alpha'(s)}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)\log^2\frac{s}{\mu^2}$$
$$+\left(n_{\text{W}}\frac{\alpha(s)}{2\pi}\beta_0+n_{\text{B}}\frac{\alpha'(s)}{2\pi}\beta_0'\right)\log\frac{s}{\mu^2}$$
$$-\frac{1}{2}\sum_{k=1}^{n_f}\left(\frac{\alpha(s)}{4\pi}T_k(T_k+1)+\frac{\alpha'(s)}{4\pi}\left(\frac{Y_k}{2}\right)^2\right)$$
$$\times\left[\log^2\frac{s}{\mu^2}-3\log\frac{s}{\mu^2}\right]\Bigg\}$$

In region $\mu \leq M$ masses cannot be neglected.

But: Only QED contributes!

Above solution is matching condition at $\mu = M$.

The general solution is thus:

$$
\begin{aligned}
\mathcal{M}^{\perp}(p_1, ..., p_n, g, g', \mu) &= \mathcal{M}^{\perp}_{\text{Born}}(p_1, ..., p_n, g(s), g'(s)) \\
\times \exp\Bigg\{ -\frac{1}{2}\sum_{i=1}^{n_g} & \left( \frac{\alpha(s)}{4\pi} T_i(T_i+1) + \frac{\alpha'(s)}{4\pi}\left(\frac{Y_i}{2}\right)^2 \right) \log^2 \frac{s}{M^2} \\
& + \left( n_{\text{W}} \frac{\alpha(s)}{2\pi}\beta_0 + n_{\text{B}} \frac{\alpha'(s)}{2\pi}\beta'_0 \right) \log \frac{s}{M^2} \\
& - \frac{1}{2}\sum_{k=1}^{n_f} \left( \frac{\alpha(s)}{4\pi} T_k(T_k+1) + \frac{\alpha'(s)}{4\pi}\left(\frac{Y_k}{2}\right)^2 \right) \\
& \times \left[ \log^2 \frac{s}{M^2} - 3\log \frac{s}{M^2} \right] \Bigg\} \\
& \times \exp\Bigg[ -\frac{1}{2}\sum_{i=1}^{n_f} \left( w_i^f(s,\mu^2) - w_i^f(s,M^2) \right) \\
& - \frac{1}{2}\sum_{i=1}^{n_g} \left( w_i^{\text{W}}(s,\mu^2) - w_i^{\text{W}}(s,M^2) \right) \Bigg]
\end{aligned}
$$

For external $Z$ and $\gamma$ we have mixing!

$\gamma$ semi-inclusively: Assume $\mu_{exp} \leq M$:

$$d\sigma^{\perp}(p_1, \ldots, p_n, g, g', \mu_{exp}) = d\sigma^{\perp}_{\mathsf{elastic}}(p_1, \ldots, p_n, g(s), g'(s), \mu) \times \exp(w^{\gamma}_{\mathsf{exp}}(s, m_i, \mu, \mu_{exp}))$$

and thus ( $\mu$-dependence cancels):

$$d\sigma^{\perp}(p_1, \ldots, p_n, g, g', \mu_{exp}) = d\sigma^{\perp}_{\mathsf{Born}}(p_1, \ldots, p_n, g(s), g'(s)$$

$$\times \exp\left\{ - \sum_{i=1}^{n_g} \left( \frac{\alpha(s)}{4\pi} T_i(T_i + 1) + \frac{\alpha'(s)}{4\pi} \left( \frac{Y_i}{2} \right)^2 \right) \log^2 \frac{s}{M^2} \right.$$

$$+ \left( n_{\mathsf{W}} \frac{\alpha(s)}{\pi} \beta_0 + n_{\mathsf{B}} \frac{\alpha'(s)}{\pi} \beta'_0 \right) \log \frac{s}{M^2}$$

$$- \sum_{k=1}^{n_f} \left( \frac{\alpha(s)}{4\pi} T_k(T_k + 1) + \frac{\alpha'(s)}{4\pi} \left( \frac{Y_k}{2} \right)^2 \right)$$

$$\left. \times \left[ \log^2 \frac{s}{M^2} - 3 \log \frac{s}{M^2} \right] \right\}$$

$$\times \exp\left[ - \sum_{i=1}^{n_f} \left( w^f_i(s, \mu^2) - w^f_i(s, M^2) \right) \right.$$

$$\left. - \sum_{i=1}^{n_g} \left( w^{\mathsf{W}}_i(s, \mu^2) - w^{\mathsf{W}}_i(s, M^2) \right) \right]$$

$$\times \exp(w^{\gamma}_{\mathsf{exp}}(s, m_i, \mu, \mu_{exp}))$$

## Longitudinal degrees of freedom

Use Goldstone-boson equivalence theorem:

$$\mathcal{M}(W_L^{\pm}(k), \psi_{\text{phys}}) = C_{\text{W}} \mathcal{M}(\phi^{\pm}(k), \psi_{\text{phys}}) + \mathcal{O}\left(\frac{M_{\text{W}}}{\sqrt{s}}\right)$$

$$\mathcal{M}(Z_L(k), \psi_{\text{phys}}) = C_{\text{Z}} \mathcal{M}(\phi(k), \psi_{\text{phys}}) + \mathcal{O}\left(\frac{M_{\text{Z}}}{\sqrt{s}}\right)$$

$C_{\text{Z}}$, $C_{\text{W}}$ depend on wave-function renormalization constants and mass counterterms.

Thus, logarithmic corrections at NLO, but DL:

$$d\sigma^{\|}(p_1, \ldots, p_n, g, g', \mu_{exp}) = d\sigma^{\phi}_{\text{Born}}(p_1, \ldots, p_n, g, g')$$
$$\times \exp\left\{ -\sum_{i=1}^{n} \left( \frac{\alpha}{4\pi} T_i(T_i+1) + \frac{\alpha'}{4\pi}\left(\frac{Y_i}{2}\right)^2 \right) \log^2 \frac{s}{M^2} \right\}$$
$$\times \exp\left[ -\sum_{i=1}^{n} \left( w_i^{\text{DL}}(s, \mu^2) - w_i^{\text{DL}}(s, M^2) \right) + w_{exp}^{\text{DL}} \right]$$

## Comparison with explicit results

Agreement with Beenakker et al. at one loop for

$e^+ e^- \longrightarrow W^+ W^-$

transverse: NLO, longitudinal: DL

Agreement with Kühn et al. to all orders for

$e^+ e^- \longrightarrow f^+ f^-$

Disagreement with Ciafaloni/Comelli at DL for

$Z' \longrightarrow f^+ f^-$

## Conclusions

NLO Sudakov logarithms in broken gauge theories are calculated to all orders using fields of the unbroken theory and the gauge invariant IR evolution equation method.

They *factorize* and *exponentiate* (with reasonable experimental cuts, mixing).

$\log \frac{s}{M^2} \log \frac{t}{u}$ should be calculated at least at one loop.

Agreement at one loop (NLO for $\perp$, DL for $\|$) with calculation using physical fields!

The method is universal for all processes at a future collider (NLC, LHC).