RacoonWW: Radiative corrections on $e^+e^- \to WW \to 4\,\text{fermions}$

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Linear Collider Workshop, Padova, May 6, 2000

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Four-fermion production

Interesting processes: $e^+e^- \to WW \to 4$ fermions

Precise measurement of M_W:

- σ_{tot} at threshold (161 GeV)
- Invariant-mass distribution of the decay products of the W bosons

$\Rightarrow$ Better bounds on M_H and new physics

Investigation of gauge-boson self interactions:

$e^+e^- \to 4f$: γWW,ZWW

$e^+e^- \to 4f\gamma$: $\gamma\gamma$WW,γZWW

- σ_{tot} at high energies
- Angular distributions (W-production angle)
- Photon-energy distribution

$\Rightarrow$ Better bounds on non-standard couplings

Theoretical Requirements

Experimental accuracy:

	LEP2	Linear Collider
Energy	160 – 200 GeV	$\gtrsim$ 500 GeV
ΔM_{W}	30 MeV	$\lesssim$ 15 MeV
$\Delta\sigma/\sigma$	1%	$\lesssim$ 0.5%

Size of radiative corrections:
5 – 10%: LEP2 (160 – 210 GeV)
20 – 40%: Linear Collider ($\gtrsim$ 1 TeV)

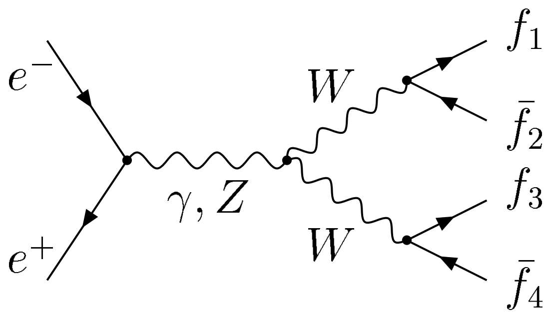
$\Rightarrow$ Radiative corrections required!

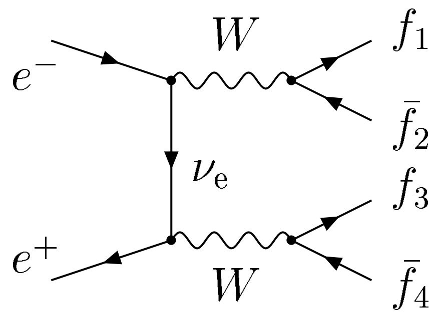
Remarks:

- 6 – 144 diagrams for $\mathrm{e^+e^-} \to 4\mathrm{f}$
- 14 – 1008 diagrams for $\mathrm{e^+e^-} \to 4\mathrm{f}\gamma$
- $\mathcal{O}(10^3 - 10^4)$ one-loop diagrams

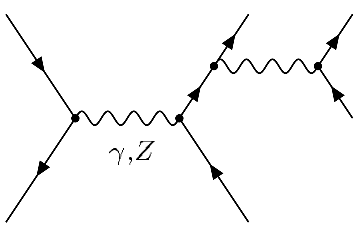
Born amplitude

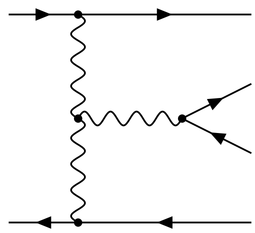
Signal diagrams: two resonant W bosons

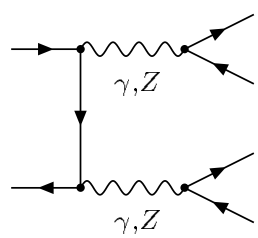




Background diagrams: at most one resonant W







etc.

$$\text{Typical size} \approx \frac{\Gamma_W}{M_W} \approx 2.5\%$$

⇒ Background diagrams required!

Virtual corrections

Double-pole approximation:

$$\mathcal{M} = \underbrace{\frac{R_{+-}(k_+^2, k_-^2)}{(k_+^2 - M^2)(k_-^2 - M^2)}}_{\text{doubly-resonant}} + \underbrace{\frac{R_+(k_+^2, k_-^2)}{k_+^2 - M^2} + \frac{R_-(k_+^2, k_-^2)}{k_-^2 - M^2}}_{\text{singly-resonant}} + \underbrace{N(k_+^2, k_-^2)}_{\text{non-resonant}}$$

$$= \frac{R_{+-}(M^2, M^2)}{(k_+^2 - M^2)(k_-^2 - M^2)} + \dots$$

with $M^2 = M_{\mathrm{W}}^2 + \mathrm{i} M_{\mathrm{W}} \Gamma_{\mathrm{W}}$.

Error estimate:

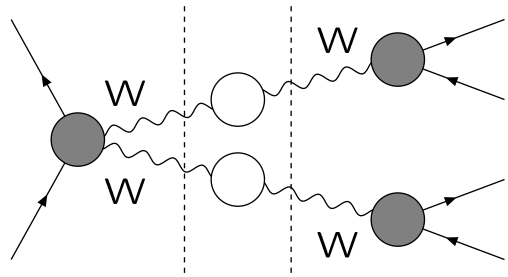
$$\sim \mathcal{O}\left(\frac{\alpha}{\pi}\frac{\Gamma_{\mathrm{W}}}{M_{\mathrm{W}}} \log(\cdots)\right) \lesssim 0.5\%$$

Remarks:

- Gauge invariant, Ward identities fulfilled
- Valid not too close to threshold
- Considerable reduction of diagram number
- Generic result for all final states

Doubly-resonant virtual corrections

Factorizable corrections:



Böhm et al. '88;
Fleischer, Jegerlehner, Zralek '89;
Bardin, S. Riemann, T. Riemann '86;
Jegerlehner '86;
Denner, Sack '90

on-shell production   on-shell decays

$\Rightarrow$ Existing results for on-shell W-pair production and on-shell W decay can be used!

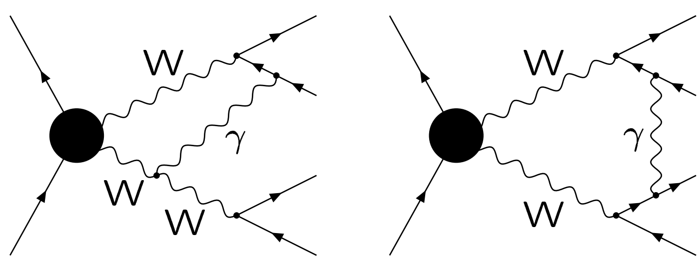
On-shell production depends only on W- production angle

$\Rightarrow$ Expansion in Legendre polynomial

Non-factorizable corrections:

$$\mathcal{M}_{\text{nf}} := \mathcal{M}_{\text{doubly resonant}} - \mathcal{M}_{\text{f}}$$

Examples:



etc.

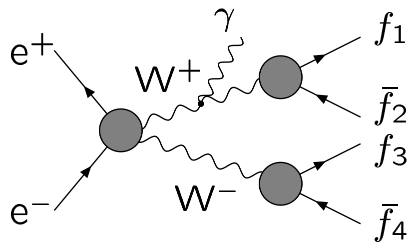
Melnikov,Yakovlev '96
Beenakker,Berends,Chapovsky '97
Denner, Dittmaier, R. '97

Only photons with $E_\gamma \lesssim \Gamma_W$ relevant q

Real corrections

Why not a DPA for real corrections?

Example:



$$\mathcal{M} \propto \frac{1}{[(k_{f_1}+k_{\bar{f}_2})^2 - M^2][(k_{f_1}+k_{\bar{f}_2}+q_\gamma)^2 - M^2]}$$

⇒ Overlapping resonances for $E_\gamma \sim \Gamma_W$

⇒ Double-pole approximation have to be defined for different observables differently

⇒ Exact matrix element calculation for $e^+e^- \to 4f\gamma$

Photon recombination:

Example: realistic invariant-mass distribution

$$\frac{d\sigma}{dk_+^2} \text{ with } k_+^2 = \begin{cases} (k_{f_1}+k_{\bar{f}_2}+q_\gamma)^2 & ,\gamma \text{ collinear to } f_1, \bar{f}_2 \\ (k_{f_1}+k_{\bar{f}_2})^2 & ,\text{otherwise} \end{cases}$$

(exception: $\mu^\pm$)

⇒ Realistic observables depend on photon resolution

⇒ Monte Carlo generator for realistic observables

Monte Carlo generator RacoonWW

$\mathcal{O}(\alpha)$ corrections with RacoonWW

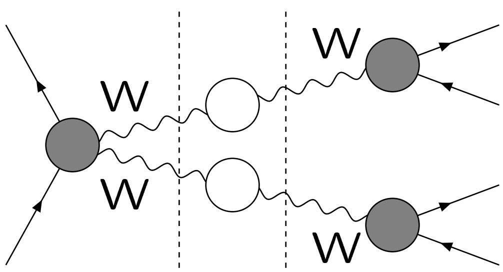
virtual

real

$e^+e^- \to 4f + \gamma$ (tree level):
Exact matrix element

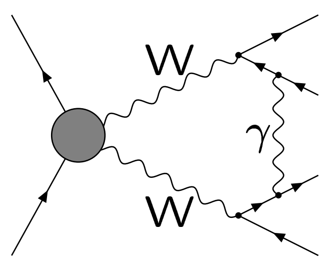
$e^+e^- \to WW \to 4f$ (one loop):
Double-pole approximation

Factorizable corrections



building blocks:
– W-pair production
– W decay

Non-factorizable corrections



etc.

γ-exchange between production- and decay-subprocesses with $E_\gamma \lesssim \Gamma_W$

Further features of RacoonWW

Higher-order corrections:

- Universal higher-order corrections via the G_μ-scheme
- Soft-photon exponentiation and leading $\mathcal{O}(\alpha^3)$ ISR via structure functions

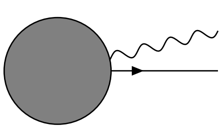
Monte Carlo integration:

- Multichannel Monte Carlo approach
- Weighted events
- Subtraction method or phase-space slicing

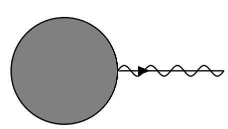
Massless final-state fermions:

$\Rightarrow$ Avoid final-state mass singularities

- Cuts between final-state $e^\pm$ and beam, and $f\bar{f}$-pairs
- Photon recombination:



γ, f $\quad (p_f + p_\gamma)^2 < M^2_{\mathrm{rec}} \quad \Rightarrow$



$f\gamma \quad p_{f\gamma} = p_f + p_\gamma$

Master formula for $\mathcal{O}(\alpha)$ corrections:

$$\begin{aligned}
&\int \mathrm{d}\Phi^{4f\gamma}\, |\mathcal{M}_{\mathrm{real}}|^2\, \mathcal{O}(\Phi^{4f\gamma}) \\
&\quad + \int \mathrm{d}\Phi^{4f} 2\,\mathrm{Re}\,(\mathcal{M}_{\mathrm{virt}}\mathcal{M}^*_{\mathrm{Born}})\, \mathcal{O}(\Phi^{4f}) \\
&= \int \mathrm{d}\Phi^{4f\gamma} \left[|\mathcal{M}_{\mathrm{real}}|^2\, \mathcal{O}(\Phi^{4f\gamma}) - \mathcal{V}^{4f\gamma}\mathcal{O}(\Phi^{4f}(\Phi^{4f\gamma}))\right] \\
&\quad + \int \mathrm{d}\Phi^{4f} \left[2\,\mathrm{Re}\,(\mathcal{M}_{\mathrm{virt}}\mathcal{M}^*_{\mathrm{Born}})\, \mathcal{O}(\Phi^{4f}) + \mathcal{V}^{4f}\mathcal{O}(\Phi^{4f})\right] \\
&\quad + \sum_{\pm} \int \mathrm{d}x \int \mathrm{d}\Phi^{4f}(x p_{\mathrm{e}^\pm}) \mathcal{V}^{4f}_{\pm}(x)\mathcal{O}(\Phi^{4f})
\end{aligned}$$

- Sum of terms with $\mathcal{V}^{4f\gamma}, \mathcal{V}^{4f}, \mathcal{V}^{4f}_{\pm} = 0$
- Fermion masses neglected except for mass singularities
- Observable $\mathcal{O}$: includes cuts and photon recombination
- $\mathcal{V}^{4f\gamma}$: $|\mathcal{M}_{\mathrm{real}}|^2 - \mathcal{V}^{4f\gamma}$ is finite in whole phase space
  (no regulator $m_\gamma = m_f = 0$)
- $\mathcal{V}^{4f}$: singularities cancel with virtual corrections
  ($\propto \ln m_\gamma \ln m_f, \ln m_\gamma, \ln^2 m_f, \ln m_f$)
- $\mathcal{V}^{4f}_{\pm}(x)$: $\ln(m_{\mathrm{e}}/Q)$ from $\mathcal{O}(\alpha)$ ISR survive!

$$\mathcal{V}^{4f}_{\pm} = [f(x)]_+ g(x), \qquad \int \mathrm{d}x [f(x)]_+ G(x) = \int \mathrm{d}x f(x)[G(x) - G(1)]$$

Subtraction method

Master formula for $\mathcal{O}(\alpha)$ corrections:

$$\int \mathrm{d}\Phi^{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2\,\mathcal{O}(\Phi^{4f\gamma})$$

$$+\int \mathrm{d}\Phi^{4f} 2\,\mathrm{Re}\,(\mathcal{M}_{\mathrm{virt}}\mathcal{M}^*_{\mathrm{Born}})\,\mathcal{O}(\Phi^{4f})$$

$$= \underbrace{\int \mathrm{d}\Phi^{4f\gamma}\left[|\mathcal{M}_{\mathrm{real}}|^2\,\mathcal{O}(\Phi^{4f\gamma}) - \mathcal{V}^{4f\gamma}\mathcal{O}(\Phi^{4f}(\Phi^{4f\gamma}))\right]}_{\text{no mass singularities}}$$

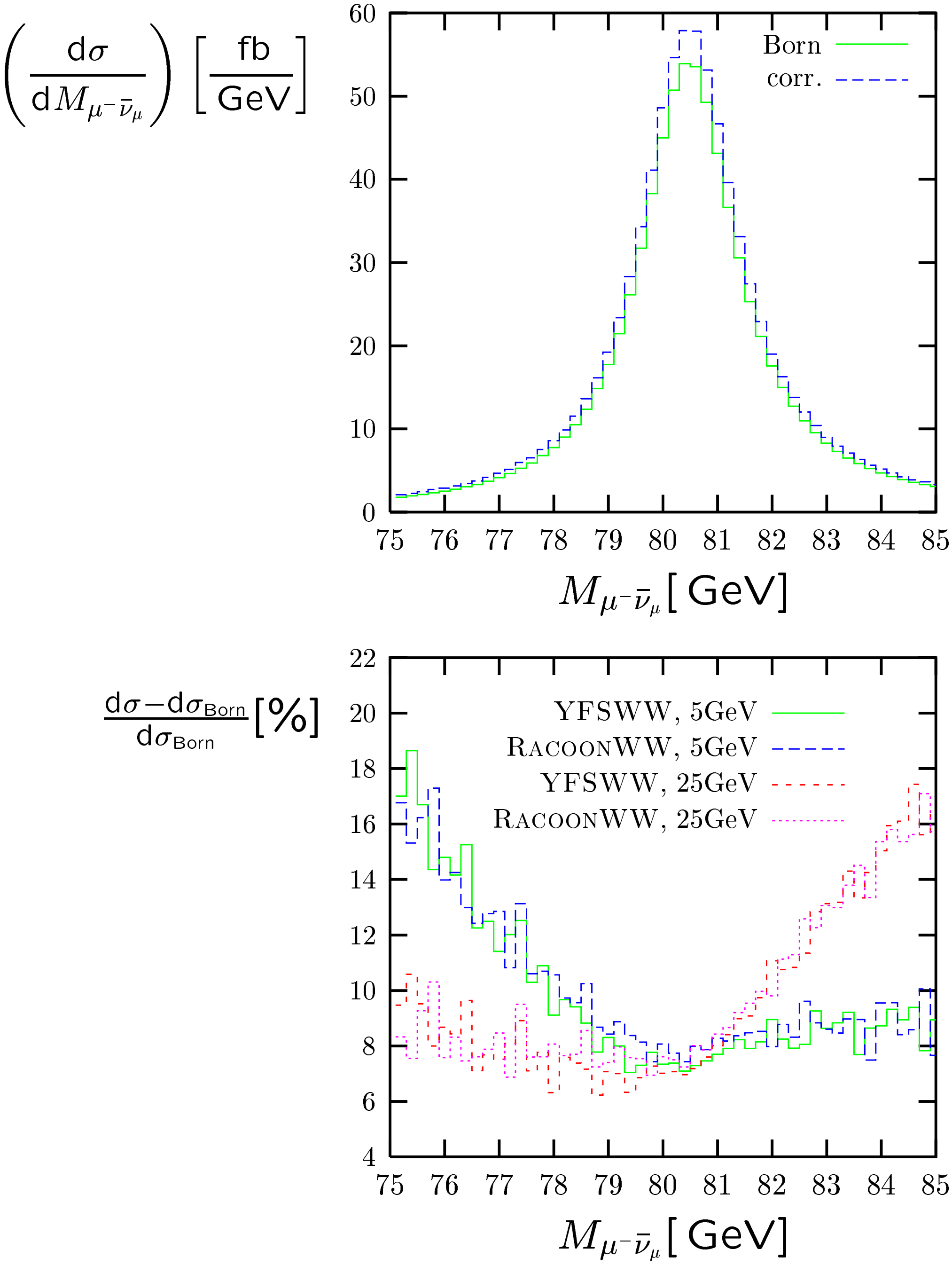
$$+\underbrace{\int \mathrm{d}\Phi^{4f}\left[2\,\mathrm{Re}\,(\mathcal{M}_{\mathrm{virt}}\mathcal{M}^*_{\mathrm{Born}})\,\mathcal{O}(\Phi^{4f}) + \mathcal{V}^{4f}\mathcal{O}(\Phi^{4f})\right]_{\mathrm{DPA}}}_{\text{no mass singularities } (G_\mu \text{ scheme})}$$

$$+\underbrace{\sum_{\pm}\int \mathrm{d}x \int \mathrm{d}\Phi^{4f}(xp_{\mathrm{e}^\pm})\mathcal{V}^{4f}_{\pm}(x)\mathcal{O}(\Phi^{4f})}_{\propto \ln(m_{\mathrm{e}}/Q)\ \Rightarrow\ \text{no DPA applied!}}$$

$\Rightarrow$ Consistent double-pole approximation!

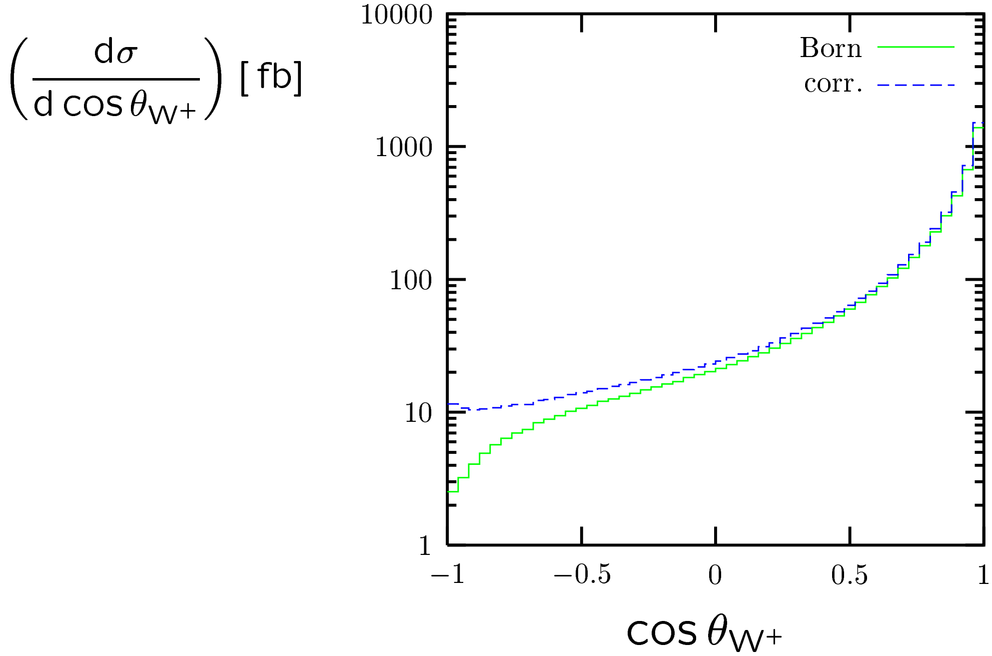
Invariant-mass distribution

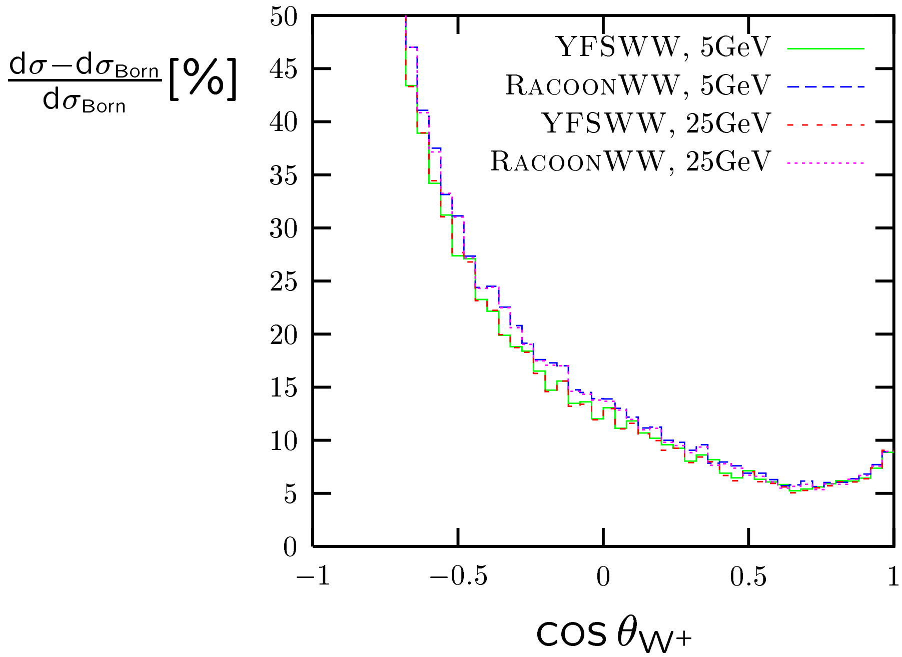
$\sqrt{s} = 500\,\mathrm{GeV}$, $\mathrm{e}^+\mathrm{e}^- \to \mathrm{u}\bar{\mathrm{d}}\mu^-\bar{\nu}_\mu(+\gamma)$, preliminary



W-production-angle distribution

$\sqrt{s} = 500\,\mathrm{GeV}$, $\mathrm{e^+e^-} \to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu(+\gamma)$, preliminary

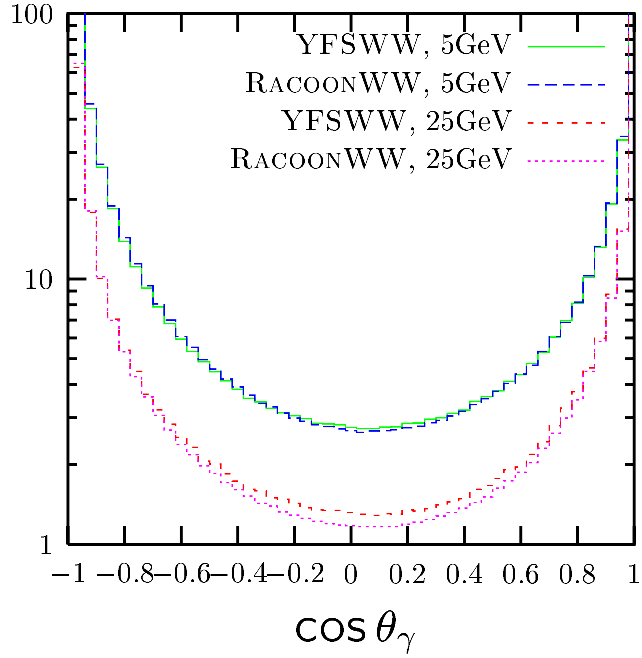




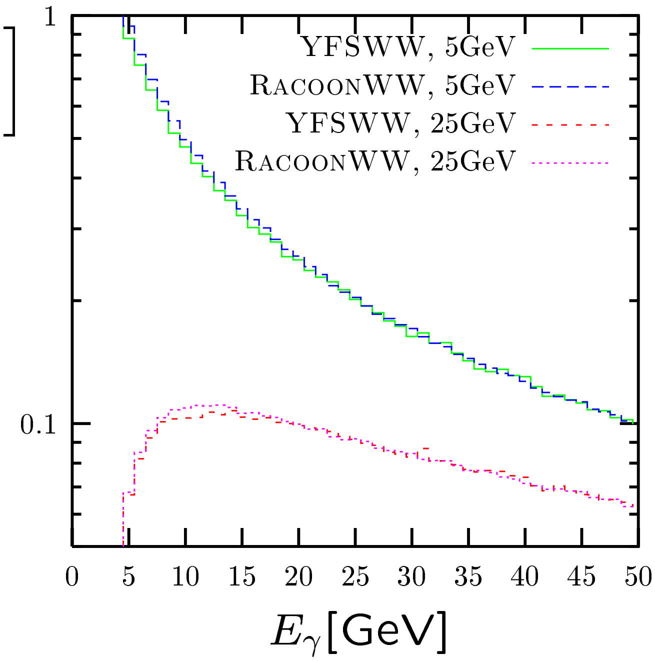
Photon distributions

$\sqrt{s} = 500\,\text{GeV},\ e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu\gamma$, preliminary

$\left(\frac{d\sigma}{d\cos\theta_\gamma}\right)$ [fb]



$\left(\frac{d\sigma}{dE_\gamma}\right)\left[\frac{\text{fb}}{\text{GeV}}\right]$



Summary and Outlook

Monte Carlo generator RacoonWW

- Lowest order: full amplitude for $e^+e^- \to 4f$
- Real corrections: full amplitude for $e^+e^- \to 4f\gamma$ and ISR beyond $\mathcal{O}(\alpha)$
- Virtual corrections: consistent DPA for $e^+e^- \to WW \to 4f$

$\Rightarrow$ First generator with complete DPA for $\mathcal{O}(\alpha)$ corrections

Work in progress:

- Public version of RacoonWW
- Comparison within the LEP2 MC workshop
  - MC generator YFSWW (Jadach et al.)
    $\Rightarrow$ $\lesssim 0.5\%$ difference in total cross section
  - semi-analytical approach of Beenakker/Berends/Chapovsky
    $\Rightarrow$ agreement for leptonic final state