Flavor Symmetry Breaking and Vacuum Alignment on Orbifolds [1]

Yuji Omura[a,b,2]

[a]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

[b]*INFN, Sezione di Padova, via F Marzolo 8, I-35131, Padova, Italy*

Abstract

In this paper we present a framework for handling flavor symmetry breaking where the symmetry breaking is triggered by boundary conditions of scalar fields in extra-dimensional space. We discuss an illustrative lepton mass model with A_4 flavor symmetry.

1 Introduction

The masses and mixing angles of quarks and leptons have been long-standing and inspiring problems in particle physics. Flavor symmetry has been widely investigated as a solution to the fermion mass problems. Among various flavor groups, a particular attention has recently been paid for non-abelian discrete groups such as S_3 and A_4. These non-abelian discrete symmetries have several advantages that they provide a definite meaning for the generation and also link up different generations. Flavor symmetry breaking is caused, e.g., by non-vanishing vacuum expectation values of scalar fields, which are determined by the minimization of scalar potential. For a non-abelian discrete symmetry to leave some imprints in low-energy symmetry-broken theory, the expectation values must take specific directions, namely, the vacuum alignment is required.

In this paper, we present a framework to realize aligned flavor symmetry breaking without any elaborated potential analysis. A central ingredient is the global property of bulk scalar fields in higher-dimensional space. At the boundaries of extra space, dynamical conditions for bulk fields are specified to fix the model. That reduces the number of degrees of freedom of symmetry groups [3]. In this paper we discuss bulk scalar fields charged under the flavor symmetry, whose expectation values are aligned to cause discrete patterns of symmetry breaking. The vacuum alignment is archived in our approach by adopting the boundary conditions such that only one light mode survives and it governs all vacuum expectation values in low-energy theory.

We consider as the simplest example a five-dimensional orbifold theory on the flat gravitational background. The generalization to higher dimensions is straightforward. The five-dimensional coordinates are denoted by (x^μ, x^5) where x^5 corresponds to the fifth dimension. There are two types of operations on this space; the reflection $\hat{Z} : x^5 \to -x^5$ and the translation $\hat{T} : x^5 \to x^5 + 2\pi L$, where L is a constant. They trivially satisfy $\hat{T}\hat{Z} = \hat{Z}\hat{T}^{-1}$. On field variables living in the bulk, these operations are expressed in terms of matrices in the field space:

$$\hat{Z}\phi(x^5) \,=\, Z^{-1}\phi(-x^5), \qquad \hat{T}\phi(x^5) \,=\, T^{-1}\phi(x^5 + 2\pi L). \tag{1.1}$$

Let us suppose the extra space is compactified on the S^1/Z_2 orbifold with the radius L. This is achieved by the identifications $\hat{Z}(x^5) = x^5$ and $\hat{T}(x^5) = x^5$. On this orbifold, there are two fixed points (boundaries) at $x^5 = 0$ and $x^5 = \pi L$. The boundary conditions of bulk fields in compactifying the extra dimension are then defined by the identifications $\hat{Z}\phi(x^5) = \phi(x^5)$ and $\hat{T}\phi(x^5) = \phi(x^5)$.

Here we give a general recipe for non-abelian discrete symmetry breaking in the fifth dimension S^1/Z_2. The bulk theory has a flavor symmetry acting on the three-generation quarks and leptons. We introduce bulk scalar fields ϕ_i $(i = 1, 2, 3)$ which transform as a triplet under the flavor symmetry. The triplet representation can be reducible. The boundary conditions for the scalars ϕ_i are given by

$$\hat{Z}: \qquad \phi(-x^5) \,=\, Z\phi(x^5), \tag{1.2}$$

$$\hat{T}: \qquad \phi(x^5 + 2\pi L) \,=\, T\phi(x^5). \tag{1.3}$$

Here Z and T are the unitary representation matrices of flavor symmetry group and act on the generation space. The consistency relations require $Z^2 = (ZT)^2 = 1$. This implies that for each choice of the matrix Z,

[1]This paper is based on the paper [1] collaborated with T. Kobayashi and K. Yoshioka.

[2]E-mail address: omura@scphys.kyoto-u.ac.jp

it can be rotated to the diagonal form; $Z = \mathrm{diag}(p_1, p_2, p_3)$ with $p_i = \pm 1$. For a positive (negative) parity element $p_i = +1$ ($p_i = -1$), a corresponding bulk field has an even (odd) wavefunction about the extra-dimensional coordinate. Therefore the number of zero modes is given by that of positive parity elements when one takes into account the $\hat{Z}$ boundary condition only. We fix $p_1 = +1$ in the following discussion. To include another boundary condition taken into account, it is useful to express the translation matrix as $T = e^{\pi i W}$ where W is the hermite 3×3 matrix written in the basis where Z is diagonal. From the boundary condition (1.3), the mode expansion is found to be

$$\phi_i(x^\mu, x^5) = \sum_n \Big(\exp \Big[i \Big(\frac{W}{2} + n \Big) \frac{x^5}{L} \Big] \Big)_{ij} \phi_j^{(n)}(x^\mu). \tag{1.4}$$

The even (cosine) or odd (sine) function part is chosen depending on the parity assignment for ϕ_i. From this mode expansion, we find that massless zero modes are obtained when W has zero eigenvalues (mod 2). For example, we consider the boundary condition that the number of massless mode is reduced to 1 with either Z or T only. This is the case that the matrix Z has one positive parity, i.e., $p_1 = +1$ and $p_2 = p_3 = -1$. In this basis, W should take the following form:

$$W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}. \tag{1.5}$$

The right-bottom 2×2 sub-matrix (denoted hereafter as w) is hermite and has integer eigenvalues so that the consistency relation $(ZT)^2 = 1$ is satisfied.

In a four-dimensional effective theory below the compactification scale $1/L$, the physics is described by zero modes. Assuming that these zero modes develop their vacuum expectation values at low energy, we find that they contribute to Yukawa operators for quarks and leptons, and the vacuum alignment is established.

2 A_4 flavor symmetry

As one example of flavor symmetry, we discuss A_4 which consists of the even permutations of four objects. The A_4 group is composed of twelve elements generated by two fundamental elements P, R and their derivatives: PR, RP, R^2, PR^2, PRP, RPR, R^2P, RPR^2, and R^2PR. They satisfy the characteristic relations for the A_4 group: $P^2 = (PR)^3 = R^3 = 1$.

In addition to the trivial singlet, the A_4 group has three non-trivial representations, the triplet and two pseudo singlets (denoted as $1'$ and $1''$), the latters of which are respectively multiplied by the complex numbers χ and χ^2 under the R operation ($\chi = e^{2\pi i/3}$). The representation matrices for the triplet are built up from

$$P = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}, \qquad R = \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}. \tag{2.1}$$

Let us consider a bulk scalar ϕ in the triplet representation of A_4 symmetry and examine the boundary conditions which leave a single zero mode in four-dimensional effective theory.

We consider the $T^2/(Z_2 \times Z_3)$ ($\simeq T^2/Z_6$) orbifold. It is useful to denote the extra-dimensional coordinates (x^5, x^6) by a complex one $z = x^5 + ix^6$. The extra-dimensional space has four types of operations; the translations $\hat{T}$ and $\hat{T}'$, the reflection $\hat{Z}_2$, and the rotation $\hat{Z}_3$. They act on z as

$$\begin{aligned} \hat{T} \;&:\; z \to z + 2\pi L, \qquad & \hat{Z}_2 \;&:\; z \to -z, \\ \hat{T}' \;&:\; z \to z + 2\pi L\chi, \qquad & \hat{Z}_3 \;&:\; z \to \chi z, \end{aligned} \tag{2.2}$$

with χ being the cubic root of the unity ($\chi = e^{2\pi i/3}$). The T^2/Z_6 orbifold is obtained by the identifications with these operations. The $\hat{Z}_2$ parity has the four fixed points $z = 0$, πL, $\pi L\chi$ and $\pi L(1+\chi)$. The $\hat{Z}_3$ action leaves the three points $z = 0$, $(1+2\chi)/3$ and $(2+\chi)/3$ unchanged. Therefore the T^2/Z_6 orbifold has one fixed point at the origin $z = 0$. We will later consider that the standard model fields live on this four-dimensional fixed point. On the field variables $\phi(x^\mu, z)$, the compactification is performed with the identification;

$$\begin{aligned} \phi(z+2\pi L) &= T\,\phi(z), & \phi(-z) &= Z_2\,\phi(z), \\ \phi(z+2\pi L\chi) &= T'\phi(z), & \phi(\chi z) &= Z_3\,\phi(z), \end{aligned} \tag{2.3}$$

where T, T' and $Z_{2,3}$ are the triplet representation matrices of A_4 group.

It is important that these matrices should satisfy several consistency relations associated with the property of orbifold. From the transformation rules (2.2), we find the following independent relations for the T^2/Z_6 orbifold, such as $(Z_2)^2 = (Z_3)^3 = 1$. We find that only two types of non-trivial boundary conditions are viable in the A_4 theory on T^2/Z_6:

$$\text{(i)}\ \ Z_2 = P, \qquad Z_3 = T = T' = I \tag{2.4}$$

$$\text{(ii)}\ \ Z_3 = R, \qquad Z_2 = T = T' = I \tag{2.5}$$

where the matrices P and R are given in (2.1) and I is the unit matrix. The other sets of matrices with exchanging generation labels also become the solutions.

For the boundary condition (i), i.e., $\phi(-z) = P\phi(z)$, it is easily found from the matrix form (2.1) that we have a single zero mode in ϕ_1 due to a positive parity. The zero mode has a constant wavefunction profile. Then the vacuum expectation value becomes

$$\langle\phi\rangle \,\propto\, (1,\,0,\,0). \tag{2.6}$$

On the other hand, the boundary condition (ii) is explicitly written by $\phi(\chi z) = R\,\phi(z)$. The vacuum expectation value of the original field ϕ becomes

$$\langle\phi\rangle \,\propto\, (1,\,1,\,1). \tag{2.7}$$

We apply the above result for the A_4 flavor symmetry to constructing of an explicit orbifold model for lepton masses and generation mixing. A higher-dimensional extra space is introduced for flavor symmetry invariance, while the vacuum alignment itself is viable in lower dimensions.

Let us consider an eight-dimensional theory on the orbifold $T^2/Z_2 \times T^2/Z_3$. The standard-model fields are assumed to live on a fixed point of the orbifold, which we here choose the origin of extra-dimensional space. The three families of left-handed lepton doublets ℓ_i transform as a triplet of A_4 while the right-handed charged leptons e_1, e_2, e_3 are assigned to three different singlets, 1, $1'$, $1''$, respectively. We introduce two gauge-singlet bulk scalars ϕ and ϕ' of the triplet representation, which give effective Yukawa and mass operators for leptons in low-energy theory. The fifth and sixth dimensions are compactified on T^2/Z_2 and the seventh and eighth ones on T^2/Z_3. The scalars ϕ and ϕ' live on T^2/Z_2 and T^2/Z_3, respectively. The basic building block is the same as the models presented in [2]. The bulk theory has the flavor A_4 and Z_3' symmetries (other than the operation $\hat{Z}_3$). The field content and the flavor-symmetry charges are summarized below:

$$\begin{array}{c|cccccc|cc} & \ell & e_1 & e_2 & e_3 & h & \eta & \phi & \phi' \\ \hline A_4 & 3 & 1 & 1' & 1'' & 1 & 1 & 3 & 3 \\ Z & \chi & \chi & \chi & \chi & 1 & \chi & \chi & 1 \end{array} \tag{2.8}$$

Here h and η are the electroweak doublet and singlet Higgs fields, respectively. We have boundary interactions between the bulk and boundary fields which are invariant under the flavor symmetry:

$$\mathcal{L}_Y \,=\, y_1\bar{e}_1\ell\phi h + y_2\bar{e}_2\phi'\ell h + y_3\bar{e}_3\ell\phi' h + w_1\phi\overline{\ell^c}\ell h^2 + w_2\eta\overline{\ell^c}\ell h^2 + \cdots\,, \tag{2.9}$$

where $w_{1,2}$ and $y_{1,2,3}$ are the coupling constants and the wavefunctions of bulk scalar fields are evaluated at the fixed point on which the standard-model fields reside.

To fix the model, the boundary conditions for bulk fields must be specified. According to the result obtained in the above, we have the following non-trivial boundary conditions on the scalar fields:

$$\phi(-z) \;=\; P\,\phi(z), \tag{2.10}$$

$$\phi'(\chi z') \;=\; R\,\phi'(z'), \tag{2.11}$$

where $z = x^5 + ix^6$ and $z' = x^7 + ix^8$, and the other boundary conditions are trivial. Under these conditions, the previous analysis says that the zero modes are found in ϕ_1 and $\phi'_1 + \phi'_2 + \phi'_3$ which develop the expectation values of the form

$$\langle\phi\rangle = a\,(1,\,0,\,0), \qquad \langle\phi'\rangle = a'(1,\,1,\,1), \tag{2.12}$$

in low-energy effective theory below the compactification scale. It is noticed that these aligned forms of expectation values were required in the models of Ref. [2]. Inserting the scalar expectation values in (2.9), we obtain the operators for charged-lepton Dirac masses and neutrino Majorana masses; $\mathcal{L}_M = \bar{e}_i(M_e)_{ij}\ell_j + \overline{\ell_i^c}(M_L)_{ij}\ell_j$:

$$M_e = a'v\begin{pmatrix} y_1 & y_1 & y_1 \\ y_2 & \chi^2 y_2 & \chi y_2 \\ y_3 & \chi y_3 & \chi^2 y_3 \end{pmatrix}, \qquad M_L = v^2\begin{pmatrix} w_2 b & & \\ & w_2 b & w_1 a \\ & w_1 a & w_2 b \end{pmatrix}, \tag{2.13}$$

where $v = \langle h\rangle$ and $b = \langle\eta\rangle$. These forms of mass matrices are known [2] to be well fitted to the present experimental values. The lepton generation mixing is exactly given by the tri-bimaximal mixing. The neutrino mass eigenvalues are predicted as $\Delta m^2_{21} = -\mathrm{Re}[w_1 a(w_1 a + 2w_2 b)]v^2/2$ and $\Delta m^2_{31} = -4\mathrm{Re}(w_1 w_2 ab)v^2$. The hierarchy of charged-lepton masses is proportional to that of y_i. It may be implemented by, for example, the Froggatt-Nielsen mechanism [4], where the three right-handed charged leptons have different properties.

Finally, we comment on the boundary conditions (2.10)–(2.11) and the symmetry invariance of the boundary Lagrangian $\mathcal{L}_Y$. The consistent boundary conditions for T^2/Z_2 and T^2/Z_3 have wider possibilities than (2.4) and (2.5). An important point is that the vacuum alignment itself is not affected by such choices of twisting as long as the light-mode physics is concerned. This fact can, in turn, be utilized to protect boundary terms by the flavor symmetry. Namely, flavor symmetry breaking occurs within the hidden sector which is separate in the extra-dimensional space from the visible sector where the standard-model fields reside. Bulk scalar fields interact both of these sectors and induce effective mass operators by connecting up the symmetry-invariant terms on the visible boundary and the aligned vacuum expectation values on the hidden boundary.

3 Summary

In this paper we have presented the scenario for breaking flavor symmetry with Scherk-Schwarz twisted boundary conditions on bulk scalar fields, where only one zero mode from multiple scalar fields survives in low-energy effective theory. Our scheme of vacuum alignment needs no elaboration of analyzing complicated scalar potential which generally involves multiple scalar fields and requires extra symmetries to realize the vacuum alignment.

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