Braneworlds with Broken Lorentz Invariance

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Introduction

We investigate braneworlds with broken Lorentz invariance

$$ISO(1,3) \quad \text{broken to} \quad ISO(3)\times\mathbb{R}$$

for example

$$ds^2 = e^{-2a(z)}dt^2 - e^{-2b(z)}d\mathbf{x}^2 - dz^2 \tag{1}$$

where $a(z)$ and $b(z)$ are smooth functions. The $3+1$ dimensional brane is located at $z=0$.

Our goal is to study spectra of field perturbations localized on the brane in the above backgrounds.

Consider static gravitational setup (Plank units)

$$G_{AB} = T_{AB}^{\text{bulk}} + \delta(z)T_{AB}^{\text{brane}}$$

The r.h.s. is shown to obey the ...

...No-Go Theorem

Let the following conditions hold

- bulk NEC $\quad T_{AB}\xi^A\xi^B \geq 0, \quad g_{AB}\xi^A\xi^B = 0$
- brane NEC $\quad T_{b,\mu\nu}\xi^\mu\xi^\nu \geq 0, \quad g_{b,\mu\nu}\xi^\mu\xi^\nu = 0$
- positiveness of brane energy density $\quad \rho_b + \sigma \geq 0$
- 3d brane flatness $\quad \kappa = 0$
- bulk Lorentz invariance violation $\quad a(z) \neq b(z)$.

Then a generic background of type (1) without physical singularities in the bulk does not exist [2].

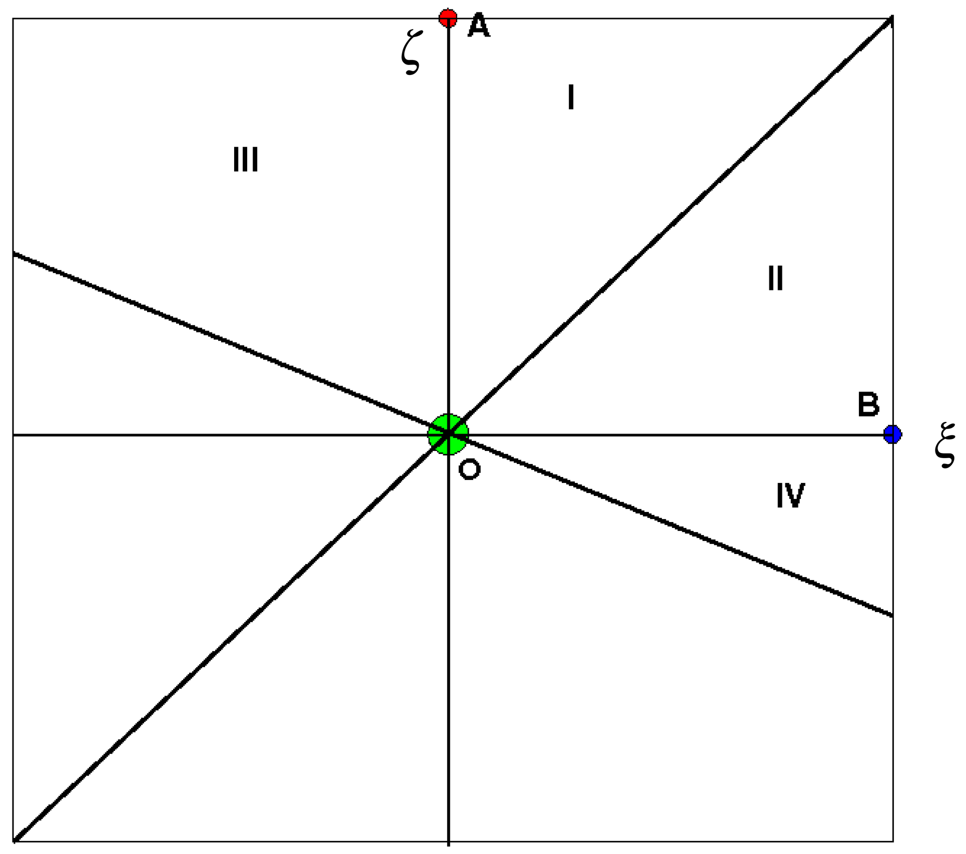
The statement implies the alternative of either having a naked singularity in the bulk or violations of null energy conditions on the brane or in the bulk.

Solution

Macroscopic solution $a(z) = \xi k|z|$, $b(z) = \zeta k|z|$

$$ds^2 = e^{-2\xi k|z|}dt^2 - e^{-2\zeta k|z|}d\mathbf{x}^2 - dz^2\,.$$

(ξ,ζ) form a parameter space



Minkowski (the origin O), AdS_5 (the diagonal), type I (upper triangle) and type II (lower triangle).

Matter – perfect anisotropic fluid

$$\begin{aligned}
T_0^0 &= ((1+\omega)u_0u^0 - \omega)\rho \\
T_1^1 &= ((1+w)u_1u^1 - w)\rho \\
T_5^5 &= ((1+\omega)u_5u^5 - \omega)\rho \\
T_5^0 &= (1+\omega)\rho u^0 u_5
\end{aligned}$$

where

$$\begin{aligned}
\rho &= -\Lambda - 6k^2\zeta^2 \\
w &= -1 + \frac{3\zeta^2 - 2\zeta\xi - \xi^2}{\Lambda/k^2 + 6\zeta^2} = -1 + k^2\frac{(3\zeta+\xi)(\xi-\zeta)}{\rho} \\
\omega &= -1 + \frac{3\zeta(\zeta-\xi)}{\Lambda/k^2 + 6\zeta^2} = -1 + k^2\frac{3\zeta(\xi-\zeta)}{\rho}
\end{aligned}$$

with $u_1 = u_5 = 0$.

Localization/Delocalization of Fluctuations

Effective potential governing fluctuation's behavior $(y = y(z))$ [1]

$$V = -\frac{1}{4}\left(\frac{\partial a}{\partial y}\right)^2 + \frac{9}{4}\left(\frac{\partial b}{\partial y}\right)^2 + p^2 e^{2(b-a)} - \frac{1}{2}\left(\frac{\partial^2 a}{\partial y^2} + 3\frac{\partial^2 b}{\partial y^2}\right)\,.$$

The above potential can be classified by the following properties

- Behavior at infinity which is controlled by the Lorentz invariance violation (the sign of $b-a$). The potential V increases/decreases as $y \to \infty$.
- Sign of the delta-function term. The potential may have either delta-well or delta-peak at the origin depending on this sign.

This rough classification enables us to describe the behavior of fluctuations in each case.

- If the momentum-dependent term increases as $y \to \infty$ the potential has a box-type shape and the excitation spectrum is discrete. The potential may have local minima and maxima, but qualitative pattern of the spectrum is determined by its behavior at infinity. On the contrary, if the potential decays at infinity, then we have continuous spectrum of plane waves propagating along y-direction. Some combination of these two scenarios is possible if $V \to V_\infty = \text{const}$ as $z \to \infty$. Then those modes with $E^2 < V_\infty$ belong to discrete spectrum and modes with $E^2 > V_\infty$ contribute to continuous spectrum.
- The sign of the delta-function term affects zero mode existence. In a delta-well there might be a zero-mode and its existence is very unlikely in a configuration with a delta-peak.

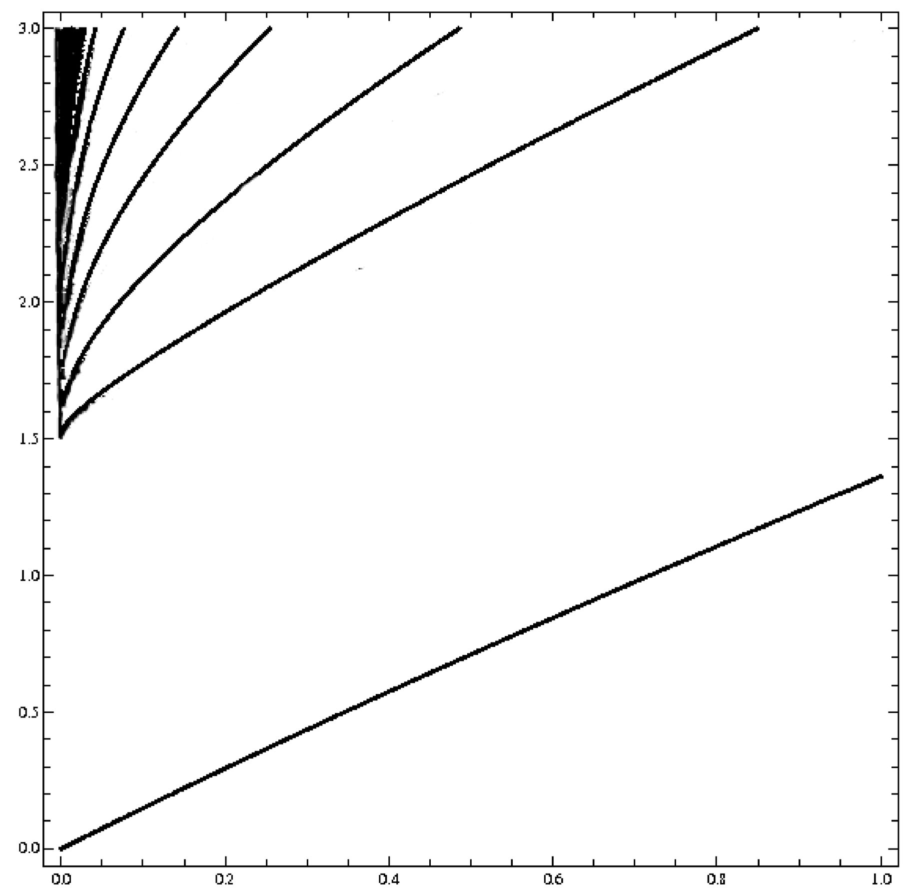
Spectrum of Perturbations (Model A)

see [3] for model B

Consider Dirac field in the background (1) for with kink-type mass term

$$S = \int dt d\mathbf{x} \int_{-\infty}^{+\infty} dz\sqrt{g}\left(i\bar{\Psi}\nabla\!\!\!/\,\Psi - m\,\text{sign}(z)\bar{\Psi}\Psi\right)\,,$$

If $m > k/2$ the spectrum $E(p)$ of the theory has the following pattern



Straight line starting from the origin shows the zero mode dispersion relation at small p

$$E \simeq \left(\frac{2m}{2m-k}\right)p\,.$$

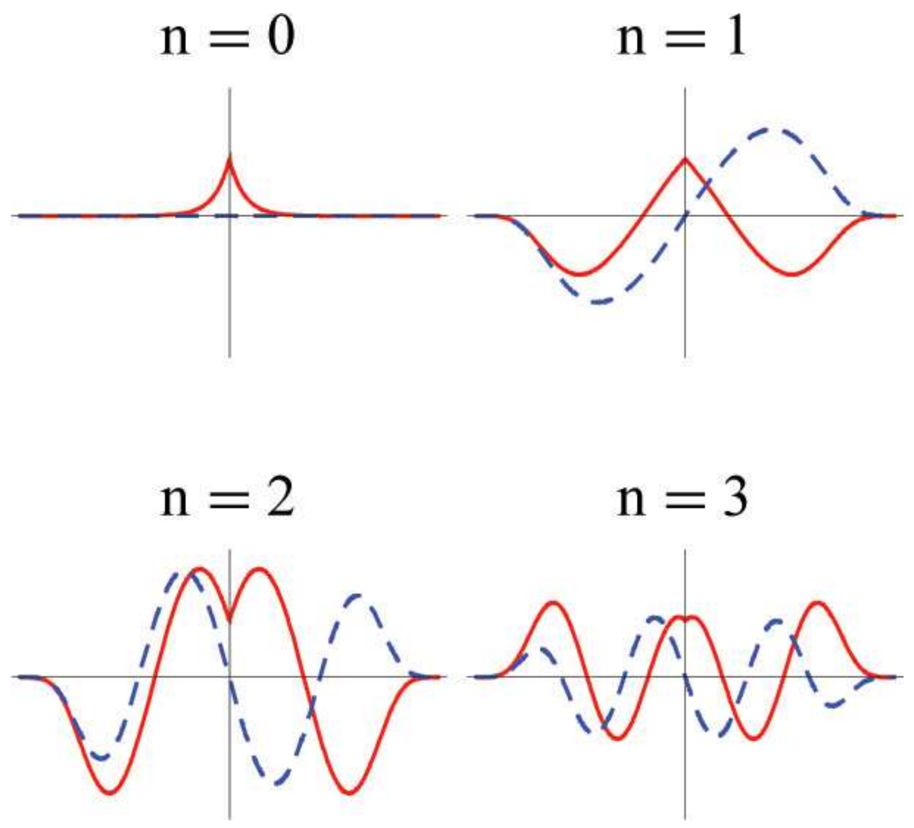
Other curves originating from the point $(0, 3/2)$ show dispersion laws for higher modes at different $n = 1, 2, \ldots$ at small p

$$E_n = m\sqrt{1 + \left(\frac{\pi n k}{2\Psi\left(\frac{1}{2}\right) m - \gamma k - 2m \log \frac{p}{2k}}\right)^2}\,.$$

At large p the theory becomes Lorentz invariant

$$E = p\,.$$

Mode profiles at $n = 0, 1, 2, 3$ for $m = 2k$, $p = 0.01$. Red solid curves correspond to left-handed and dashed blue curves correspond to right-handed spinors. On the upper left plot (n=0) the right-handed component is strongly suppressed.



Modified Newton Gravity (Model A)

On the brane static potential between two sources gets modified

$$G(r) = \frac{k}{4\pi r}\left(1 + \frac{2}{\pi k r}\right),$$

thereby acquiring different to RS2 model power correction at short distances.

Conclusions and Outlook

- Study time dependent backgrounds. Maybe the No-Go theorem [2] can be evaded
- Embedding the story into string theory and studying gauge/gravity duality for backgrounds with broken Lorentz invariance [5, 4]

References

[1] P. Koroteev and M. Libanov, Phys. Rev. D **79**, 045023 (2009) [arXiv:0901.4347 [hep-th]].

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[4] I. Gordeli and P. Koroteev, arXiv:0904.0509 [hep-th].

[5] S. Kachru, X. Liu and M. Mulligan, Phys. Rev. D **78**, 106005 (2008) [arXiv:0808.1725 [hep-th]].