Invisible Particle Mass Determinations at Hadron Colliders

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Introduction

- It's strongly anticipated that new physics will appear at the LHC. Among all possible models, new physics containing invisible particle(s) at the TeV scale is very well-motivated.

  - WIMP dark matter

  - Precision electroweak constraints: A parity symmetry for new particles can forbid tree-level exchanges of new particles.

- Many popular models belong to this category, including Supersymmetry, Universal Extra Dimensions, Little Higgs models with T-parity, etc. They all contain a new parity symmetry. As a result, the lightest parity-odd particle is stable, and can be a good dark matter candidate if it's neutral.

Introduction

- At colliders such models all give similar signatures: jets/leptons + missing energy.

- To distinguish different models and to identify the underlying new physics, we need to reconstruct the signal events and measure the properties of the new particles, including masses, spins and couplings. However, with 2 or more missing particles in each event, this is quite challenging:

    - No invariant mass peak if there are missing particles.

    - Most observables are more sensitive to mass differences instead of the overall mass scale.

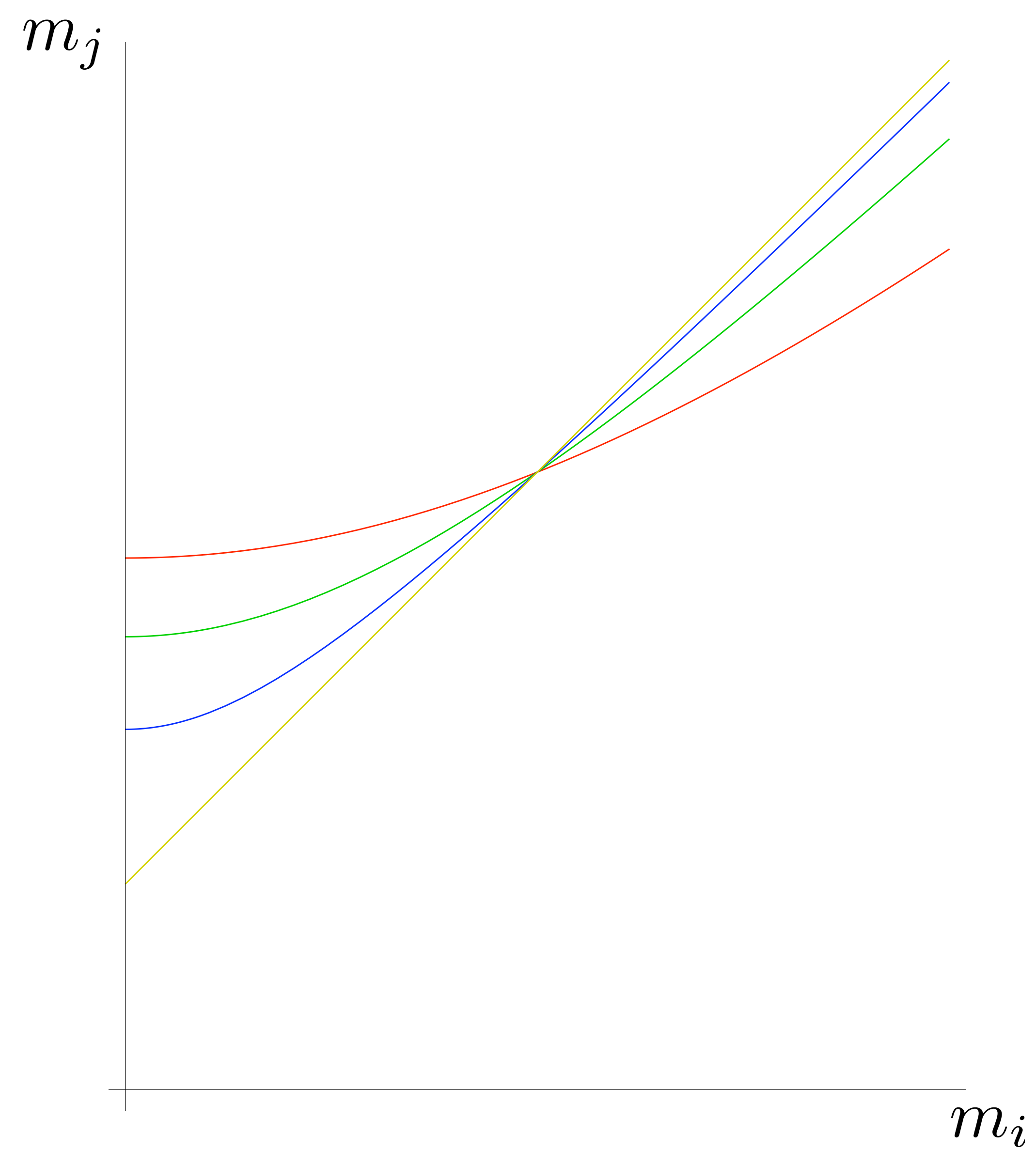
    - Total cross section and the likelihood method are model-dependent. One need to know the model first.

Mass Measurements from kinematics

- Looking for kinematic observables which depend on masses in different ways: For *N* unknown masses, we need *N* or more independent constraints:

$$\mathcal{O}_1(m_i, m_j, \ldots) = f_1(\mathbf{m_i} - \mathbf{m_j}, \ldots; m_i, \ldots),$$
$$\mathcal{O}_2(m_i, m_j, \ldots) = f_2(\mathbf{m_i} - \mathbf{m_j}, \ldots; m_i, \ldots),$$
$$\ldots$$

  - End point/edges of invariant mass distributions.
  - New kinematic variables, e.g., M_{T2}.
  - Kinematic constraints from mass shell conditions.



Mass Measurements from kinematics

- Experimental smearing, backgrounds, and combinatorics are important issues. We should use as many kinematic variables/constraints as possible to over-constrain the system. In particular, we should find variables which
  - can be measured more accurately:
    - large statistics.
    - leptons are better measured than jets.
    - peaks are generally easier to determine than edges/end points (which are more vulnerable to smearing, backgrounds, and poor statistics.)
  - intersect at bigger angles (less correlated).

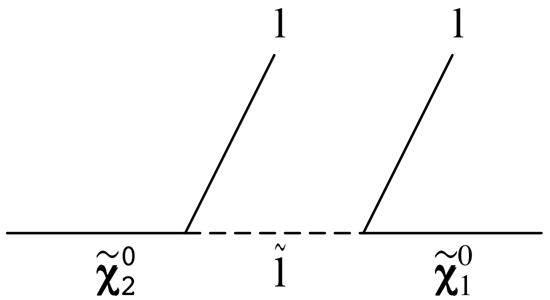
Kinematic Variables and Constraints

- End point of an invariant mass distribution.

  Hinchliffe et al, hep-ph/9610544, and many others

  - Need more than one end points (longer decay chains).
  - Does not use the information that there are 2 chains in each event.
  - The invariant mass can be viewed as a kinematic constraint on the mass parameter space on an event by event basis. The end point defines a boundary between the allowed and the forbidden regions in the mass space.

Example: the dilepton edge.



$$\text{Edge at } M_{ll} = \frac{\sqrt{(M^2_{\tilde{\chi}^0_2} - M^2_{\tilde{l}})(M^2_{\tilde{l}} - M^2_{\tilde{\chi}^0_1})}}{M_{\tilde{l}}}$$

$$\frac{(m^2_{\tilde{\chi}_2} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\tilde{\chi}_1})}{m^2_{\tilde{l}}} \geq m^2_{ll}$$

Kinematic Variables and Constraints

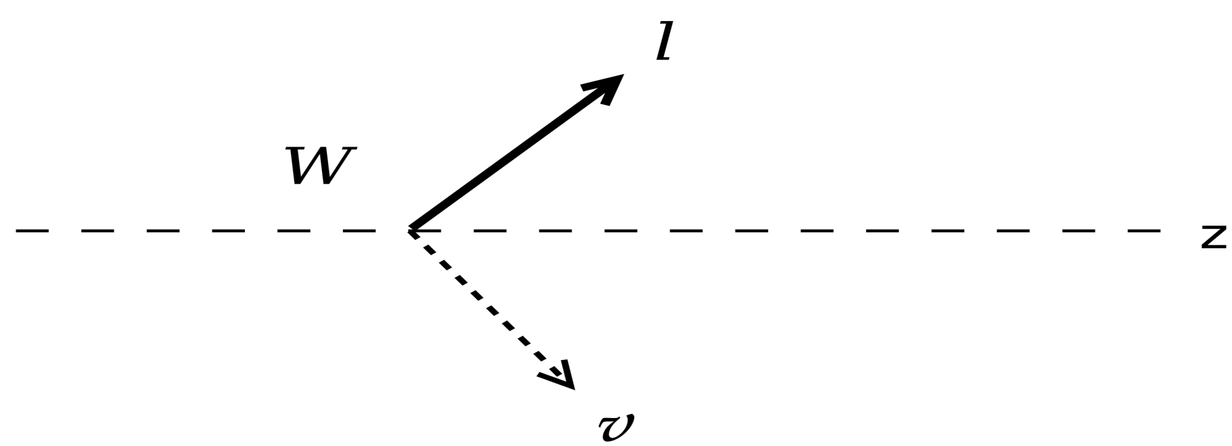
- Constrains/variables from two decay chains: Each event has two decay chains. There is useful information contained in the correlations of the two chains.

  - Not all information can be expressed in terms of Lorentz invariant quantities, as the missing transverse momentum is not Lorentz invariant.

  - It's useful to find kinematic variables/constraints invariant under the allowed subset of transformations: independent longitudinal boosts of the two chains and rotations on the transverse plane [+ back to back transverse boost of the two chains if there is no initial state radiation or upstream transverse momentum (UTM)]. A useful example is the stransverse mass variable M_{T2}. (Lester & Summers, hep-ph/9906349)

Kinematic Variables and Constraints

- Transverse mass M_T:



$$\alpha_\ell = (E_T^\ell, p_x^\ell, p_y^\ell), \quad \alpha_\nu = (E_T^\nu, p_x^\nu, p_y^\nu)$$

$$E_T^\ell = \sqrt{(p_x^\ell)^2 + (p_y^\ell)^2 + m_\ell^2}, \quad E_T^\nu = \sqrt{(p_x^\nu)^2 + (p_y^\nu)^2 + m_\nu^2}.$$

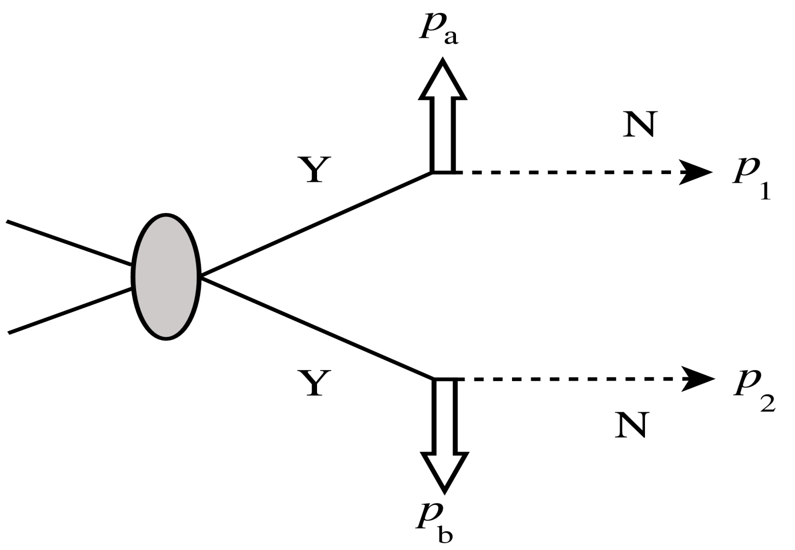
Transverse mass defined by

$$M_T^2 = (\alpha_\ell + \alpha_\nu)^2.$$

The end point of M_T distribution is M_W (for correct m_ν), which happens when *l* and *v* have the same rapidity.

Kinematic Variables and Constraints

- Stransverse mass M_{T2}:

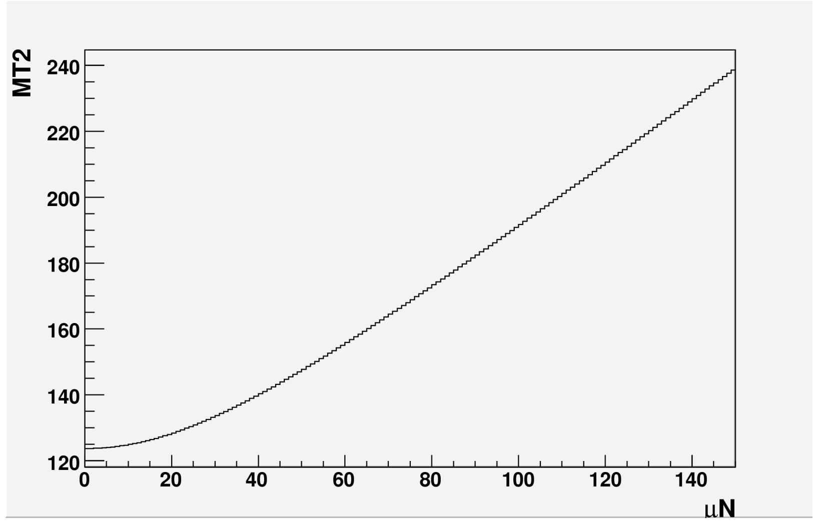


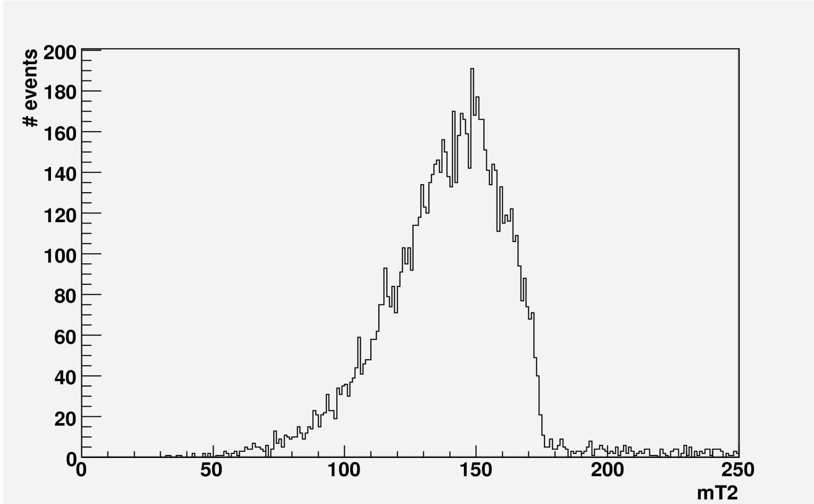
  - Trial N mass, μ_N
  - Consider all partitions of $\not{p}_T = \mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)}$.

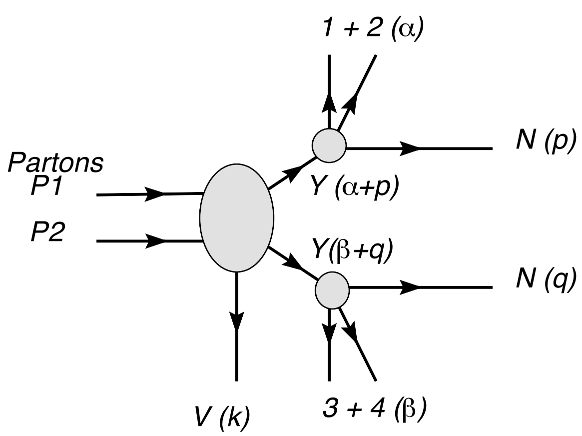
$$M_{T2}(\mu_N) \equiv \min_{\mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)} = \not{\mathbf{p}}_T} \left[\max\{ M_T(1, a; \mu_N), M_T(2, b; \mu_N) \} \right]$$

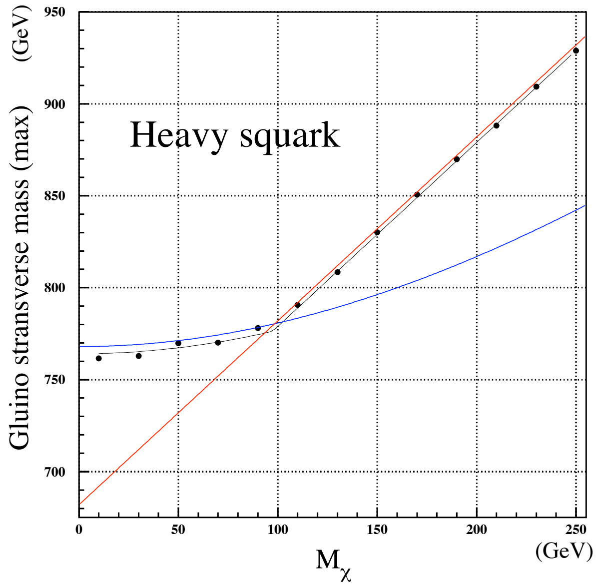
Kinematic Variables and Constraints

- Properties of M_{T2}:
  - A function of the missing particle mass.
  - End point of M_{T2} gives the correct mother particle mass M_Y, for the correct input missing particle mass M_N.
  - When there are 2 or more visible particles on each chain, $M_{T2,max}$ has 2 branches and exhibits a kink at the correct mass point. (Cho, et al, arXiv: 0709.0288, 0711.4526; see also Barr, Gripaios, Lester, 0711.4008)



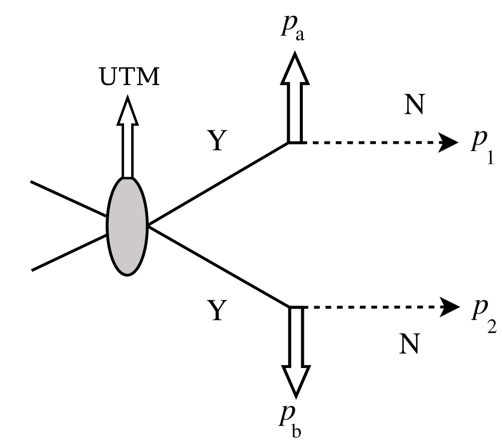




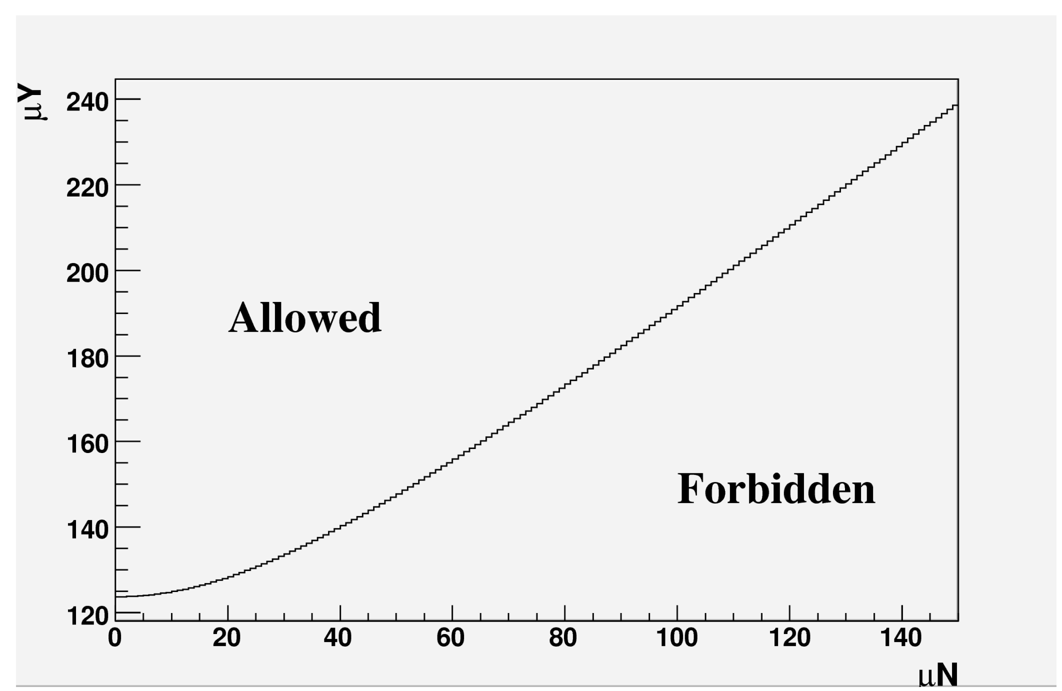


Kinematic Variables and Constraints

- M_{T2} can be understood as the minimal kinematic constraints for a given event: (HC & Z. Han, arXiv:0810.5178)
  - Minimal kinematic constraints: mass shell constraints of the decaying mother particles and the missing particles + missing transverse momentum constraint.
  - $M_{T2}(m_N)$ of a single event is the boundary of the allowed and the forbidden regions in the 2-dim mass space based on the minimal kinematic constraints of that event.

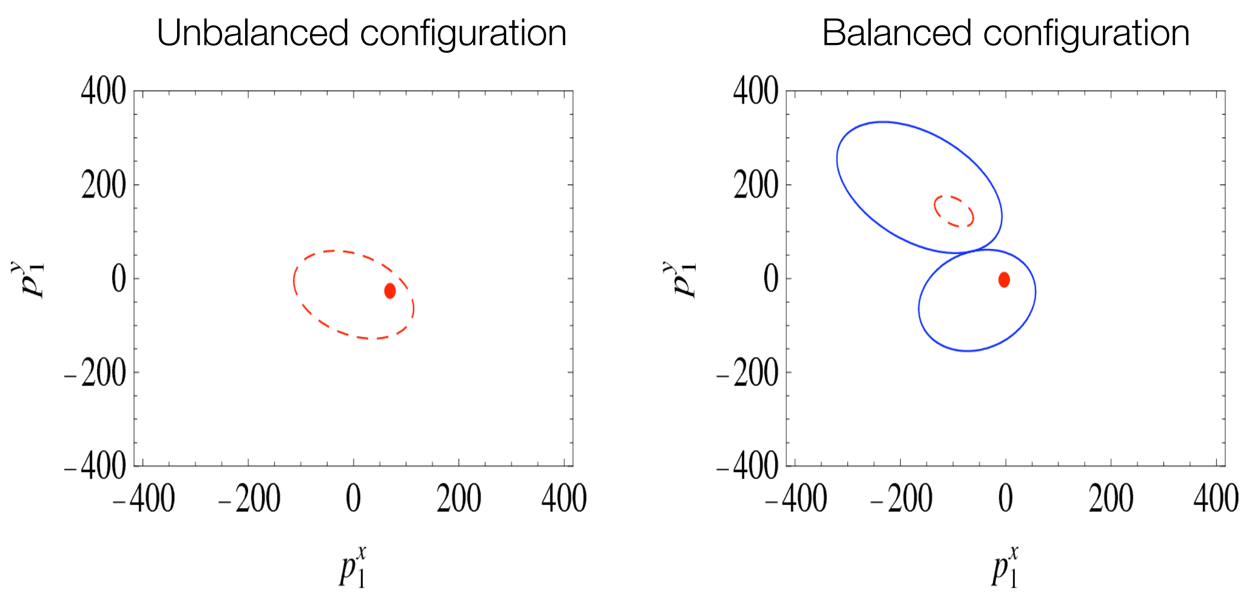


$$
\begin{aligned}
p_1^2 &= p_2^2 = \mu_N^2, \\
(p_1 + p_a)^2 &= (p_2 + p_b)^2 = \mu_Y^2, \\
p_1^x + p_2^x &= \not{p}^x,\ p_1^y + p_2^y = \not{p}^y,
\end{aligned}
$$



Kinematic Variables and Constraints

- Most kinematic variables can be understood as kinematic constraints which separate the allowed and forbidden regions in the mass parameter space. It may provide better ways to calculate certain kinematic variables. For example, a new algorithm to calculate M_{T2} based on solving the minimal kinematic constraints is 5-9 times faster (and more accurate) than the original algorithm based on scanning over all possible divisions of missing momentum.

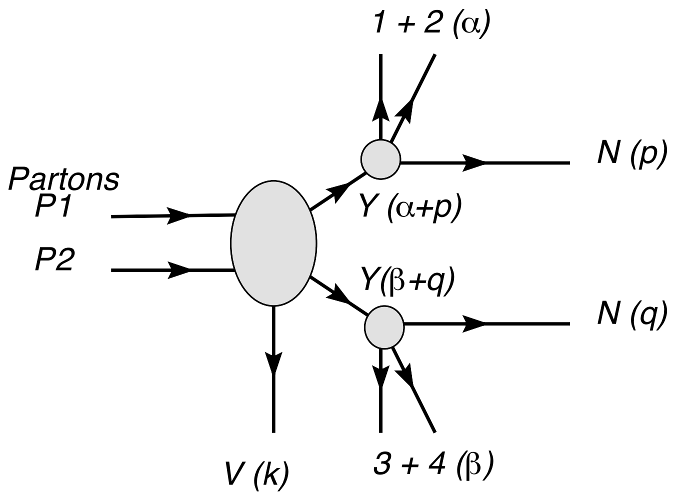


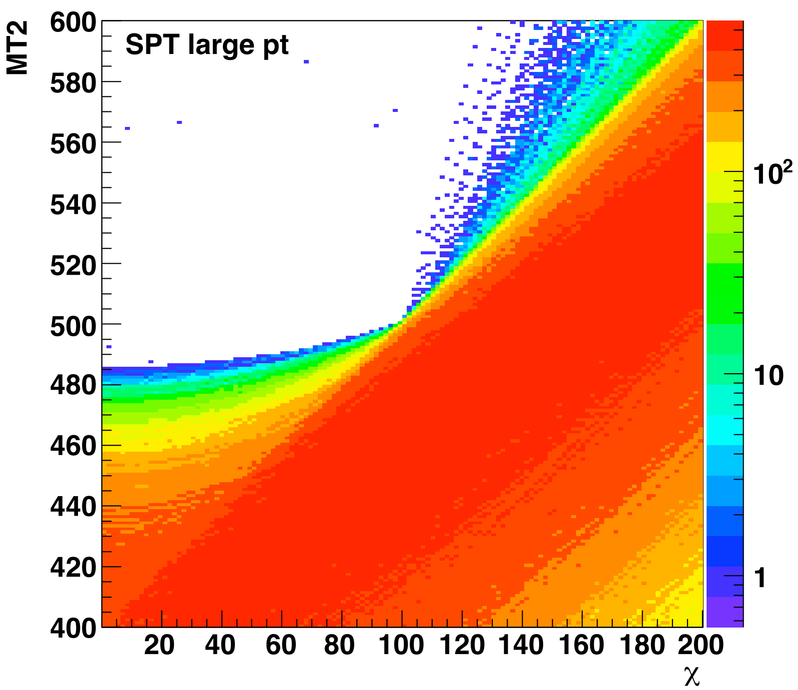
- Examining the allowed region in the mass space consistent with all signal events provide a convenient way to combine constraints from various kinematic variables. The correct mass point lies at the special point (kink) on the boundary of the allowed region.

Mass Determination - One-Step Decay

- For one-step decays (2 on-shell particles) of both chains, M_{T2} and the invariant mass (M_{12} or M_{34}) of all visible particles from a single chain are all the kinematic constraints of the process.

- Correct masses can be in principle determined by the kink of the $M_{T2,max}$. In practice, experimental smearing, backgrounds, and wrong combinations can cause errors in extracting $M_{T2,max}$ for each input missing particle mass M_N, which could make the determination of the kink position difficult.

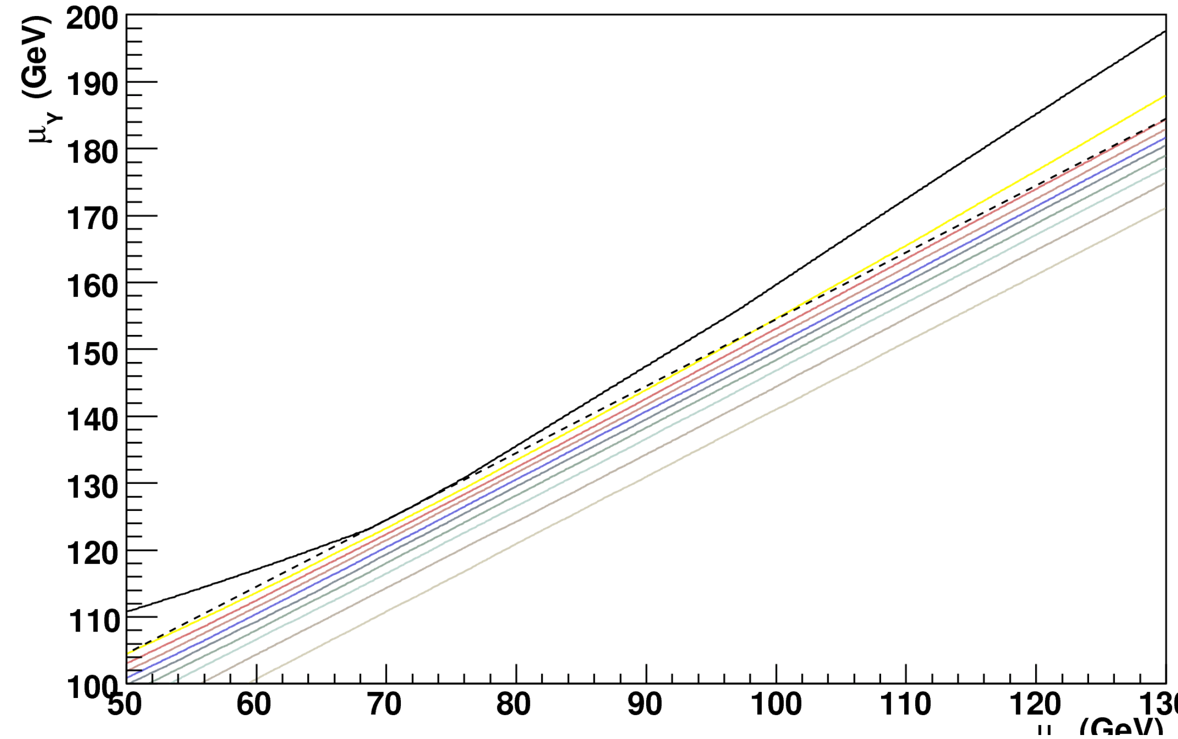




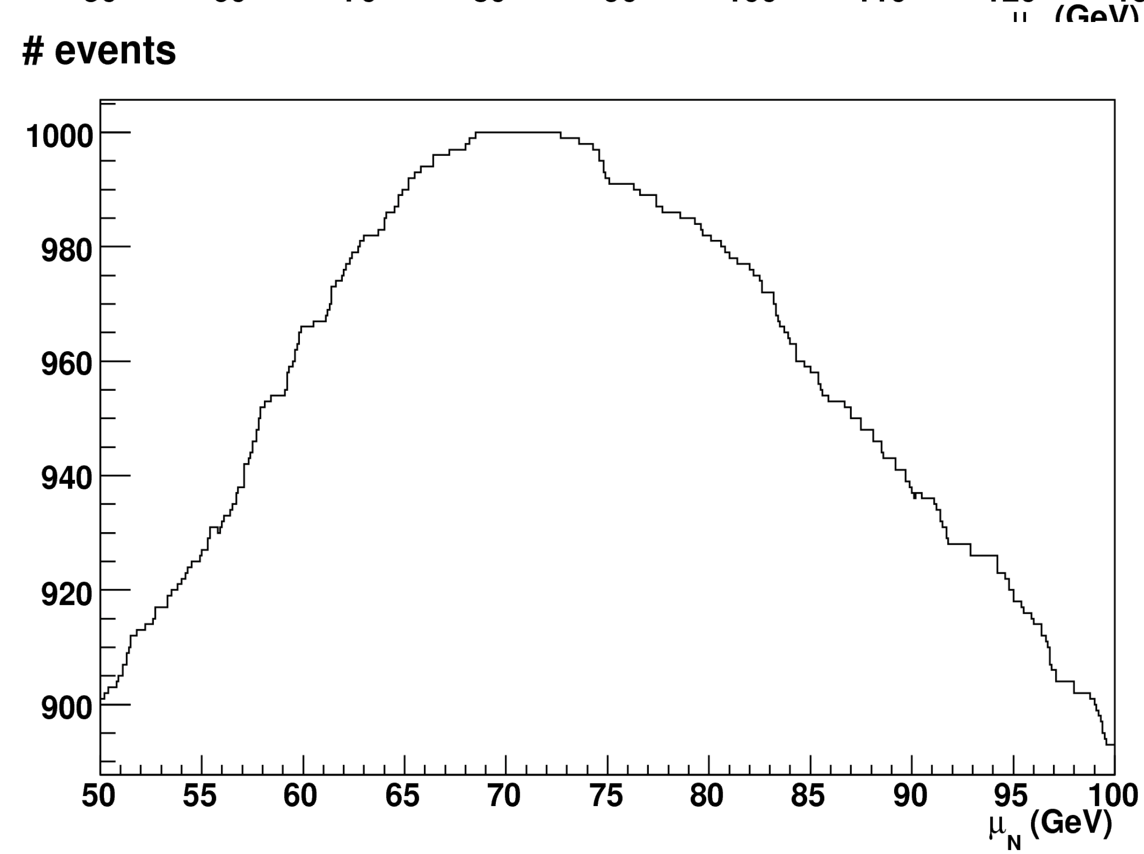
Barr, Gripaios & Lester, arXiv: 0711.4008

Mass Determination - One-Step Decay

- The end point of the invariant mass distribution of one chain (M_{12} or M_{34}) may be easier to determine (only one number, and only one chain is needed). It gives the mass difference of the two new particles, M_Y-M_N.



- One can count the number of solvable events along the constant mass difference line (determined from the end point of the invariant mass). The correct mass is at the maximum due to the kink nature of the kinematic constraint boundary from M_{T2}. (This is equivalent to the M_{2C} variable method by Barr, Ross & Serna, arXiv:0806.3224.)

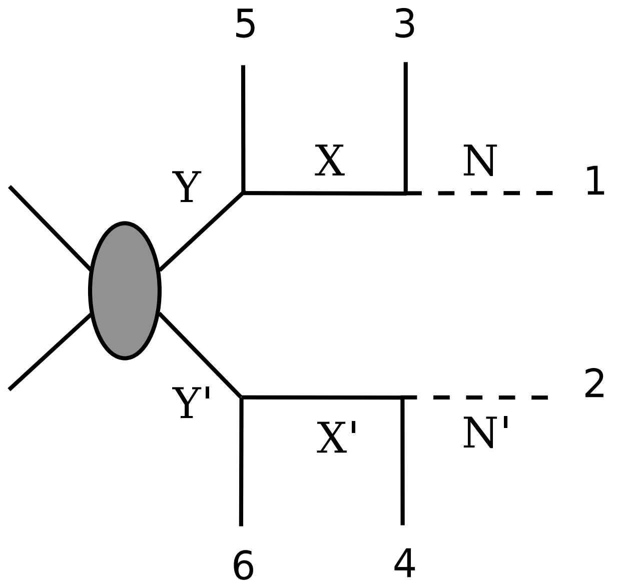


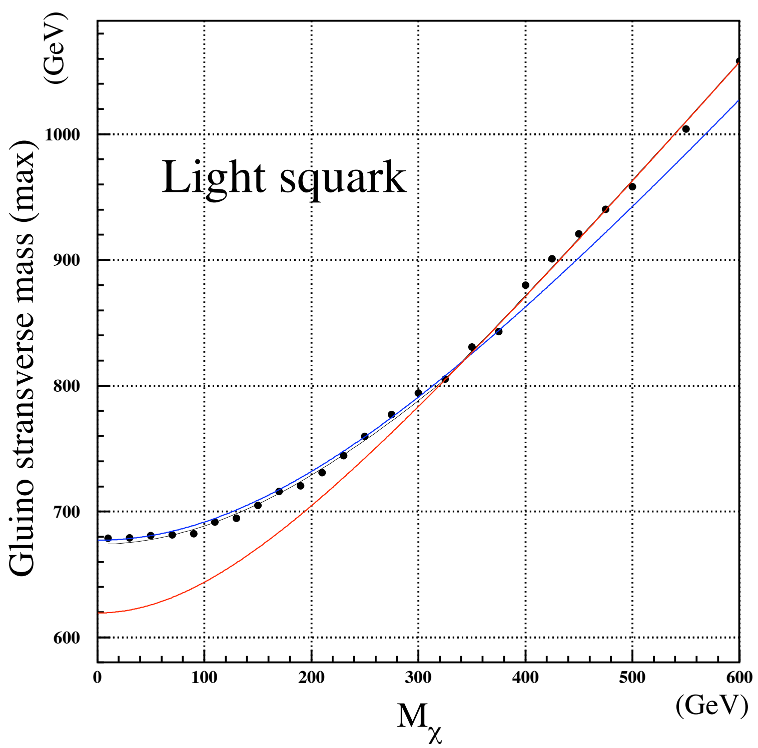
Mass Determination - Two-Step Decay

- There are 3 on-shell new particles if each chain has two-step decays. One more on-shell constraint should allow a better mass determination than the one-step decay case.

- $M_{T2,max}$ still exhibits a kink at the correct mass point. However, the kink is less distinct compared with the case of one-step decays.

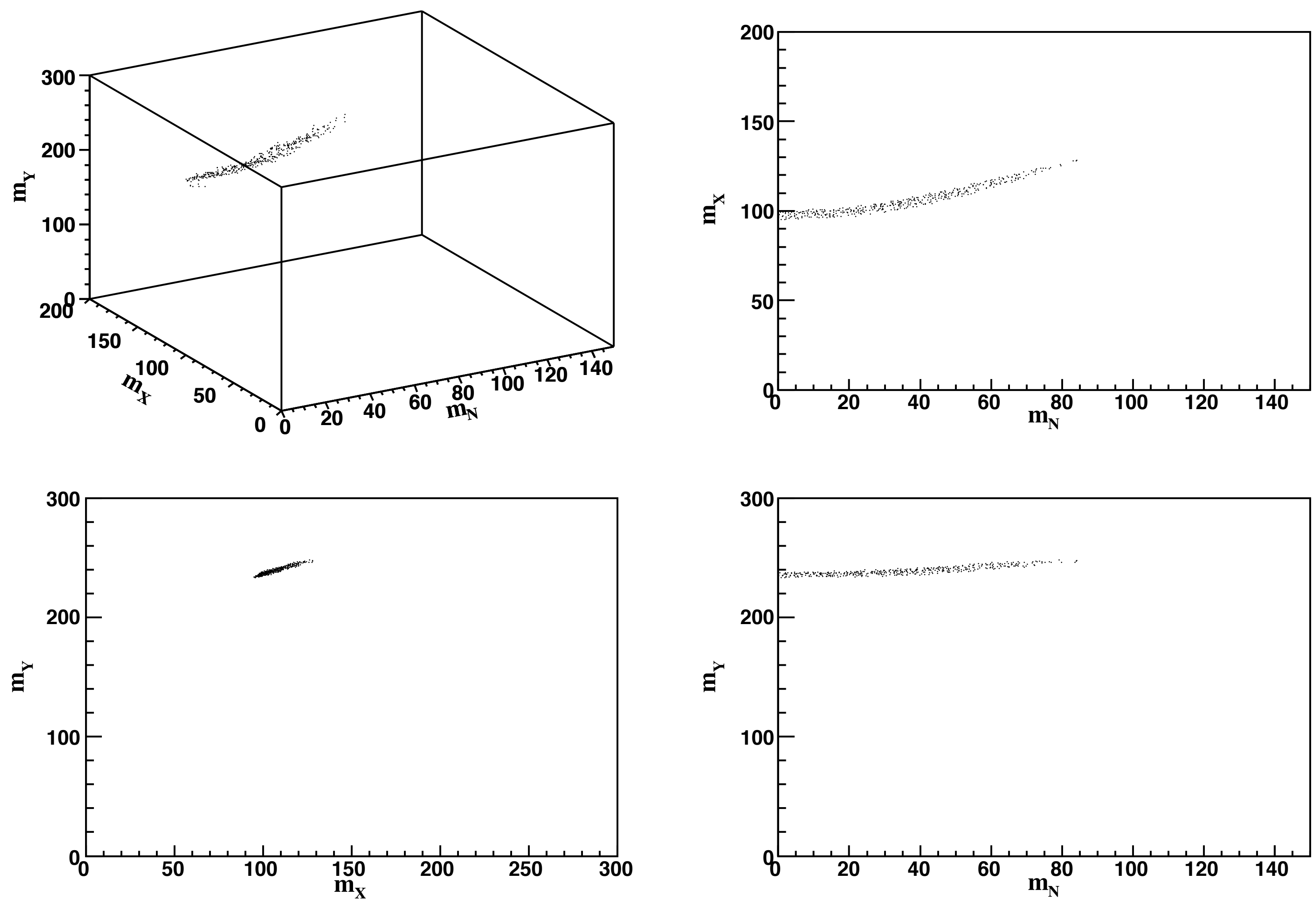
- One can find $M_{T2,max}$ for several subprocesses as considered by Burns, Kong, Matchev, Park, 0810.5576; Barr, Pinder, Serna, 0811.2138. This corresponds to consider several subsets of the kinematic constraints and apply each subset at one time.





Mass Determination - Two-Step Decay

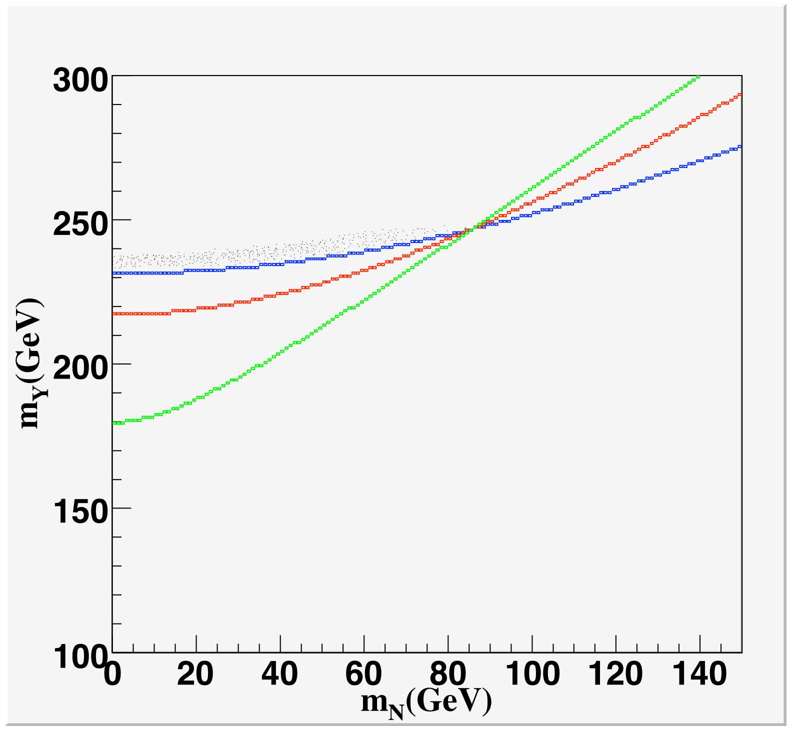
- It should be more powerful to apply all kinematic constrains for each event simultaneously. One then obtains a much more restricted allowed region in the mass parameter space.

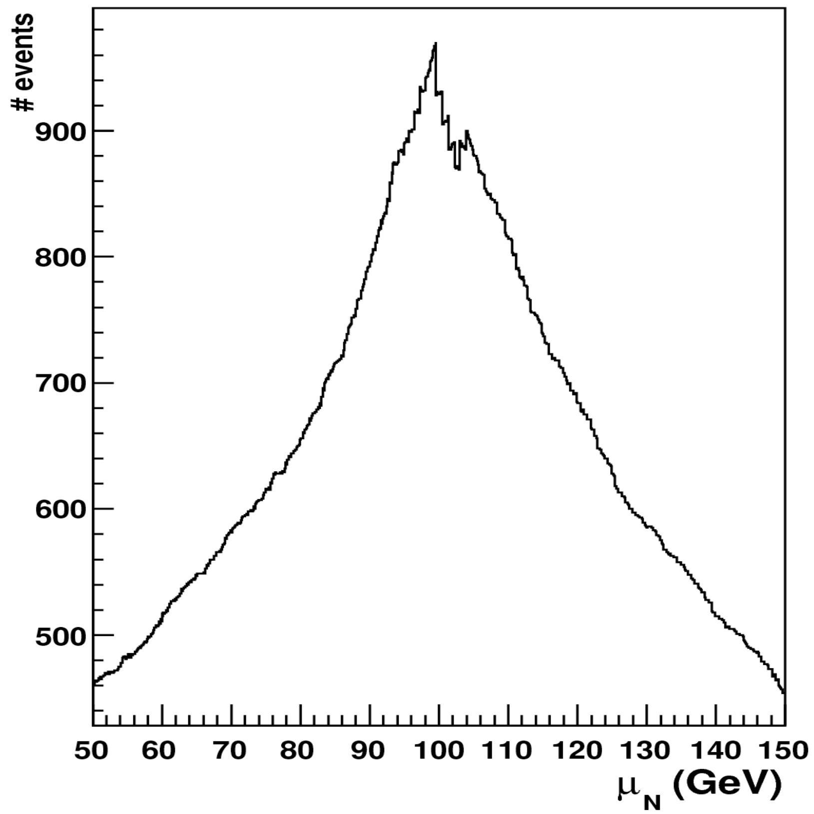


$(m_Y, m_X, m_N)=(246.6, 128.4, 85.3)$ GeV, 500 events, no smearing, correct combination
(HC, Gunion, Han, Marandella, McElrath, arXiv: 0707.0030)

Mass Determination - Two-Step Decay

- The consistent region in the mass space lies along the constant mass-squared differences.
  $$m_Y^2 - m_X^2 = m_{Y,\mathrm{true}}^2 - m_{X,\mathrm{true}}^2$$
  $$m_X^2 - m_N^2 = m_{X,\mathrm{true}}^2 - m_{N,\mathrm{true}}^2$$
- The correct mass point lies at the end point of the consistent region, which can be vulnerable to smearing and backgrounds.
- However, the consistent region alone gives 2 relations among 3 masses. They intersect with the relations from other kinematic variables at bigger angles. By combining other variables such as $M_{T2,\mathrm{max}}$ and the invariant mass end point, we can over-constrain this system and obtain a good determination of masses.

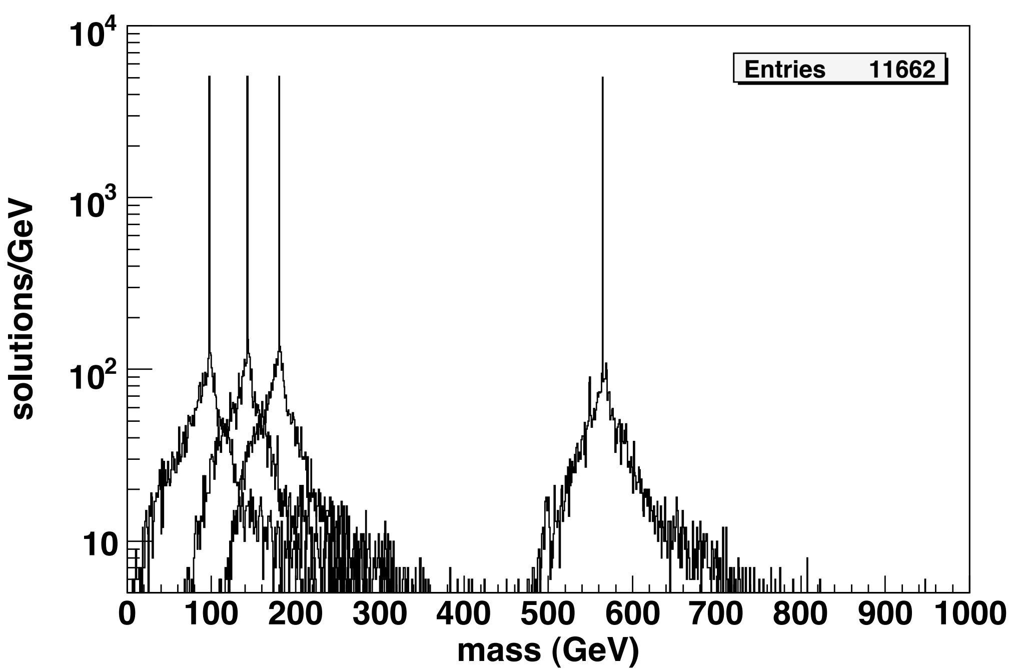




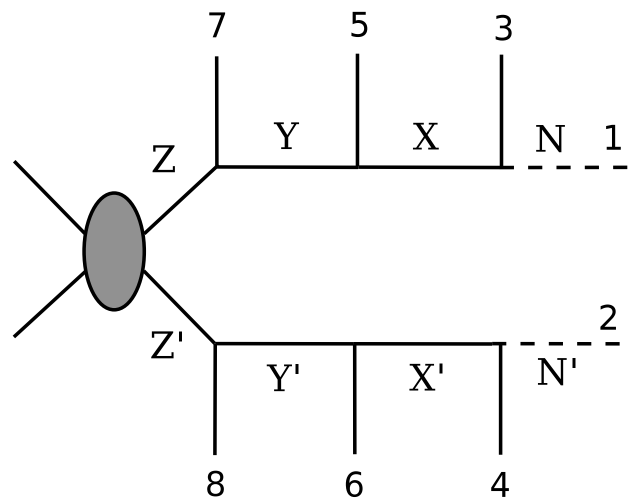
Mass Determination - Three-Step Decay

- With one more mass shell constraint, (up to 8) discrete solutions for the masses can be obtained by just combining two such events. (HC, Engelhardt, Gunion, Han, McElrath, arXiv:0802.4290, 0905.1344)

Example: $\tilde{q}\tilde{q} \to q\tilde{\chi}_2^0 q\tilde{\chi}_2^0 \to q\tilde{l}lq\tilde{l}l \to q\tilde{\chi}_1^0 llq\tilde{\chi}_1^0 ll$
SPS1a, masses: (97.4, 142.5, 180.3, 564.8) GeV



Signals only with correct combinations and no smearing

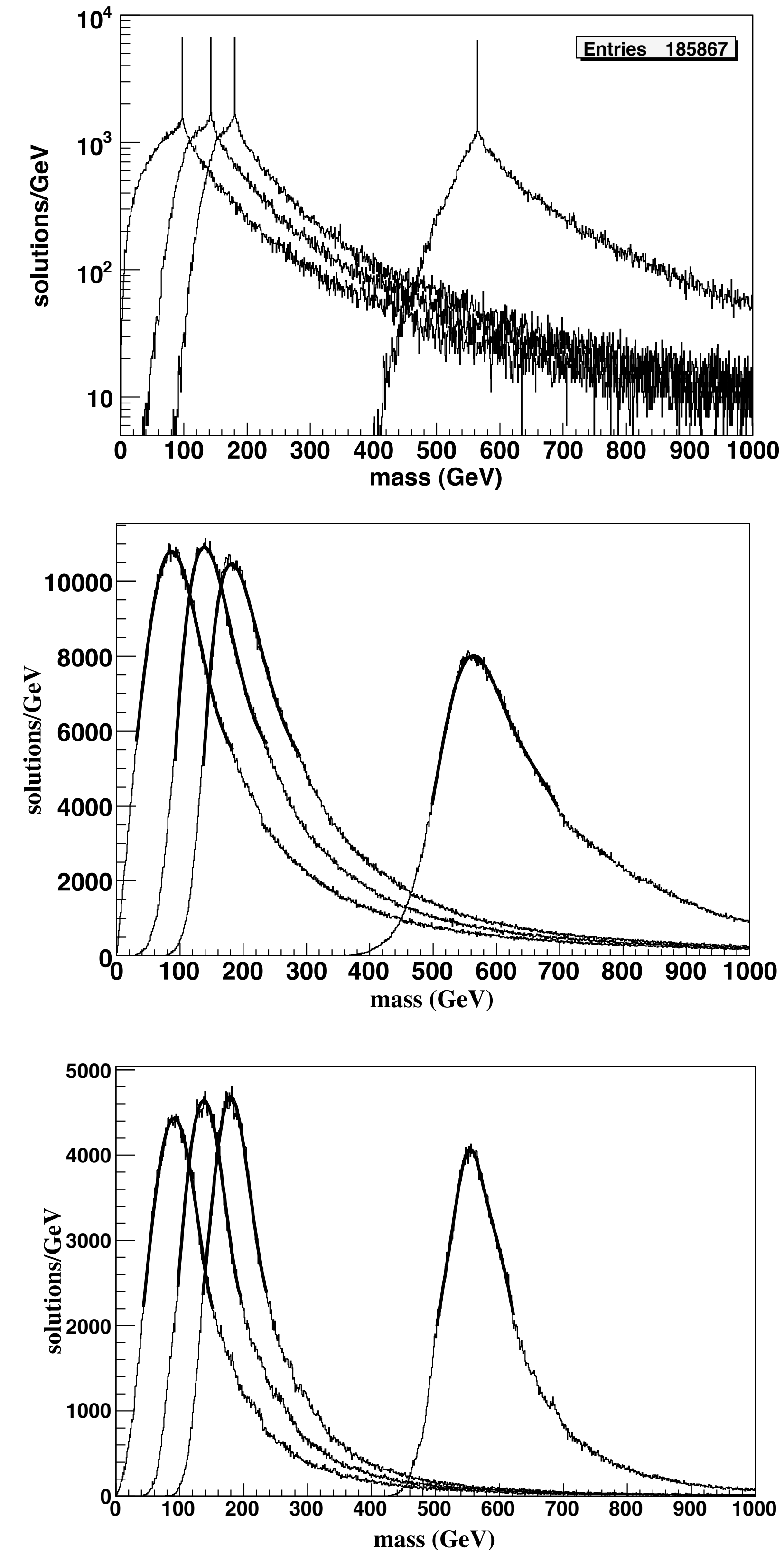


$$p_1^2 = p_2^2$$
$$(p_1 + p_3)^2 = (p_2 + p_4)^2$$
$$(p_1 + p_3 + p_5)^2 = (p_2 + p_4 + p_6)^2$$
$$(p_1 + p_3 + p_5 + p_7)^2 = (p_2 + p_4 + p_6 + p_8)^2$$
$$p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y$$

8 unknowns (4-momenta of 2 missing particles with 6 equations => Each event is consistent a 2-dim surface in the 4-dim mass space.

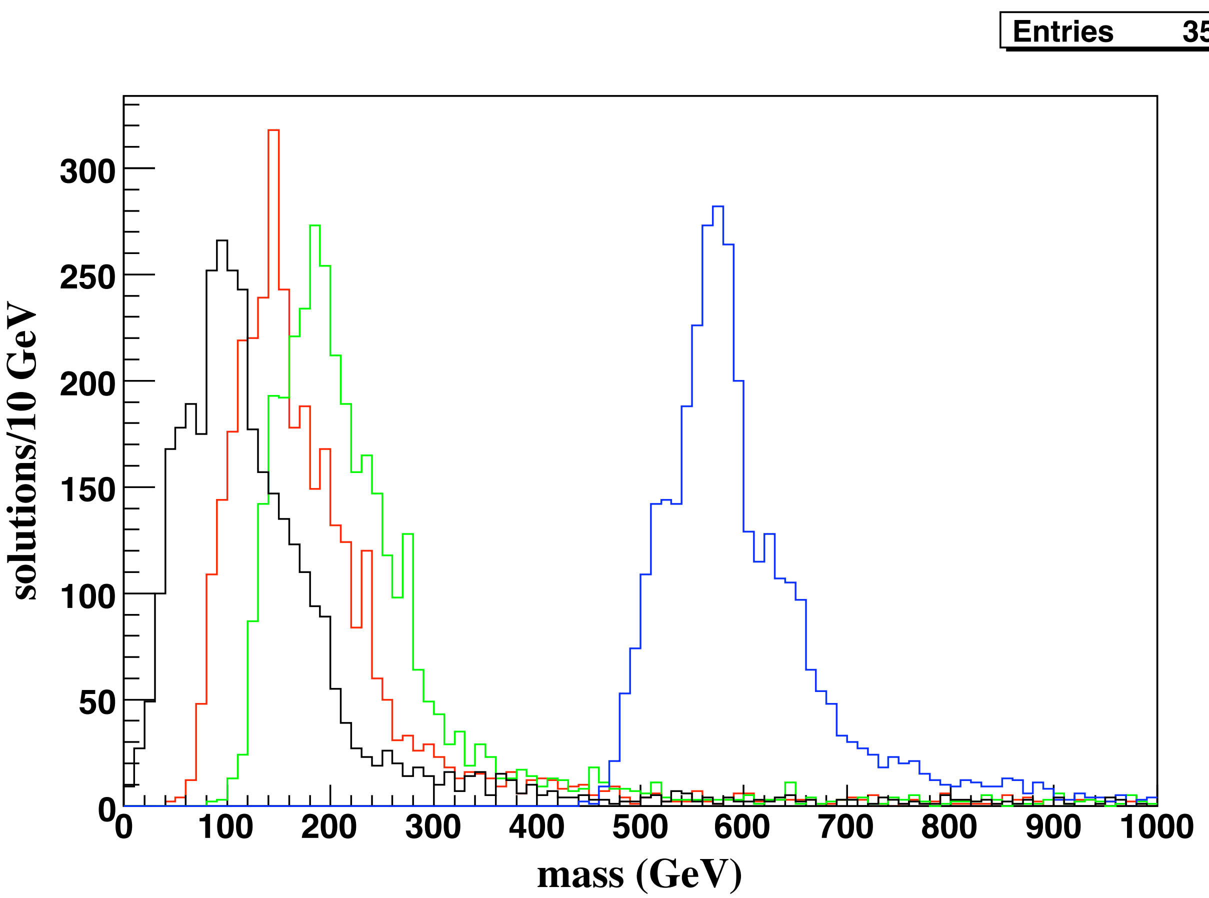
Mass Determination - Three-Step Decay

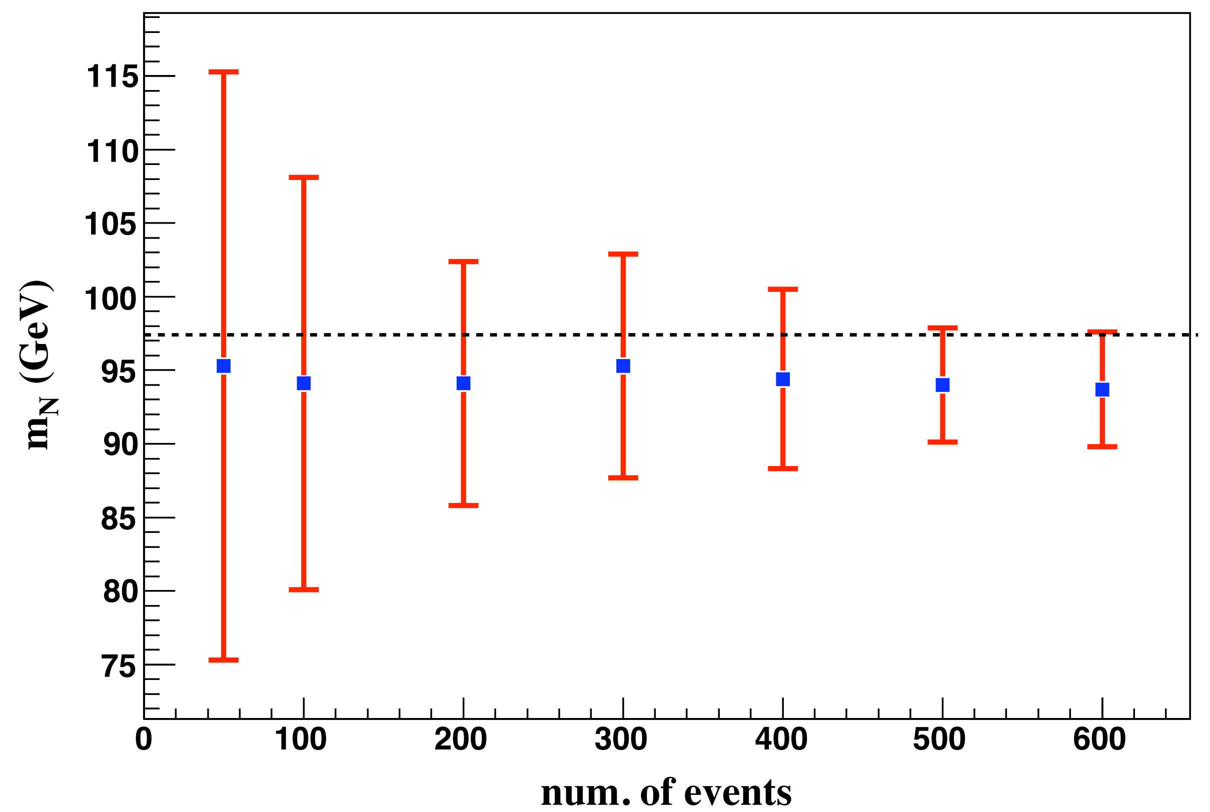
- Wrong combinations are an important issue for accurate mass determinations.
- There are many more wrong solutions than the correct solutions. After smearing, the wrong solutions induce biases of the peak positions.
- Wrong combinations and background events in general can pair with less other events to give valid solutions. Cuts on that and on mass differences reduce wrong solutions significantly.
- A cut on the invariant mass end point is very effective in removing biases without sacrificing much statistics.



Mass Determination - Three-Step Decay

- A reasonable determination of masses can be achieve with very few events, even including the experimental resolutions and combinatorics.





50 events, all possible combinations of visible particles with experimental smearing.

Conclusions

- Significant progress in model-independent mass determination of invisible particles has been made recently. Many new kinematic variables and constraints are found to be useful for such determination.

- Relations among various kinematic variables are better understood now. Constraints on the mass space provide a uniform view of these observables. Best mass determination can be achieved by combining all possible kinematic constraints.

- Once masses are measured, one can attempt to reconstruct the whole kinematics of each event. It can help to determine other properties (e.g., spins and interactions) of the new particles, which are necessary steps towards establishing the underlying new physics.