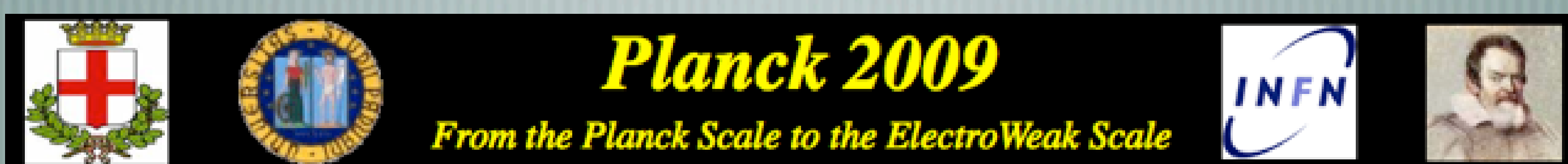
# The Higher Derivative Standard Model

Benjamin Grinstein

Padova, May 25, 2009



# with Donal O'Connell and Mark Wise

## Incomplete list of references

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Phys. Rev. D77:085002,2008 (+ J.R.Espinosa)
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C. Carone & R. Lebed, Phys. Lett.B668: 221-225,2008
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Y-F Cai, T-t Qiu, R. Brandenberger & X-m Zhang, arXiv:0810.4677 [hep-th]
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C. Carone, arXiv:0904.2359 [hep-ph]

EW constraints

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Lee&Wick: Finite Theory Of Quantum Electrodynamics Phys.Rev.D2:1033-1048,1970
Cutkosky et al (CLOP): A Non-Analytic S Matrix Nucl.Phys.B12:281-300,1969
Boulware&Gross: Lee-Wick Indefinite Metric Quantization: A Functional Integral Approach, Nucl.Phys.B233:1,1984
Antoniadis&Tomboulis: Gauge Invariance And Unitarity In Higher Derivative Quantum Gravity,Phys.Rev.D33:2756,1986
Fradkin&Tseytlin: Higher Derivative Quantum Gravity: One Loop Counterterms And Asymptotic Freedom, Nucl.Phys.B201:469-491,1982
Stelle: Renormalization of Higher Derivative Quantum Gravity, Phys. Rev D16:953,1977

Lee, Wick, Coleman, Gross.... not everyone who has worked on this is a crackpot

R. Rattazzi

# Introduction

- Often said:
  symmetry+field content+renormalizability+unitarity = SM
- Higher Derivative (HD) terms:
  - can be made of same fields and preserve symmetries
  - renormalizability preserved
  - unitarity?? Lee-Wick says yes
- Should be decided by experiment

# Summary of main results

- No quadratic divergences (solves "big" fine-tuning problem)
- No flavor problem
- EW precision OK, if mass of new resonances few TeV
- Problematic for GUT ?
- Old: Acausal (or maybe more like "spooky non-locality")
  - One of my main personal motivations:
    Consistent platform to test short distance acausality (in covariant QFT)

# The Price we pay

- Defined with special rules as a perturbation expansion

  Lee & Wick
  CLOP (Cutkowski et al)

- No known non-perturbative formulation

  Boulware & Gross
  (but see VanTonder)

- Not a normal field theory (no spectral decomposition)
- Numbers of particles doubled
- Acausal/non-local on short scales (is this a Price?)

# The Price we pay

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- Acausal/non-local on short scales (is this a Price?)

String theorists should not complain!

# Related: $R^2$ gravity

- Renormalizable
- Asymptotically Free
- But, unitary?

Antoniadis & Tomboulis

Fradkin & Tseytlin

Stelle

# Outline

- Minimalistic presentation of six results:
    - No "big" fine-tuning problem
    - No flavor problem
    - EW precision OK, if mass of new resonances few TeV
    - Renormalization and GUTs
    - High energy vector-vector scattering: the special operators
    - Thermal HD theory and causality
- LW-CLOP unitarity (by simple example)

# The HD SM

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm HD}$$

$$\mathcal{L}_{\rm HD} = \frac{1}{2M_1^2}\left(D^\mu F_{\mu\nu}\right)^a \left(D^\lambda F_\lambda{}^\nu\right)^a - \frac{1}{2M_2^2}\left(D_\mu D^\mu H\right)^\dagger \left(D_\nu D^\nu H\right) - \frac{1}{M_3^2}\bar{\psi}_L (i\not{D})^3 \psi_L$$

(one for each group factor)

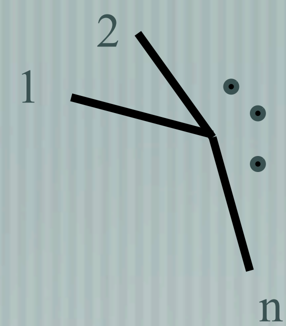
Gauge fixing can be as usual $\mathcal{L}_{\rm GF} = \frac{1}{2\xi}(\partial \cdot A)^2$

or can include HD's, eg,
(convenient for power counting) $\mathcal{L}_{\rm GF} = \frac{1}{2\xi}(\partial \cdot A)(1 + \frac{\partial^2}{M_3^2})(\partial \cdot A)$

# Naive degree of divergence, naively done (but correct!)

propagators $\sim \dfrac{i}{p^2 - p^4/M^2}$

vertices




$\sim p^{6-n} \quad \text{(leading)}$

$$\begin{aligned} L &= \text{\# of loops} \\ V_n &= \text{\# of vertices with n lines} \\ I &= \text{\# of internal propagators} \\ E &= \text{\# of external lines} \end{aligned}$$

naive degree of divergence:

$$D = 4L + \sum_n (6-n)V_n - 4I$$

topological identities

$$L = I - \sum_n V_n + 1 \qquad \sum_n nV_n = 2I + E$$

$$\Rightarrow \boxed{D = 6 - 2L - E}$$

possible divergences:

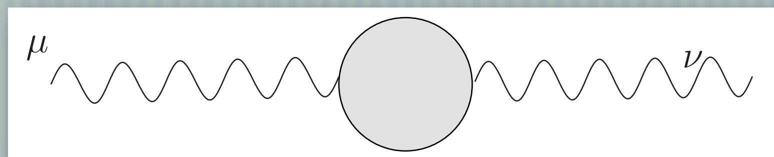
$$D = \begin{cases} 4 - E & L = 1 \\ 2 - E & L = 2 \end{cases}$$

quadratic only for L=1, E=2

Note: renormalizability straightforward

# 1. Quadratic divergences?

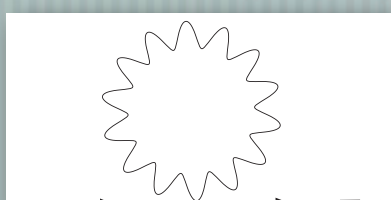
## (i) Gauge fields: gauge invariance decreases divergence to $D = 0$

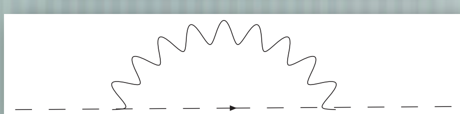



$$= i(p_\mu p_\nu - g_{\mu\nu} p^2)\Pi(p^2)$$

## (ii) Higgs field: quadratic divergence from vertex with two derivatives

$$(D^2H)^\dagger (D^2H) \qquad D^2 H = [\partial^2 + 2igA\cdot\partial + ig(\partial\cdot A)]H$$





Choose gauge $\partial\cdot A = 0$ and integrate by parts: there are at least two derivatives on external field

$$\Rightarrow \boxed{\delta m_H^2 \sim m^2 \ln \Lambda^2}$$

Notes:

1. Physical mass is gauge independent. Quadratic divergences found in unphysical quantities
2. Result checked by explicit calculation (arbitrary $\xi$-gauge)

2. FCNC's

There is no need for artificially imposed restrictions
(ie, no need to impose MFV couplings for the HDs)
nor an additional huge superstructure to deal with this
(like in SUSY with gauge mediation).

This merits more study, only studied superficially so far.

# Notation: SM Yukawas:

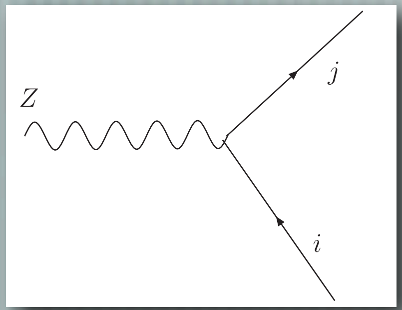
$$\mathcal{L}_{\rm SM} \supset \lambda_U H \bar{q}_L u_R + \lambda_D H^* \bar{q}_L d_R + \lambda_E H^* \bar{\ell}_L e_R$$

# Use EOM on HD terms:

$$\frac{1}{M^2} r_{ij} \bar{q}^i_L (i\not{D})^3 q^j_L = \frac{1}{M^2} (\lambda^\dagger_U r \lambda_U)_{ij}\, \bar{u}^i_R H^* i\not{D}(H u^j_R)$$

completely arbitrary matrix (order(1))

# :: There are off-diagonal tree level Z couplings, but suppressed




$$\sim \delta_{ij} + \Delta_{ij} \qquad \Delta_{ij} \sim \frac{m_i m_j r_{ij}}{M^2}$$

So, for example, with M = 1 TeV

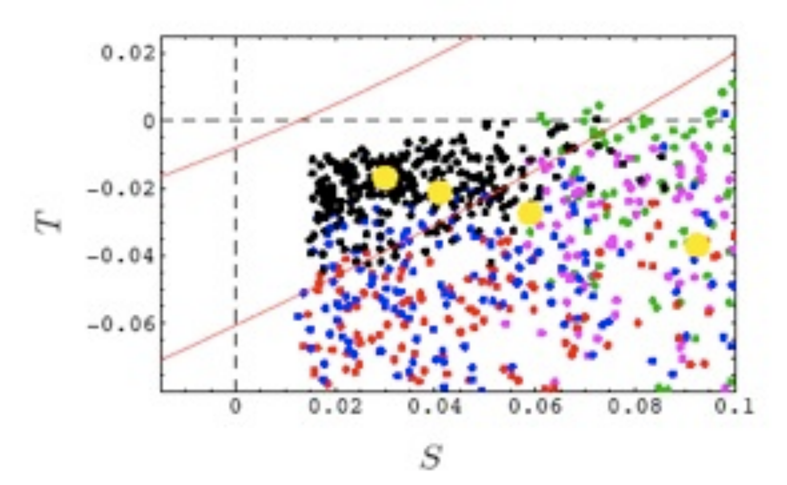
$$\Delta_{bs} \sim \frac{m_b m_s r_{bs}}{M^2} \sim 10^{-6}$$

Even for LFV, this mass suppression is sufficient

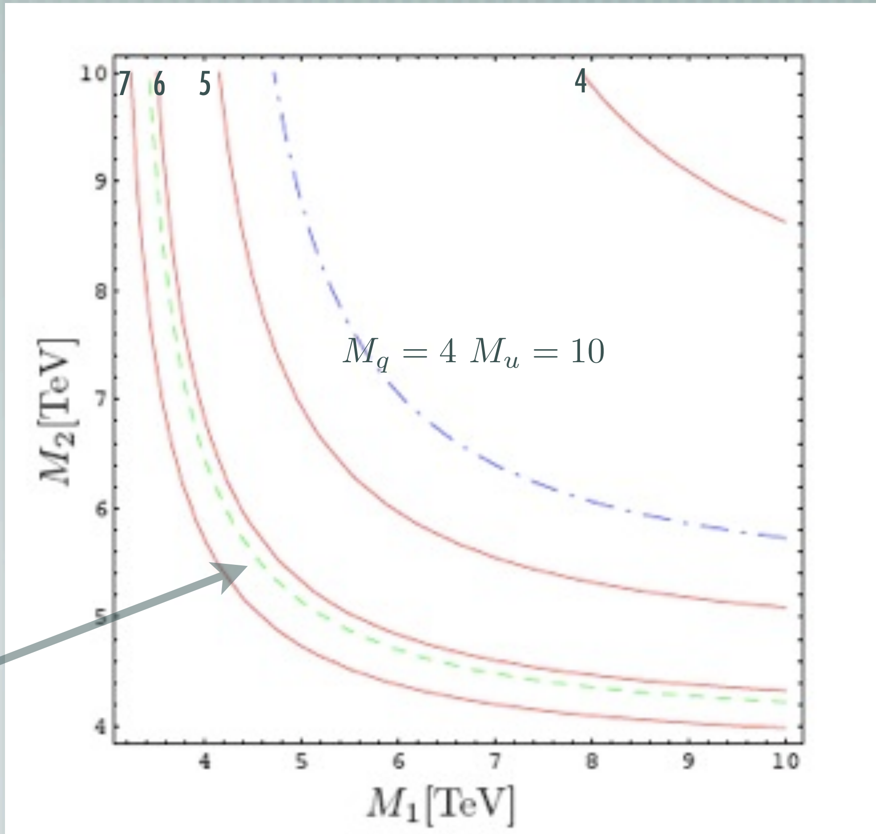
(HD-2HDM at large tan $\beta$ ?  no details)

# 3. EW precision

Alvarez, Da Rold, Schat & Szynkman, JHEP 0804:026,2008
see also: Underwood & Zwicky, Phys. Rev. D79:035016,2009
which disagrees, and
Carone & Lebed, Phys. Lett.B668: 221-225,2008




black: all M over 4TeV, colored: one M <4TeV




(a little detail in back up slides)

# Digression: LW-fields

(useful for beta-function and later for unitarity)

$$\frac{i}{p^2 - p^4/M^2} = \frac{i}{p^2} - \frac{i}{p^2 - M^2}$$

HD is like Pauli-Villars, only taking regulator fields as real, physical.
We call these Lee-Wick (LW) fields (and LW particles or resonances)

Three equivalent Lagrangians:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \hat{\phi})^2 - \frac{1}{2M^2}(\partial^2 \hat{\phi})^2 - V(\hat{\phi})$$

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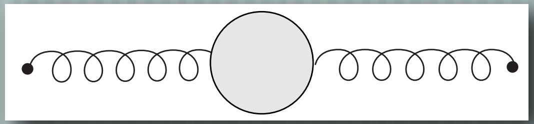
- Wrong metric problem explicit (unitarity... later)
- Looks more familiar, easy to compute
- But improved convergence non-explicit

back on plan:

# 4. YM-beta function

## Background-Field Gauge



1-loop, normal $$\beta = -\frac{g^3}{16\pi^2} C_2 \left( \frac{10}{3} + \frac{1}{3} \right)$$

1-loop, HD$^2$ theory $$\beta = -\frac{g^3}{16\pi^2} C_2 \left( 2 \times \frac{10}{3} + \frac{1}{3} + \frac{1}{6} \right)$$

1/6 is easy to understand: doubling obvious only when longitudinal and transverse modes all have same power counting. Need HD GF. But then get determinant from exponentiation trick:

$$\sqrt{\det(1 + D^2/M^2)} \int [d\alpha] e^{\frac{i}{2\xi} \int d^4x\, \alpha \left(1 + \frac{D^2}{M^2}\right) \alpha} \delta(\partial \cdot A - \alpha)$$

This det is, for UV, same as usual ghosts in BFG. The sqrt gives an additional 1/2

1-loop, HD$^3$ theory $$\beta = -\frac{g^3}{16\pi^2} C_2 \left( 2 \times \frac{10}{3} + \frac{1}{3} + \frac{1}{6} + 1 \right)$$

C. Carone, arXiv:0904.2359

More generally, in HD[2] $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\psi + \mathcal{L}_\phi,$

$$\mathcal{L}_A = -\frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{1}{m^2}\mathrm{Tr}(D^\mu F_{\mu\nu})^2 - \frac{i\gamma g}{m^2}\mathrm{Tr}(F^{\mu\nu}[F_{\mu\lambda}, F_\nu{}^\lambda])$$

$$\mathcal{L}_\psi = \bar{\psi}_L i \not{D}\psi_L + \frac{i}{m^2}\bar{\psi}_L\left[\sigma_1 \not{D}\not{D}\not{D} + \sigma_2 \not{D} D^2 + ig\sigma_3 F^{\mu\nu}\gamma_\nu D_\mu + ig\sigma_4 (D_\mu F^{\mu\nu})\gamma_\nu\right]\psi_L$$

$$\mathcal{L}_\phi = -\phi^* D^2\phi - \frac{1}{m^2}\phi^*\left[\delta_1 (D^2)^2 + ig\delta_2 (D_\mu F^{\mu\nu})D_\nu + g^2\delta_3 F^{\mu\nu}F_{\mu\nu}\right]\phi$$

$$\beta(g) = -\frac{g^3}{16\pi^2}\left[\left(\frac{43}{6} - 18\gamma + \frac{9}{2}\gamma^2\right)C_2 - n_\psi\left(\frac{\sigma_1^2 - \sigma_2\sigma_3 + \frac{1}{2}\sigma_3^2}{(\sigma_1+\sigma_2)^2}\right) - n_\phi\left(\frac{\delta_1 + 6\delta_3}{3\delta_1}\right)\right]$$

$$\gamma_\psi(g) = -\frac{g^2}{16\pi^2}\frac{3}{4}C_1\left(\frac{2\sigma_1(2\sigma_2+\sigma_3-2\sigma_4) + \sigma_2(2\sigma_2+2\sigma_3-\sigma_4) - \sigma_3^2 - \sigma_4^2 + \sigma_3\sigma_4}{\sigma_1+\sigma_2}\right)$$

$$\gamma_\phi(g) = -\frac{g^2}{16\pi^2}\frac{3}{8}C_1\left(\frac{8\delta_1^2 - \delta_2^2 - 4\delta_1\delta_2}{\delta_1}\right)$$

$$\mu\frac{\partial\gamma}{\partial\mu} = 0 \qquad \mu\frac{\partial(g^2\sigma_i)}{\partial\mu} = 2(g^2\sigma_i)\gamma_\psi(g) \quad \text{and} \quad \mu\frac{\partial(g^2\delta_i)}{\partial\mu} = 2(g^2\delta_i)\gamma_\phi(g).$$

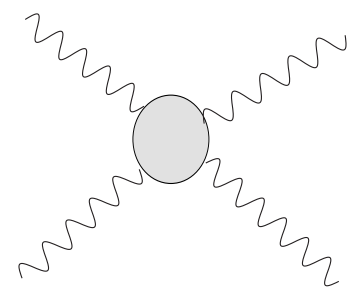
This is for general HD terms, but not all have good high energy behavior (next section)

# GUT (Carone): some fields have $HD^2$, others $HD^3$

| model | $N=3$ fields | $(b_3, b_2, b_1)$ | $\alpha_3^{-1}(m_Z)$ | error |
|---|---|---|---|---|
| SM | - | $(-7, -19/6, 41/10)$ | 14.04 | $+50.8\sigma$ |
| MSSM | - | $(-3, 1, 33/5)$ | 8.55 | $+2.9\sigma$ |
| $N=2$ 1H LWSM | none | $(-19/2, -2, 61/5)$ | 14.03 | $+50.6\sigma$ |
| $N=3$ 1H LWSM | all | $(-9/2, 25/6, 203/10)$ | 13.76 | $+48.3\sigma$ |
| $N=2$ 8H LWSM | none | $(-19/2, 1/3, 68/5)$ | 7.76 | $-4.01\sigma$ |
| $N=3$ 6H LWSM | all | $(-9/2, 20/3, 109/5)$ | 7.85 | $-3.16\sigma$ |
| $N=2$ 1H LWSM, | gluons | $(-25/2, -2, 61/5)$ | 7.81 | $-3.55\sigma$ |
| $N=2$ 1H LWSM | gluons, 1 gen. quarks | $(-59/6, 0, 41/3)$ | 8.40 | $+1.55\sigma$ |
| $N=2$ 1H LWSM | 1 gen. LH fields | $(-49/6, 2/3, 191/15)$ | 8.03 | $-1.66\sigma$ |
| $N=2$ 2H LWSM | LH leptons | $(-19/2, 1/3, 68/5)$ | 7.76 | $-4.01\sigma$ |
| $N=2$ 2H LWSM | gluons, quarks, 1H | $(-9/2, 9/2, 169/10)$ | 8.21 | $-0.06\sigma$ |

TABLE I: Predictions for $\alpha_3^{-1}(m_Z)$ assuming one-loop unification. The experimental value is $8.2169 \pm 0.1148$ [10]. The abbreviations used are as follows: H=Higgs doublets, gen.=generation, LH=left handed.

but $M_{GUT}$ low, proton decay a problem. Fermions at orbifold fixed points in Higher-dim's where wave-function vanishes?



# 5. V V -scattering and special HD terms

Consider VV-scattering, first in non-HD case:

- if described by massive vector boson lagrangian, $\mathcal{A} \sim E^2 \quad E >> m$

  unitarity violated (perturbatively)

- growth could be E$^4$, $\epsilon_L^\mu(p) = 1/M(p, 0, 0, E)$

but approximate GI at large E reduces growth by E$^2$, since $\epsilon_L^\mu(p) = p^\mu/M + (M/2E)n^\mu$

$(n^2 = 0)$

- HD:

  + GI is maintained, exact ward identities

  + Use LW-form: amplitude has no inverse powers of M

$$\Rightarrow \quad \mathcal{A} \sim E^0$$

Unacceptable growth is controlled by GI and absence of 1/M terms in lagrangian.

- HD with no LW formulation, eg, F$^3$ , does have E$^2$ growth at tree level (verified by explicit calculation)

# 6. Thermal HD theory

For unitarity need LW-CLOP prescription.

No Green functions, only S-matrix

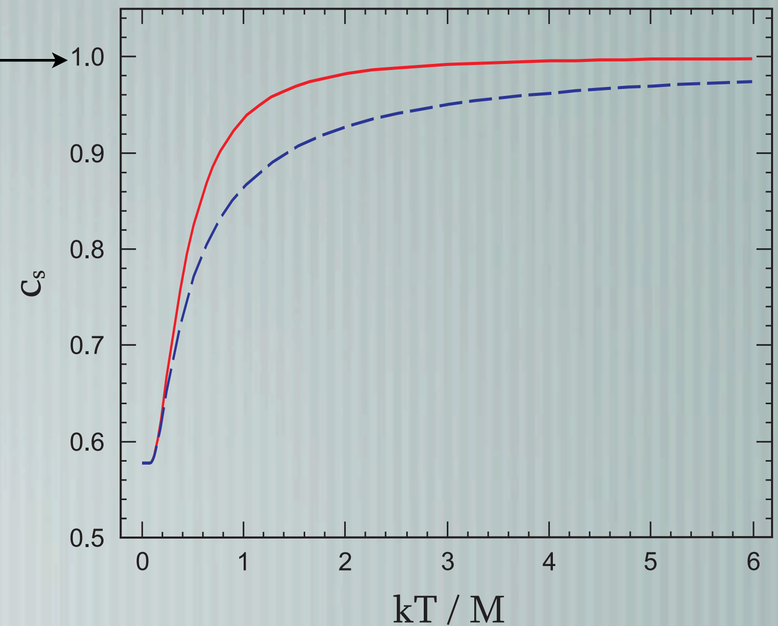
Thermodynamics from S-matrix possible (Dashen, Ma & Bernstein, Phys. Rev. 187: 345, 1969)

At weak coupling scalar HD theory gives extra contribution to thermodynamic potential:

$$\Omega_{\mathrm{LW}} = -\frac{V}{\beta}\int \frac{\mathrm{d}^3P}{(2\pi)^3}\ln\left(1 - e^{-\beta\sqrt{M^2+\mathbf{P}^2}}\right)$$

Hence negative density and negative pressure

causal limit




speed of sound $c_s = \sqrt{dp/d\rho}$

High T expansions:

$$\rho_{\mathrm{LW}} = -\left[\frac{\pi^2(kT)^4}{30} - \frac{M^2(kT)^2}{24}\right] + \dots$$

Cancel normal

Equal and positive e.o.s.: $w = 1$

$$p_{\mathrm{LW}} = -\left[\frac{\pi^2(kT)^4}{90} - \frac{M^2(kT)^2}{24} + \frac{M^3(kT)}{12\pi}\right] + \dots$$

# Unitarity

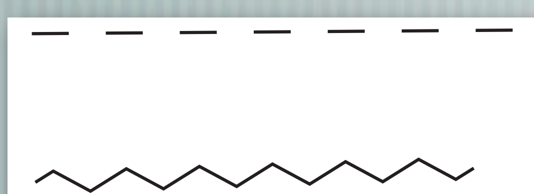
To explain basic ideas of LW-CLOP prescription we will consider only toy model for simplicity: $g\phi^3$

Recall, equivalent lagrangians

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \hat{\phi})^2 - \frac{1}{2M^2}(\partial^2 \hat{\phi})^2 - V(\hat{\phi})$$

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}M^2\chi^2 - V(\phi - \chi)$$

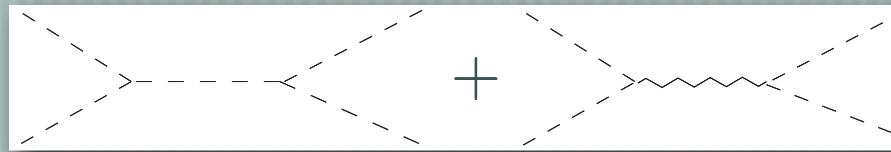
$$g\phi^3 \rightarrow g(\phi - \chi)^3$$



$$\frac{i}{p^2 - m^2}$$

$$\frac{-i}{p^2 - M^2}$$

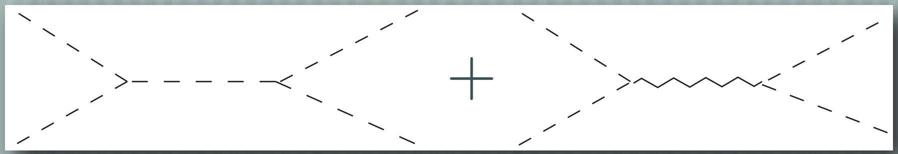
Scattering:



$$= -ig^2 \left( \frac{1}{p^2 - m^2} - \frac{1}{p^2 - M^2} \right)$$

$$\Rightarrow \quad \mathrm{Im}\, \mathcal{A}_{\mathrm{fwd}} = \pi g^2 \left[ \delta(p^2 - m^2) - \delta(p^2 - M^2) \right]$$

This is a disaster: optical theorem is violated $\quad \mathrm{Im}\, \mathcal{A}_{\mathrm{fwd}} = \pi \sqrt{s(s - 4m^2)}\sigma_T > 0$

Scattering:  $= -ig^2\left(\frac{1}{p^2-m^2} - \frac{1}{p^2-M^2}\right)$

$$\Rightarrow \quad \mathrm{Im}\,\mathcal{A}_{\mathrm{fwd}} = \pi g^2\left[\delta(p^2-m^2) - \delta(p^2-M^2)\right]$$

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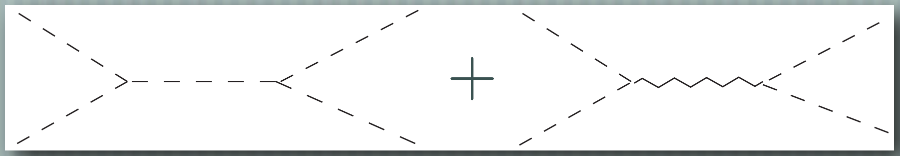
Lee & Wick observed that the ghost is unstable, not an asymptotic state; projecting the S-matrix onto the subspace of positive norm states ought to give a unitary S matrix

Here is how...

Scattering:



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---

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Here is how...

But first, here is how not:

-not by switching sign of ghost propagator (would loose convergence properties)

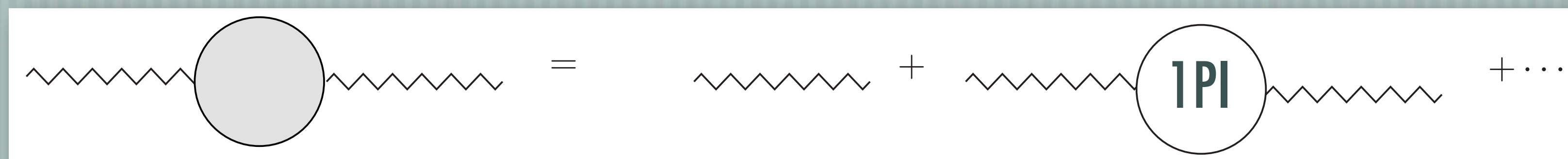
-not by turning off imaginary part of ghost (would not satisfy optical theorem still)

Reorganize perturbation theory (old school, resonances):

(i) Replace all propagators by dressed propagators (old well known way to deal with resonances)

(ii) Define amplitude by analytic continuation from positive and large Im($p^2$)

(iii) Choose rules for contours of energy integrals as necessary




$iG^{(2)}$ $\qquad$ $i\Delta$ $\qquad$ $i\Delta \quad i\Pi \quad i\Delta$

$$\Rightarrow iG^{(2)} = \frac{i}{\Delta^{-1} + \Pi}$$

very familiar, but now use $\quad i\Delta = \dfrac{-i}{p^2 - M^2} \quad$ to get the surprising

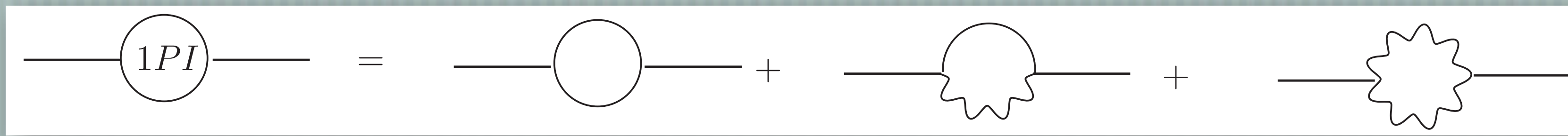
$$iG^{(2)} = \frac{-i}{p^2 - M^2 - \Pi}$$

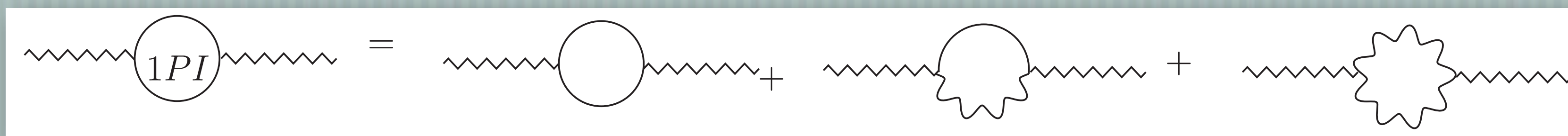
Compare this with normal case:

$$iG^{(2)} = \frac{i}{p^2 - m^2 + \Pi}$$

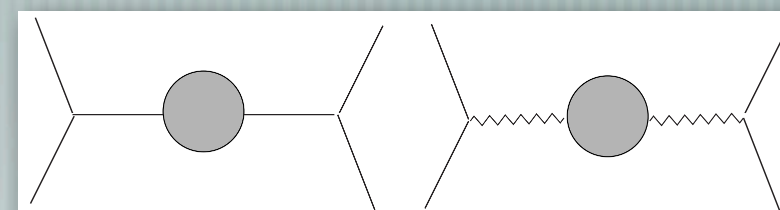
consequences on next page

## $\Pi$ itself is very "normal," it is the same for normal and LW fields:

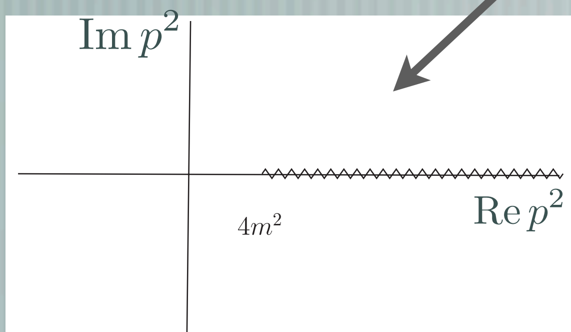


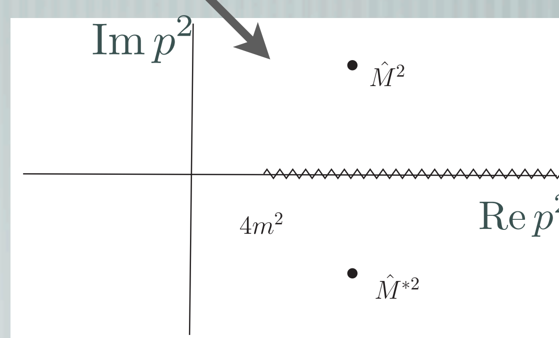




## But with this sign we have a pole in the scattering amplitude.



$$iA = -ig^2\left[\frac{1}{p^2 - m^2 + \Pi} - \frac{1}{p^2 - M^2 - \Pi}\right]$$




so in fact, the LW propagator is

$$G^{(2)} = -\frac{A}{p^2 - \hat{M}^2} - \frac{A^*}{p^2 - \hat{M}^{*2}} + \int_{4m^2}^{\infty} d\mu^2\, \frac{\rho(\mu^2)}{p^2 - \mu^2}$$

$$G^{(2)} = -\frac{A}{p^2 - \hat{M}^2} - \frac{A^*}{p^2 - \hat{M}^{*2}} + \int_{4m^2}^{\infty} d\mu^2 \, \frac{\rho(\mu^2)}{p^2 - \mu^2}$$

properties:

$$\rho(\mu^2) \geq 0 \qquad\qquad -A - A^* + \int d\mu^2 \rho(\mu^2) = -1$$

NOW compute imaginary part of forward amplitude:
the complex dipole cancels out and we end up with

$$\mathrm{Im}\, \mathcal{A}_{\mathrm{fwd}} = \pi g^2 \left[\rho_{\mathrm{normal}}(\mu^2) + \rho_{\mathrm{LW}}(\mu^2)\right]$$

This is a positive discontinuity.
You can see it is precisely the total cross section (to the order we have carried this out)

How to compute loops with internal LW propagators? Off real axis poles, so how to Wick rotate? LW-CLOP prescription (that's another talk)

How about all orders?

- Lee & Wick made general arguments, but not a proof

- Cutkosky et al (CLOP) analyzed analytic structure (particularly including the so far ignored two intermediate LW lines case) of large classes of complicated graphs

-Tomboulis solved N spinors coupled to Einstein-gravity. At large N the fermion determinant gives HD gravity and shows explicitly theory remains unitary (no need to use LW-CLOP)

- We have solved the O(N) model in large N limit. The width or LW resonance is $O(1/N)$, so positivity of spectral function easily shown. Hence example exists for which

 i) used LW-CLOP prescription

 ii) unitary shown explicitly (directly checked optical theorem)

# The End

- There exist unitary HD theories (at least large N to all orders g)
- Solves big fine tuning, is flavor OK, EWP fine
- GUT questionable... interesting open questions on UV completion
- Acausal at short distances, but does not build macroscopic acausality (at least not in thermal equilibrium)
- Other applications? Cosmology?

# Extra slides

# 3. EW precision, very rough

Use perturbation theory in HD operators, again because E << M

Then from operator analysis (eff theory; eg, Han and Skiba) know that T and S are, respectively

$$(H^\dagger D_\mu H)^2 \qquad \text{and} \qquad H^\dagger \tau^a W^a_{\mu\nu} H B_{\mu\nu}$$

Neither of these are HD ops, but we generate them using EOM.

$$(DF)_\mu = g(H^\dagger \overleftrightarrow{\partial}_\mu H) \qquad \Rightarrow \qquad \frac{g^2}{M^2}(H^\dagger D_\mu H)^2$$

Bound on boundary of total naturalness:

$$T = -\pi \frac{g_1^2 + g_2^2}{g_2^2} \frac{v^2}{M^2} \qquad \Rightarrow M \gtrsim 3 \text{ TeV}$$

while $\delta m_H^2 \sim \dfrac{g^2}{16\pi^2} M^2 \lesssim m_H^2 \Rightarrow M \lesssim 3 \text{ TeV}$

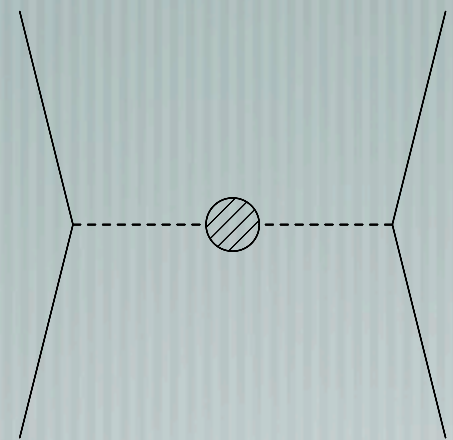
Global analysis constraints M to 3 TeV'ish.

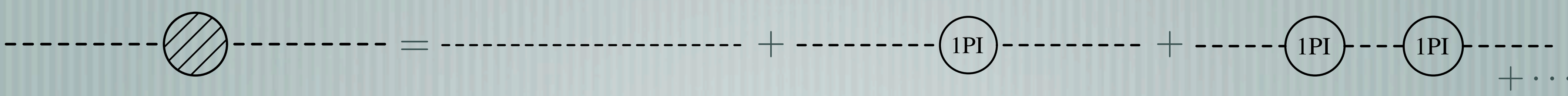
Alvarez, Da Rold, Schat & Szynkman, JHEP 0804:026,2008

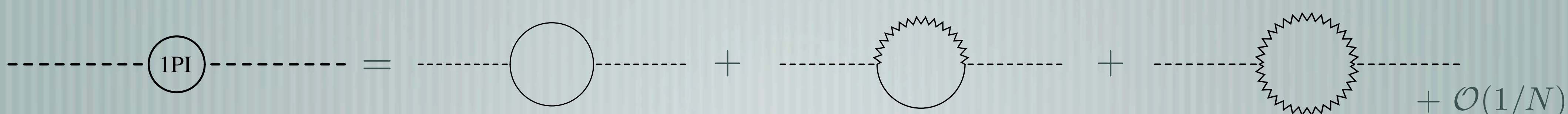
## (ii) $O(N)$ model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 - \frac{1}{2}m^2(\phi^a)^2 - \frac{1}{2}(\partial_\mu \Phi^a)^2 + \frac{1}{2}M^2(\Phi^a)^2 - \frac{1}{8}\lambda[(\phi^a - \Phi^a)^2]^2$$

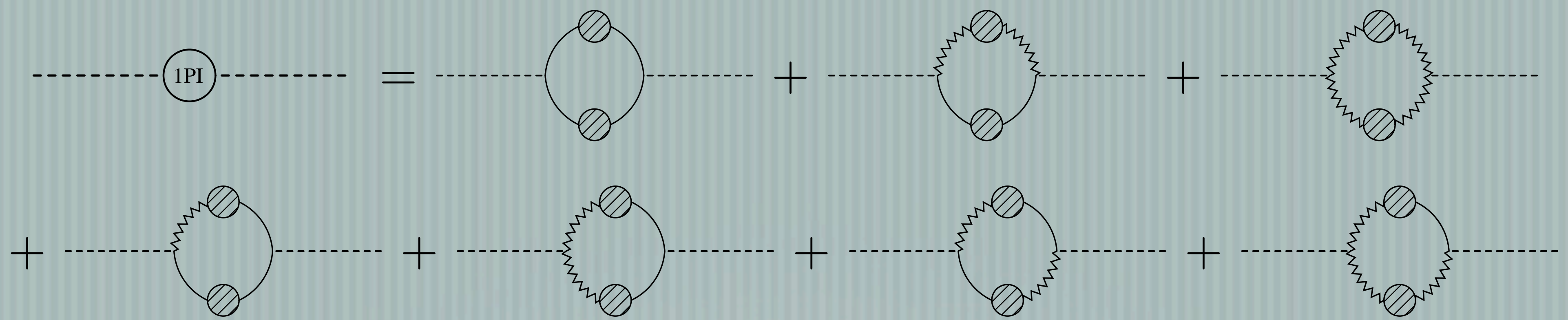
use auxiliary field $\sigma$, $\quad \mathcal{L}_{\rm int} = \frac{1}{2}\sigma^2 + \frac{1}{2}g\sigma(\phi^a - \Phi^a)^2, \quad g^2 N = g_0^2$ fixed

$i\mathcal{A} =$ 




same story as above, this does not satisfy optical theorem, need to dress propagators




but now only Im part of pole need to be kept, $\mathrm{Re}$ is a $1/N$ correction

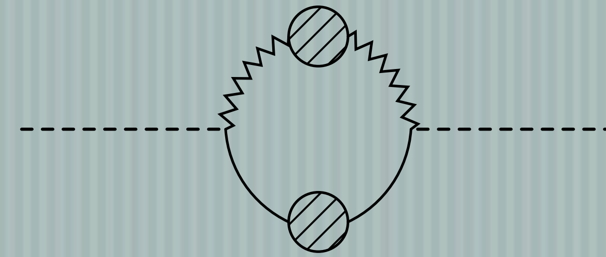
full LW propagator formally as before

$$G^{(2)} = -\frac{A}{p^2 - \hat{M}^2} - \frac{A^*}{p^2 - \hat{M}^{*2}} + \int_{9m^2}^{\infty} d\mu^2 \, \frac{\rho(\mu^2)}{p^2 - \mu^2}$$

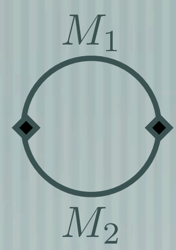
but now $A$=1+$O(1/N)$ and

$$\rho(\mu^2) \approx \frac{1}{\pi} \, \mathrm{Im} \frac{1}{\mu^2 - M^2 - iM\Gamma} \to \delta(\mu^2 - M^2)$$

We can see very explicitly how unitarity works; consider the contribution to the forward scattering amplitude from 1 normal and 1 LW



Let

$i\tilde{\mathcal{I}}(M_1, M_2) =$ 

defined with $p^0$ integral along the imaginary axis**

3 terms in LW propagator: $\mathcal{I} = -A\tilde{\mathcal{I}}(m, \hat{M}) - A^*\tilde{\mathcal{I}}(m, \hat{M}^*) + \int_{(3m)^2}^{\infty} d\mu^2\, \rho(\mu^2)\tilde{\mathcal{I}}(m, \mu)$

$$\mathrm{Im}(\mathcal{A}) = \frac{g^4 N}{16\pi} \frac{1}{|1+\Pi_\sigma(s)|^2} \int_{(3m)^2}^{\infty} d\mu^2\, \rho(\mu^2) I(s, m, \mu)$$

where $\mathrm{Im}(\tilde{\mathcal{I}}(m, \mu)) = \pi I(m, \mu)$ is the usual phase space factor

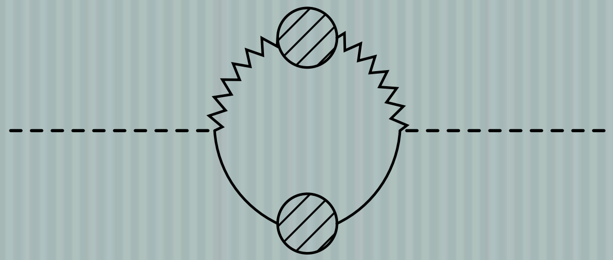
---

Replacing $\rho(\mu^2) \to \delta(\mu^2 - M^2)$ satisfies *exactly* the optical theorem

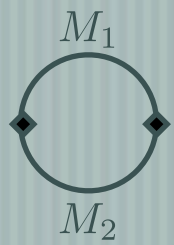
$$\sigma(\phi\phi \to \phi\Phi) = \frac{1}{\sqrt{s(s-4m^2)}} \left( \frac{g^4 N}{16\pi} \frac{1}{|1+\Pi_\sigma(s)|^2} \right) I(s, m, M) \qquad (\text{“}\Phi\text{”} = 3\phi)$$

**subtleties @ dinner tonight after wine

Physically:



recall

$i\tilde{\mathcal{I}}(M_1, M_2) =$ 

is a function of $p^2 = 4E^2$ (in CM frame)

$$\mathcal{I} = -A\tilde{\mathcal{I}}(m, \hat{M}) - A^*\tilde{\mathcal{I}}(m, \hat{M}^*) + \int_{(3m)^2}^{\infty} d\mu^2\, \rho(\mu^2)\tilde{\mathcal{I}}(m, \mu)$$

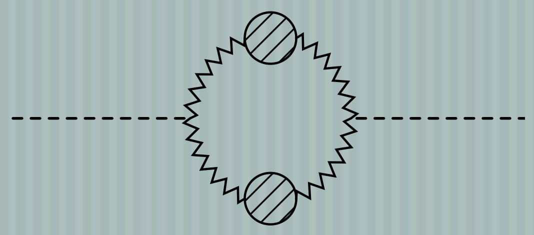
look for discontinuities in $E$ in each of three terms

discontinuity only arises from internal propagators going on shell

for first two this can only happen for complex $E$

but $E$ is external energy, always real (if external particles are the stable “normal” modes)

2 LW case is on the surface similar



3x3 terms:

$$\tilde{\mathcal{I}}(\hat{M},\hat{M}) + \tilde{\mathcal{I}}(\hat{M}^*,\hat{M}^*) + 2\tilde{\mathcal{I}}(\hat{M},\hat{M}^*) + \tilde{\mathcal{I}}(M,M) - 2\tilde{\mathcal{I}}(M,\hat{M}) - 2\tilde{\mathcal{I}}(M,\hat{M}^*)$$

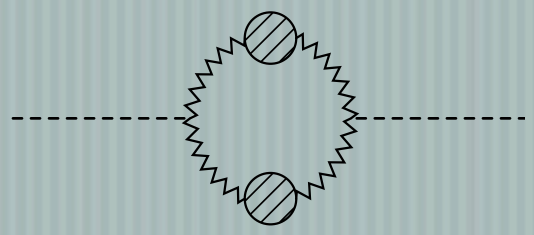
problem: both $\tilde{\mathcal{I}}(M,M)$ and $\tilde{\mathcal{I}}(\hat{M},\hat{M}^*)$ may give $\mathrm{disc}(A)$

and this one comes with wrong sign

more specifically

the integral $\tilde{\mathcal{I}}(M_1,M_2)$ as a function of $E$ has a cut with branch point at $(M_1+M_2)^2$
this is for real $E$ in both terms above

2 LW case is on the surface similar



3x3 terms:

$$\tilde{\mathcal{I}}(\hat{M},\hat{M})+\tilde{\mathcal{I}}(\hat{M}^*,\hat{M}^*)+2\tilde{\mathcal{I}}(\hat{M},\hat{M}^*)+\tilde{\mathcal{I}}(M,M)-2\tilde{\mathcal{I}}(M,\hat{M})-2\tilde{\mathcal{I}}(M,\hat{M}^*)$$

problem: both $\tilde{\mathcal{I}}(M,M)$ and $\tilde{\mathcal{I}}(\hat{M},\hat{M}^*)$ may give $\mathrm{disc}(A)$

and this one comes with wrong sign

more specifically

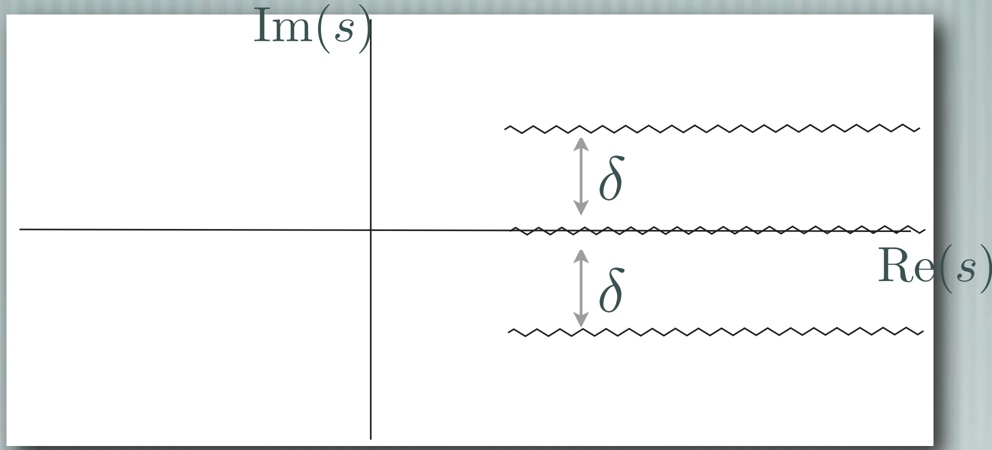
the integral $\tilde{\mathcal{I}}(M_1,M_2)$ as a function of $E$ has a cut with branch point at $(M_1+M_2)^2$
this is for real $E$ in both terms above

oopsie!

CLOP prescription:
result of integration depends on choice of contour
equivalent to taking different complex masses in the two propagators, with

$$M_2 - M_1 = i\delta$$

then letting, at the end, $\delta \to 0$
This prescription is explicitly Lorentz covariant.




"bad" cuts move off real axis,
discontinuity across real axis
is only from "good" cut

This distortion of the normal Feynman rules is what makes the non-perturbative formulation elusive

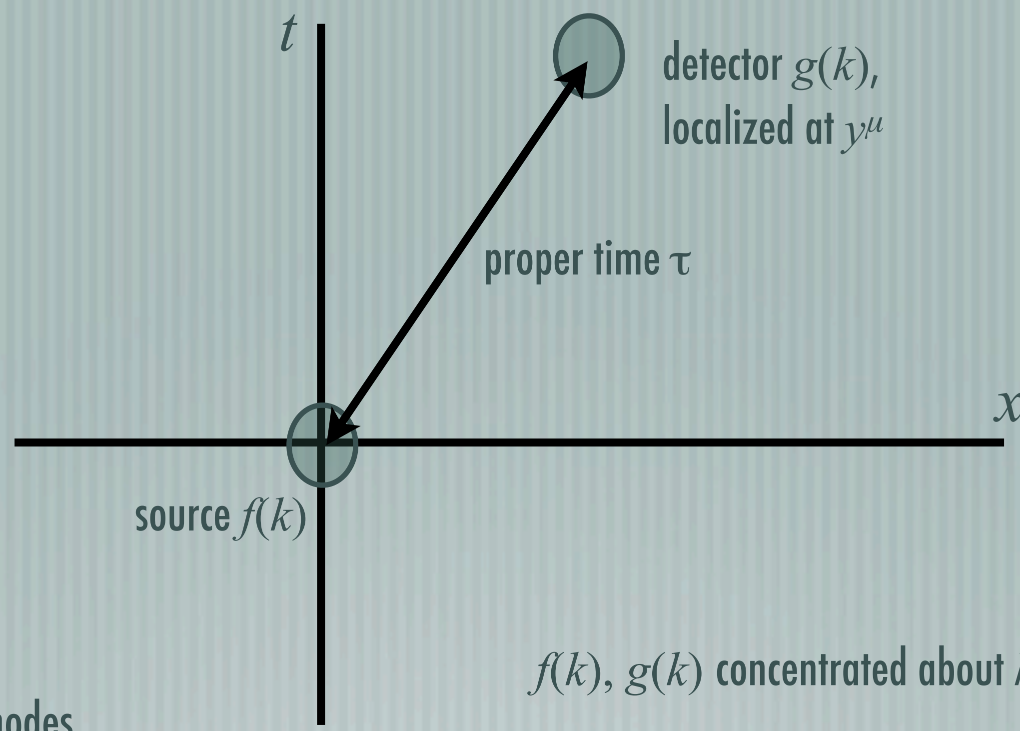
-we have checked the optical theorem for this case
-easy to generalize argument to all scattering amplitudes

# Weirdness

(going out on a limb on something I do not fully understand)

- theory quantized with canonical commutators
- in particular, all commutators vanish outside the light-cone
- causality implies analyticity in upper half plane
- we have poles in upper half plane!
- no contradiction: commutators not sufficient for causality
- HDSM is acausal, but not inconsistent (mathematics, logic, experiment)

# Recall "response theory"

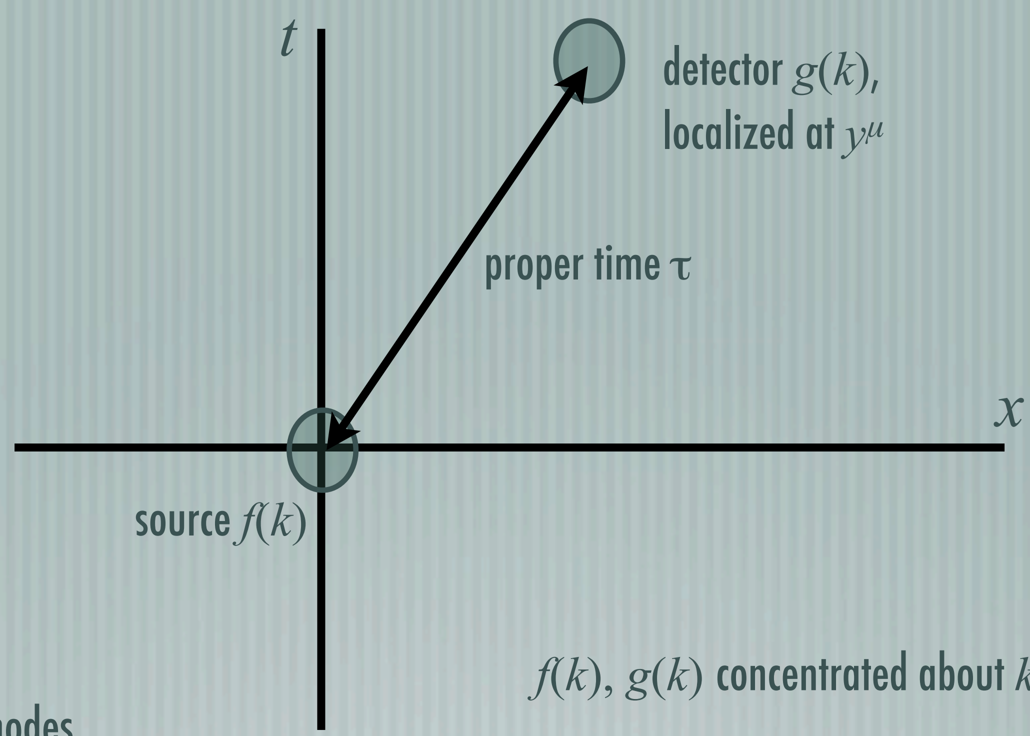



"source" can be from collision of two normal modes
"detector" from decay into normal modes

$f(k)$, $g(k)$ concentrated about $k = k_0$

$$\langle \mathrm{detector} | \mathrm{source} \rangle \sim g^*(my/\tau) f(my/\tau) \frac{1}{\tau^{3/2}} e^{-im\tau} \theta(y^0)$$

# Recall "response theory"




"source" can be from collision of
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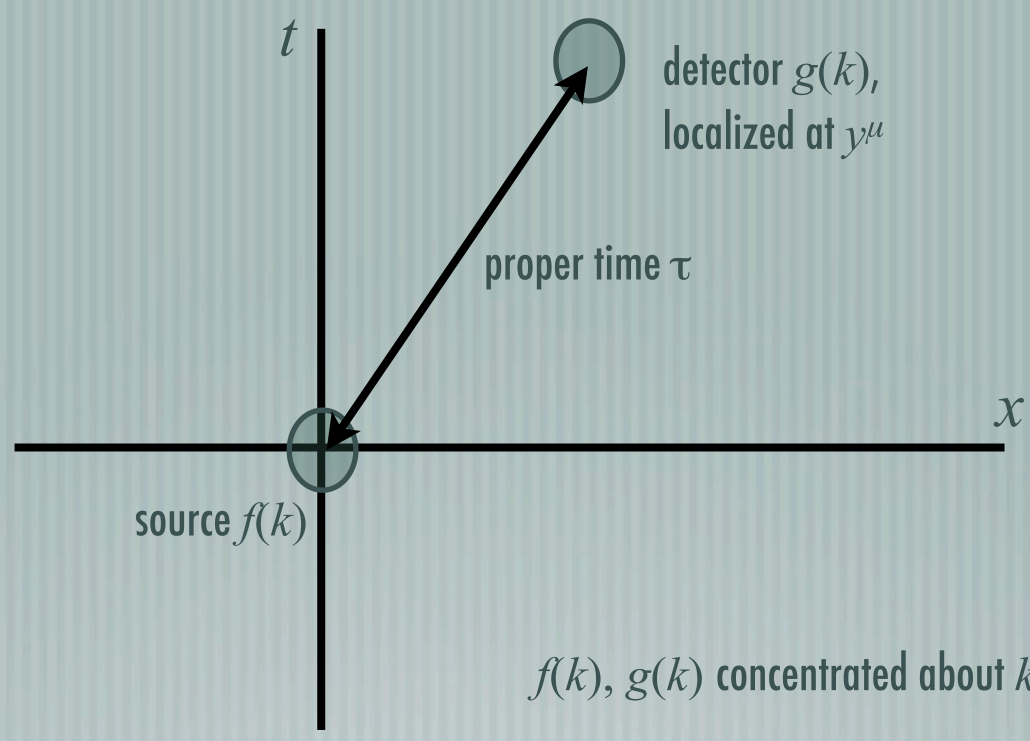
$f(k)$, $g(k)$ concentrated about $k = k_0$

$$\langle \text{detector} | \text{source} \rangle \sim g^*(my/\tau) f(my/\tau) \frac{1}{\tau^{3/2}} e^{-im\tau} \theta(y^0)$$

and for narrow resonance (pole in second sheet)

$$\langle \text{detector} | \text{source} \rangle \sim g^*(my/\tau) f(my/\tau) \frac{1}{\tau^{3/2}} e^{-im\tau} e^{-\Gamma\tau/2} \theta(y^0)$$

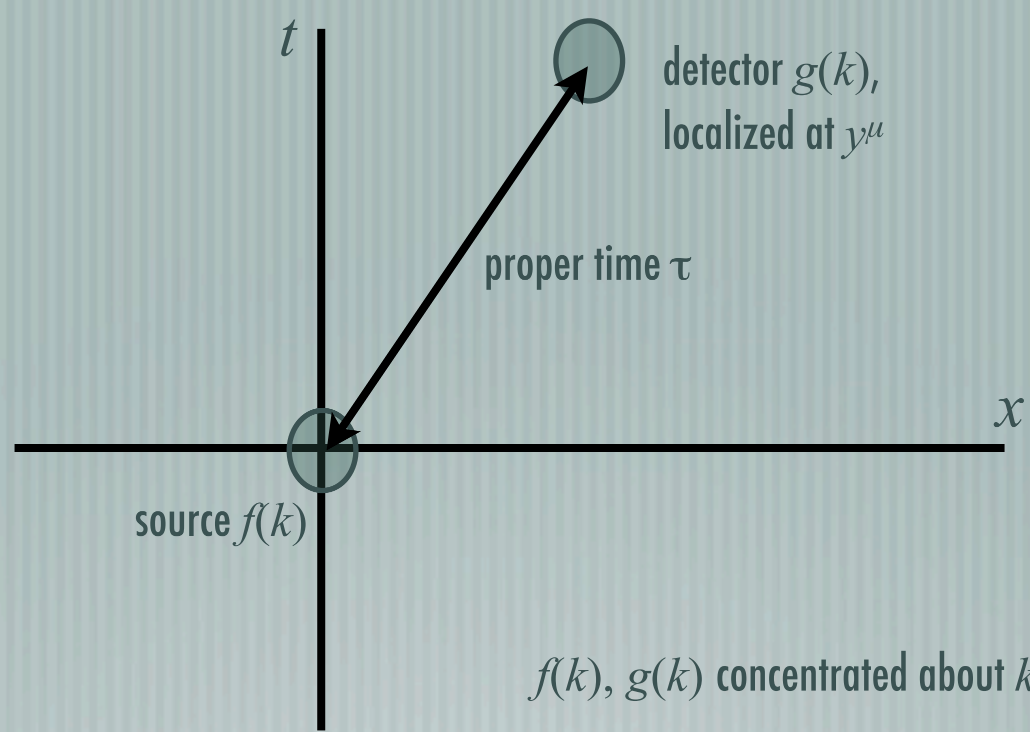
# Now for LW resonance




$f(k)$, $g(k)$ concentrated about $k = k_0$

$$\langle \text{detector} | \text{source} \rangle \sim g^*(-my/\tau) f(-my/\tau) \frac{1}{\tau^{3/2}} e^{im\tau} e^{-\Gamma\tau} \theta(-y^0)$$

# Now for LW resonance




$f(k)$, $g(k)$ concentrated about $k = k_0$

$$\langle \mathrm{detector} | \mathrm{source} \rangle \sim g^*(-my/\tau) f(-my/\tau) \frac{1}{\tau^{3/2}} e^{im\tau} e^{-\Gamma\tau} \theta(-y^0)$$

# And for LW virtual "dipole"

$$\langle \mathrm{detector} | \mathrm{source} \rangle \sim g^*(-my/\tau) f(-my/\tau) \frac{1}{\tau^3} e^{-2i\mathrm{Re}(M)\tau}$$