Automatic computation

of helicity amplitudes

and cross sections

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Padua 2000

- **Need**

  Reliable cross section computation and event generation for multiparticle (≤ 10) porcesses.

- **Result**

  - **HELAC:** A.Kanaki and C.G.Papadopoulos, hep-ph/0002082.

    Matrix element computation algorithm, based on Dyson-Schwinger equations, including: EWK, QCD, fermion masses, reliable arithmetic, running couplings and masses . . . .

  - **PHEGAS:** C.G.Papadopoulos, LEP2Y2K in preparation

    Monte-Carlo phase space integration/generation based on optimized multichannel approach.

- Give the process

$$e^- e^+ \to \mu\ \bar{\nu}_\mu\ u\ \bar{d}\ \gamma\ \gamma$$

- Define the cuts

$$E_i > E_0 \quad \cos\theta_i < c_0 \quad \cos\theta_{ij} < c_1$$

- The cross section is:

$$\sigma = 1.73(5)\ \text{fb at } \sqrt{s} = 500\ \text{GeV}$$

- **... plus any other kinematical distribution !**

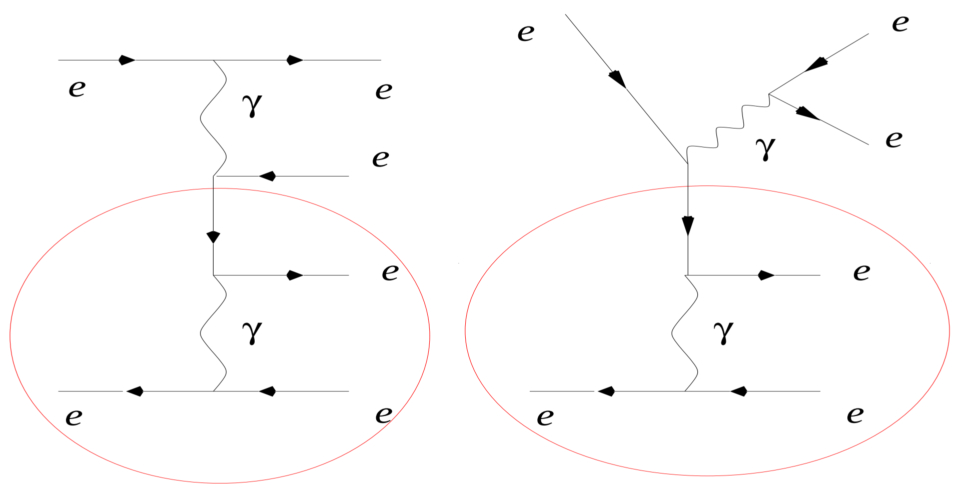
Old Feynman graphs $\rightarrow$ computational cost $\sim n!$

New Dyson-Schwinger $\rightarrow$ computational cost $\sim 3^n$

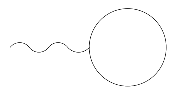
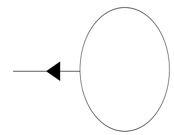
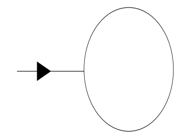
P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157

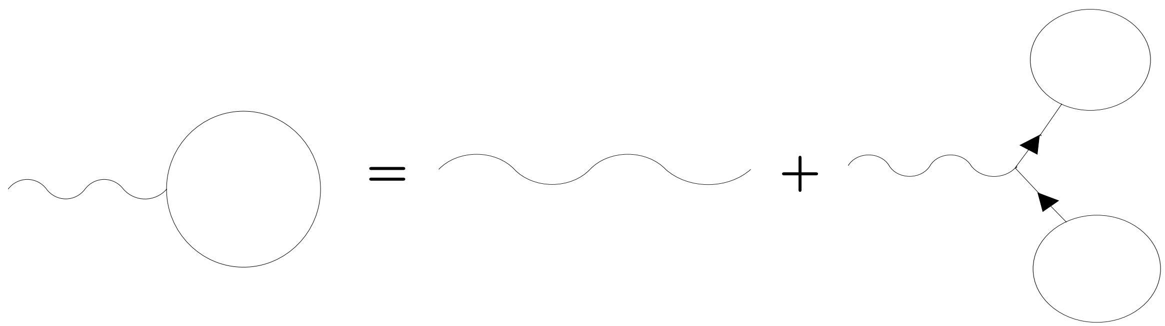
F. Caravaglios and M. Moretti, Phys. Lett. B358 (1995) 332

- ***Example:*** $e^-e^+ \rightarrow e^-e^+e^-e^+$ **in QED:**



⇒ Systematic approach:

$$b_\mu(P) \; =$$  $$\psi(P) \; =$$  $$\bar{\psi}(P) \; =$$ 



$$b^\mu(P) = \sum_{i=1}^{n} \delta_{P=p_i} b^\mu(p_i) + \sum_{P=P_1+P_2} (ig)\Pi^\mu_\nu \; \bar{\psi}(P_2)\gamma^\nu \psi(P_1)\epsilon(P_1, P_2)$$

- Let n external particles with momenta $p_i^\mu, i = 1 \ldots, n$, and define the momentum P^μ

$$P^\mu = \sum_{i \in I} p_i^\mu \ , \qquad I \subset \{1, \ldots, n\}$$

- the binary vector $\vec{m} = (m_1, \ldots, m_n)$, where its components take the values 0 or 1 :

$$P^\mu = \sum_{i=1}^{n} m_i \ p_i^\mu \ .$$

- Moreover this binary vector can be uniquely represented by the integer

$$m = \sum_{i=1}^{n} 2^{i-1} m_i \ , \quad 0 \le m \le 2^n - 1$$

- Replace

$$b_\mu(P) \to b_\mu(m) \ .$$

- **Convenient ordering of integers in binary representation ⇒ level l, defined by**

$$l = \sum_{i=1}^{n} m_i \; .$$

- **external momenta are of level 1**
- **the total amplitude corresponds to the unique level n integer $2^n - 1$**

$$\mathcal{A} = b(1) \cdot b(2^n - 2)$$

This ordering dictates the natural path of the computation : starting with level-1 sub-amplitudes, we compute the level-2 ones using the Dyson-Schwinger equations and so on up to level $n-1$

The solution

$$e^-(1)\ \ e^+(2) \longrightarrow e^-(4)\ \ \bar{\nu}_e(8)\ \ u(16)\ \ \bar{d}(32)$$

1	10	33	2	-2	8	1
1	12	33	4	-2	8	1
1	48	34	16	-3	32	4
2	26	-4	10	33	16	-3
			...			
2	62	-2	10	33	52	-1
2	62	-2	12	33	50	-1
2	62	-2	58	31	4	-2
2	62	-2	58	32	4	-2
2	62	-2	60	31	2	-2
2	62	-2	60	32	2	-2

- Dirac algebra simplification: 2-dim vs 4-dim and chiral representation, including $m_f \neq 0$.
- The sign factor:

$$\epsilon(P_1, P_2) \longrightarrow \epsilon(m_1, m_2)$$

  we define

$$\epsilon(m_1, m_2) = (-1)^{\chi(m_1, m_2)}$$

$$\chi(m_1, m_2) = \sum_{i=n}^{2} \hat{m}_{1i} \left(\sum_{j=1}^{i-1} \hat{m}_{2j} \right)$$

  where hatted components are set to 0 if the corresponding external particle is a boson.
- Full EWK theory, both Unitary and Feynman gauges.

A. Denner, Fortsch. Phys. 41, 307 (1993).

Colour Configuration - EWK⊕QCD

- ⋆ quarks $1 \dots n$
- ⋆ antiquarks $\sigma_i(1 \dots n)$ and
- ⋆ gluons $= q\bar{q}$

Each color-configuration amplitude is proportional to

$$D_i = \delta_{1,\sigma_i(1)}\delta_{2,\sigma_i(2)} \dots \delta_{n,\sigma_i(n)}$$

where σ_i represents the i-th permutation of the set $1, 2, \dots, n$.

$$\mathcal{C}_{ij} = \sum D_i D_j = N_c^{\alpha} \ , \ \ \alpha = \langle \sigma_1, \sigma_2 \rangle$$

♠ exact color treatment ⇒ low color charge

HELAC - the code

- **A two-phase computation**
  - **phase A: selection and tree-construction (integer arithmetic)**
  - **phase B: computation (high-precision arithmetic)**
- **Comparisons: full technical agreement**
  - EXCALIBUR $m_f = 0$ **comparable speed**
  - MADGRAPH $m_f \neq 0$ **comparable speed for** $n < 7$.
- **As expected** HELAC **shows an exponential (instead of factorial)** CPU**-time growth.** F.A.Berends, C.G.Papadopoulos and R.Pittau, hep-ph/0002249.
- **There is no a priori limitation for the number of particles that** HELAC **can treat, the only restrictions being that of memory allocation.**

Numerical reliability

- Problem due to huge 'gauge' cancellations !
- In order to have a taste of a multi-precision computation we have computed the squared amplitude for the process

$$e^-(p_1)e^+(p_2) \to e^-(p_3)e^+(p_4)e^-(p_5)e^+(p_6)$$

at two phase space points.

- Phase space point (A) is just a randomly generated one by the phase-space generator `RAMBO`.
- Phase space point (B)

$$p_1^0/\ \mathrm{GeV} = 100,\ \vec{p}_1 + \vec{p}_2 = 0,$$
$$p_3^0/p_1^0 = 0.9,\ \theta_3 = 0,$$
$$(p_5+p_6)^2/(p_4+p_5+p_6)^2 = 0.1,\ \ \theta_4 = \phi_4 = \theta_5 = \phi_5 = 0$$

with $m_e = 0.511 \times 10^{-3}$ GeV.

- **Results are provided for these points, by using the `real*8` (DP), `real*16` (QP) and 34-digit multi-precision version of the code.**

David M. Smith, Transactions on Mathematical Software 17 (1991) 273.

(A)	(B)
1.539728523150595E-008	1.256276706229023E+023
1.5397285231505885415676300282501 3D-08	3.0716260109371091513413692497308 9D+22
1.5397285231505885415676300282501 1853M-8	3.0716260109371091512795010924177 0808M+22

- **In the `MP`-version of the code the precision is defined by the user.**

PHEGAS

- Phase space

$$d\Phi_n = (2\pi)^{4-3n} \prod_{i=1}^{n} \frac{d^3 p_i}{2E_i} \delta\left(\sum E_i - w\right) \delta^3\left(\sum \vec{p}_i\right)$$

- RAMBO, VEGAS nice but completely inefficient!

$$d\sigma_n = \text{FLUX} \times |\mathcal{M}_{2\to n}|^2 d\Phi_n$$

need appropriate mappings of peaking structures, plus optimization!

- Efficiency $\Rightarrow$ to a large number of generators, each one for a specific class of processes.

Multichannel approach

$$\mathcal{I} = \int f(\vec{x}) d\mu(\vec{x}) = \int \frac{f(\vec{x})}{p(\vec{x})} p(\vec{x}) d\mu(\vec{x})$$

$$p(\vec{x}) = \sum_{i=1}^{M_{ch}} \alpha_i \, p_i(\vec{x}) \qquad \sum_{i=1}^{M_{ch}} \alpha_i = 1$$

$$\mathcal{I} \rightarrow \left\langle \frac{f(\vec{x})}{p(\vec{x})} \right\rangle \qquad \mathcal{E}^2 N \rightarrow \left\langle \left(\frac{f(\vec{x})}{p(\vec{x})} \right)^2 - \mathcal{I}^2 \right\rangle$$

⋆ **Optimize $\alpha_i \Rightarrow$ Minimize $\mathcal{E}$** ⋆

R.Kleiss and R.Pittau, Comput. Phys. Commun. 83, 141 (1994).

New **Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!**

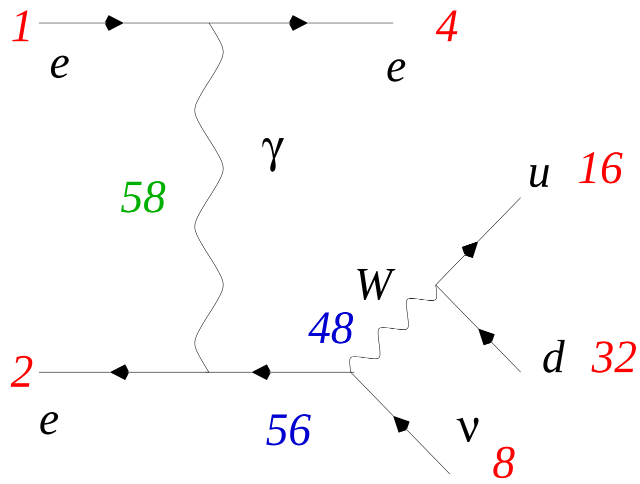
problem unsolved

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

Old **Feynman graphs: exhibit single peaking structure!**

problem solved

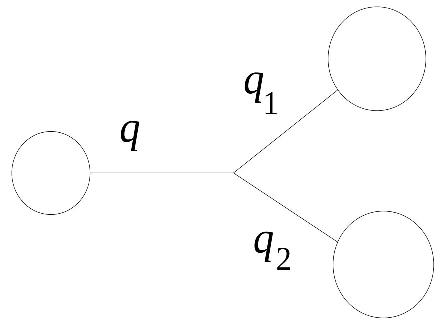
Back to Feynman graphs:



The corresponding intrinsic representation looks like

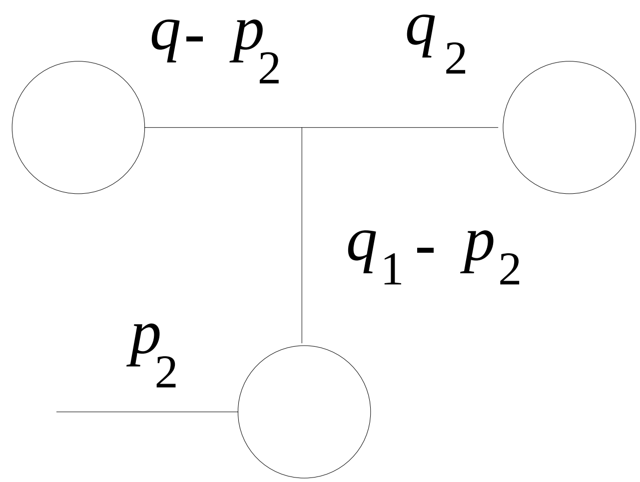
62	**-2**	**4**	**-2**	**58**	**31**
58	**31**	**2**	**-2**	**56**	**2**
56	**2**	**48**	**33**	**8**	**1**
48	**33**	**16**	**-3**	**32**	**4**

Time-like momenta $q^2 \geq 0$



$$
\begin{aligned}
d\Phi_n &= \ldots \frac{dq_1^2}{2\pi}\,\frac{dq_2^2}{2\pi}\,d\Phi_2(q \to q_1, q_2)\ldots \\
&= \ldots \frac{dq_1^2}{2\pi}\,\frac{dq_2^2}{2\pi}\,d\cos\theta\,d\phi\,\frac{\lambda(q^2, q_1^2, q_2^2)^{1/2}}{32\pi^2\,q^2}\ldots
\end{aligned}
$$

Space-like momenta



$$d\Phi_n = \ldots \frac{dq_1^2}{2\pi}\ \frac{dq_2^2}{2\pi}\ dt\ d\phi\ \frac{1}{32\pi^2\ q\ p_2}$$

$$t = (q_1 - p_2)^2 = m_2^2 + q_1^2 - \frac{E_2}{q}(q^2 + q_1^2 - q_2^2) + \frac{\lambda^{1/2}}{q} p_2 \cos\theta$$

- **Find limits of $t(q_1^2, \cos\theta)$:**

$$t_\pm = m_2^2 + q_1^2 - \frac{E_2}{q}(q^2 + q_1^2 - q_2^2) \pm \frac{\lambda^{1/2}}{q} p_2$$

$$\frac{\partial t_\pm}{\partial q_1^2} = 1 - \frac{E_2}{q} \mp \frac{p_2}{q} \frac{q^2 + q_2^2 - q_1^2}{\lambda^{1/2}}$$

- **t_{max}:**

$$\frac{\partial t_+}{\partial q_1^2} < 0 \quad \text{then} \quad t_{max} = t_{+,max} = t_+(q_1^2 = q_{1,min}^2)$$

$$\frac{\partial t_+}{\partial q_1^2} > 0 \quad \text{then} \quad t_{max} = t_+(q_1^2 = x_-)$$

$$x_- = q^2 + q_2^2 - 2\, q\, q_2 \frac{1 - E_2/q}{\sqrt{\alpha}}\ , \quad \alpha = \left(1 - \frac{E_2}{q}\right)^2 - \left(\frac{p_2}{q}\right)^2 > 0$$

- t_{min}:

$$\frac{\partial t_-}{\partial q_1^2} > 0 \quad \text{then} \quad t_{min} = t_{-,min} = t_-(q_1^2 = q_{1,min}^2)$$

$$\frac{\partial t_-}{\partial q_1^2} < 0 \quad \text{then} \quad t_{min} = t_-(q_1^2 = x_-)$$

$$x_- = q^2 + q_2^2 - 2\, q\, q_2 \frac{E_2/q - 1}{\sqrt{\alpha}}$$

♠ q_1^2-limits:

$$\left(t - q_1^2 - m_2^2 + \frac{E_2}{q}(q^2 + q_1^2 - q_2^2)\right)^2 \leq \lambda \left(\frac{p_2}{q}\right)^2$$

1. $a > 0$ so $x_- < q_1^2 < x_+$ with $x_- = \max(x_-, q_{1,min}^2)$ and $x_+ = \min(x_+, q_{1,max}^2)$.
2. $a < 0$ then we have to satisfy two conditions $q_1^2 < x_\pm$ or $x_\pm < q_1^2$ and $q_{1,min}^2 < q_1^2 < q_{1,max}^2$

- At the end we get:

$$d\Phi_n \to \prod ds_i \; p_i(s_i) \prod dt_j \; p_j(t_j) \prod d\phi_k \prod d\cos\theta_l$$

- $p(x)$ are chosen so that 'singularities' are smoothed out!

  - ♦ $(s - m^2)^2 + m^2\Gamma^2$ for massive unstable particles, $W^\pm$, Z.
  - ♦ s^ν for time-like massless propagators, e.g. γ, gluons, fermions.
  - ♦ $|t|^\nu$ for space-like massless propagators.

Final states	number of FG	$\sqrt{s}$ (GeV)	Cross section (fb)
$u\ \bar{d}\ s\ \bar{c}\ \gamma$	90(74)	200	199.75 (16)
$e^-\ \bar{\nu}_e\ \mu^+\ \nu_\mu\ \gamma$	108(100)	200	29.309 (25)
$\mu^-\ \bar{\nu}_\mu\ u\ \bar{d}\ \gamma\ \gamma$	587(210)	500	1.730 (58)
$\mu^-\ \bar{\nu}_\mu\ u\ \bar{d}\ c\ \bar{c}$	209(102)	500	0.1783 (20)
$\mu^-\ \bar{\nu}_\mu\ u\ \bar{d}\ c\ \bar{c}\ \gamma$	2142(339)	500	0.02451 (65)

Outlook

PHEGAS/HELAC: a framework for high-energy phenomenology

- **Standard Model fully included**

⋆ **High color charge processes: multijet production**

P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157

⇒ **New Physics effects**

  - **Trilinear Gauge Couplings + Quartic**
  - **SUSY and new particles**

⋆ **Higher order corrections**

  - **Direct approach**
  - **Running couplings and masses**