The Effective Theory of Quintessence: the w<-1 Side Unveiled

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Based on work with P. Creminelli, G. D'Amico, and F.Vernizzi.

JCAP 0902 (2009) 018 [arXiv:0811.0827 [astro-ph]]

Outline

- Introduction: Quintessence, ghosts.
- Framework: The effective theory of quintessence.
- Stability and the quintessence plane.
- Phenomenology.

Introduction

Quintessence $$S = \int d^4x \sqrt{-g} P(\phi, X)$$

Where $$X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad p_Q = P \qquad \rho_Q = 2XP_X - P$$

$$\Rightarrow w_Q = \frac{P}{2XP_X - P}$$

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Ghosts

$$S = \int d^4x\, a^3 \left[-\dot{\phi}^2 - V(\phi) \right] \qquad \Rightarrow w = \frac{-\dot{\phi}^2 - V}{-\dot{\phi}^2 + V} < -1$$

Carroll, Hoffman, Trodden, 2003

But a ghost suffers from quantum instability!!

Cline, Jeon, Moore, 2003

Effective Theory

Armendariz-Picon, Damour, Mukhanov, 1999

Quintessence

$$S = \int d^4x\sqrt{-g}\;\; P(\phi, X)$$

Perturb $\quad \phi(x) = \phi_0(t + \pi(x)) \;\Rightarrow \delta\phi = \dot{\phi}_0\pi$

Synchronous gauge $\quad ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$

$$S = \int d^4x a^3 \left[\frac{1}{2}(\rho_Q + p_Q + 4M^4)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 \right.$$
$$\left. -\frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi \right]$$

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$$S = \int d^4x\sqrt{-g}\left[P(\phi,X) - \frac{\bar{M}^2}{2}\left(\Box\phi + 3H(\phi)\right)^2\right]$$

Quintessence
+
Higher derivs

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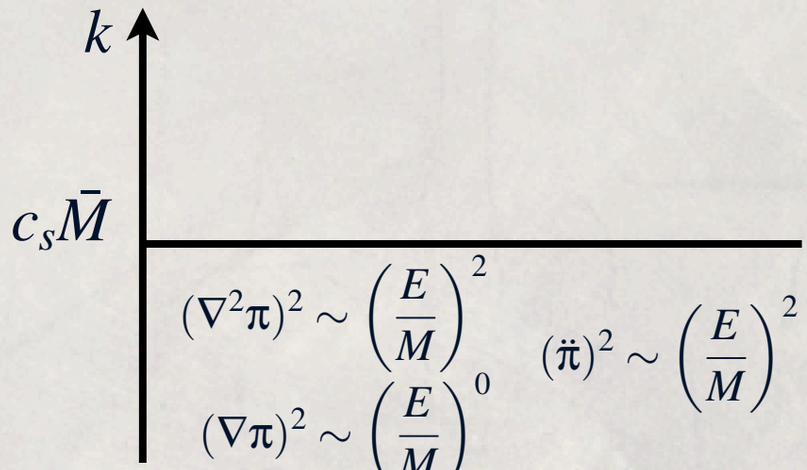
Creminelli, Luty, Nicolis, Senatore, 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore, 2007
Arkani-Hamed, Cheng, Luty, Mukohyama, 2004

Effective Theory

$k \ll c_s \bar{M}$ ⟶ Relativistic

$$t \sim x \sim E^{-1}$$



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$$c_s^2 = \frac{(\rho_Q + p_Q)}{(\rho_Q + p_Q) + M^4}$$

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$k \gg c_s \bar{M}$ ⟶ Non-relativistic

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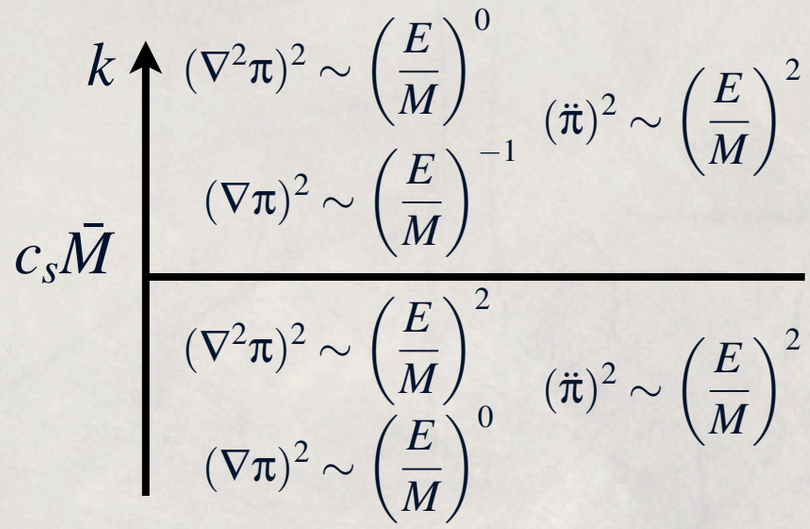

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Irrelevant

Effective Theory

$k > \bar{M}$ ⟶ Effective th. breaks down $\frac{\bar{M}}{H} > \left(\frac{M_p}{H}\right)^{1/2} \sim 10^{30}$

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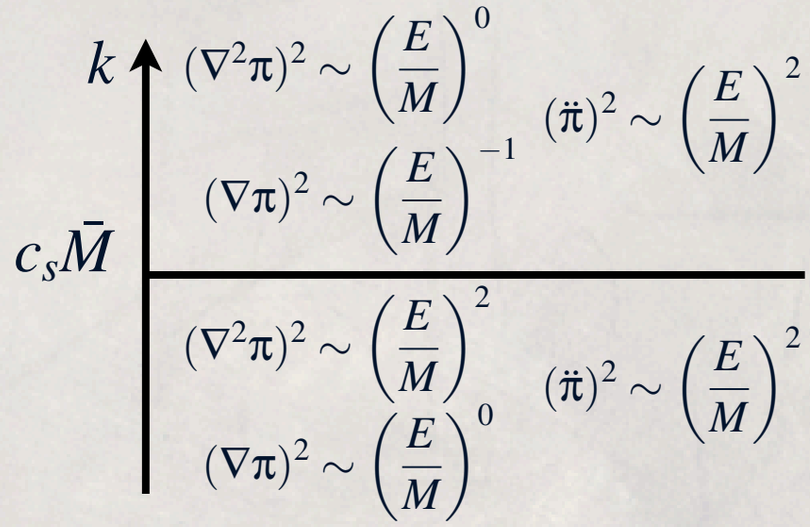

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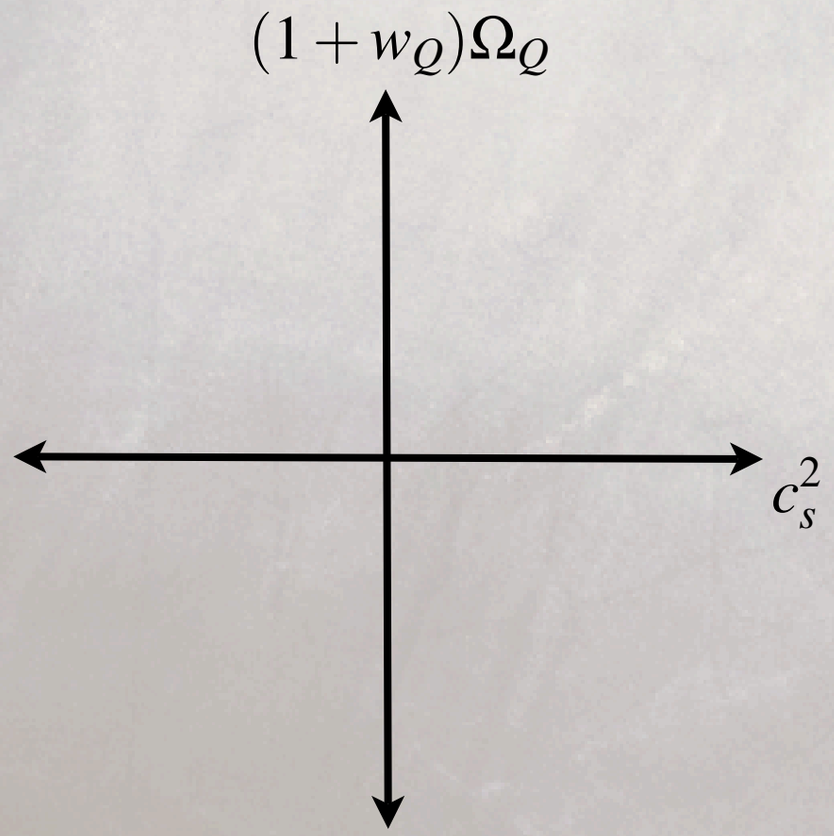
Irrelevant Subdominant

$$c_s^2 = \frac{(\rho_Q + p_Q)}{(\rho_Q + p_Q) + M^4}$$

The Quintessence Plane

$$(\rho_Q + p_Q + 4M^4)\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$$

$$c_s^2 = \frac{(\rho_Q + p_Q)}{(\rho_Q + p_Q) + M^4} \quad \text{and} \quad (1 + w_Q)\Omega_Q = \frac{(\rho_Q + p_Q)}{H^2 M_P^2}$$



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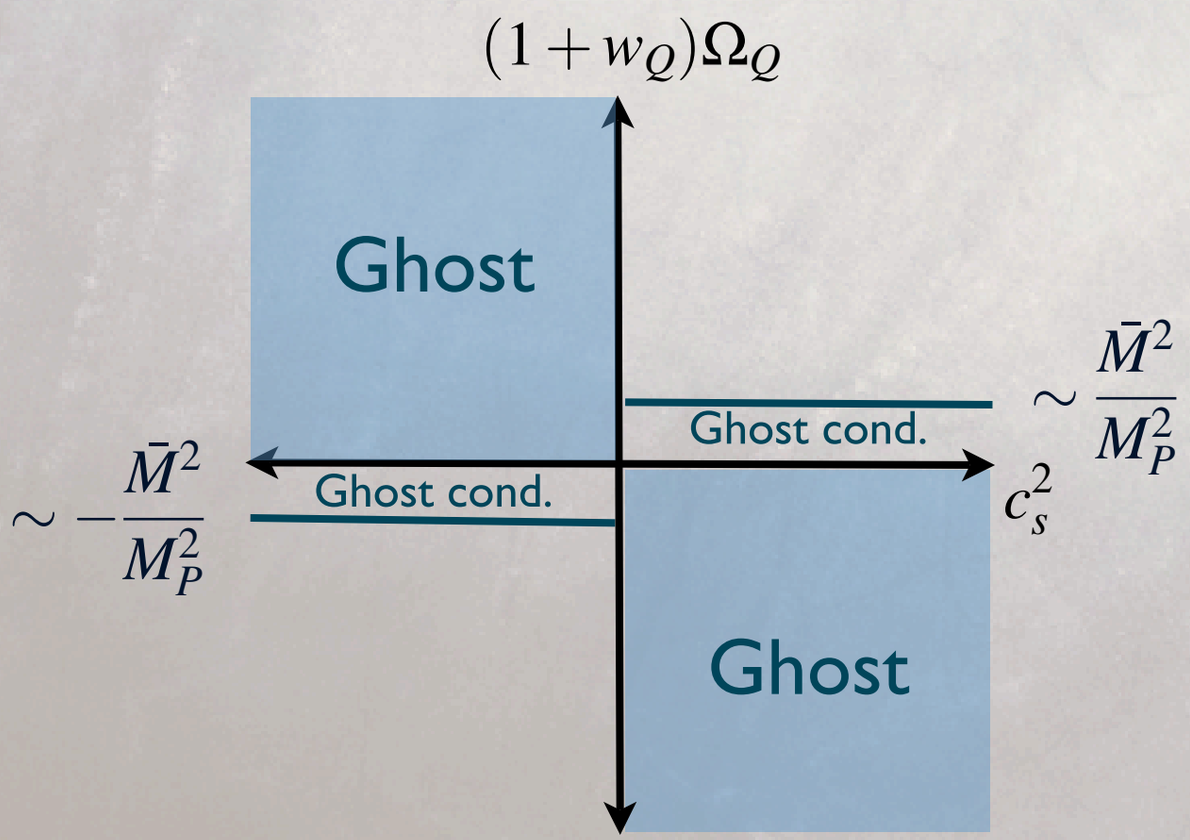

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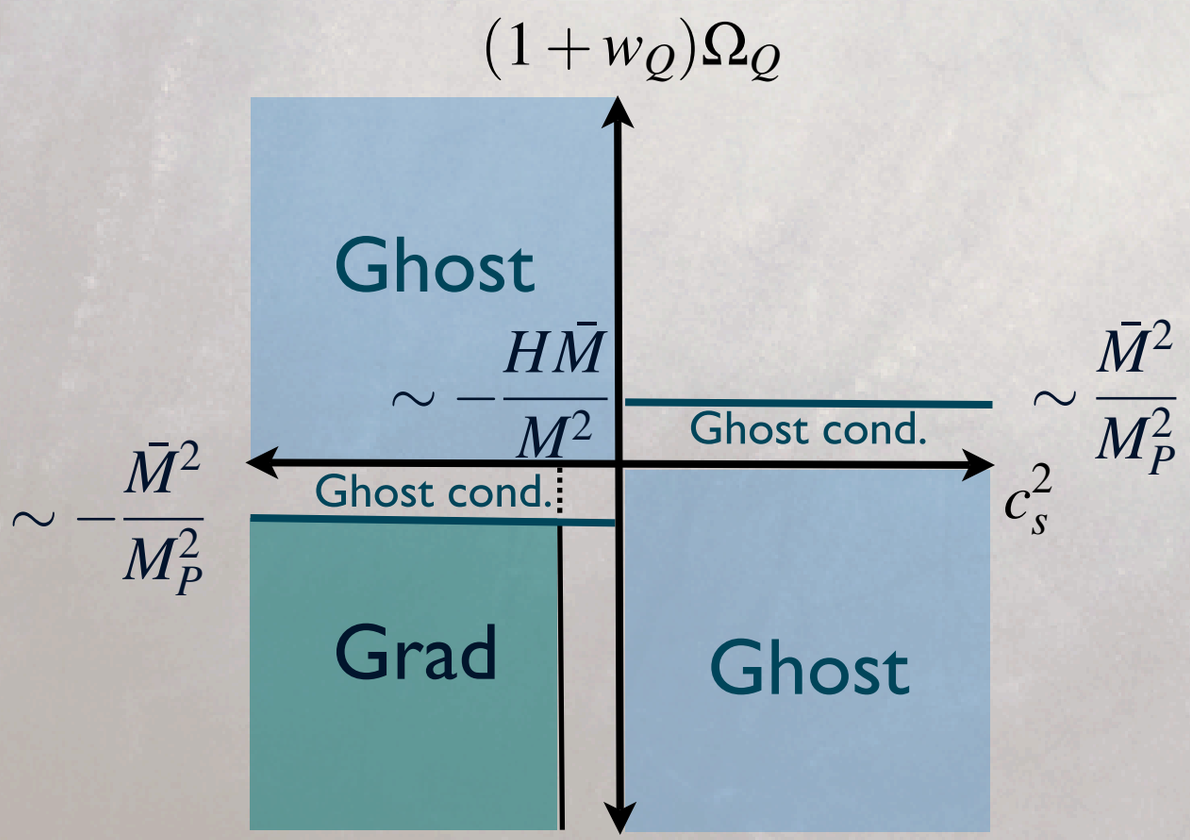

The fastest grad. instability

$$\omega_{grad} \approx i\frac{c_s^2 M^2}{\bar{M}} < iH$$

$$\Rightarrow |c_s^2| < \frac{\bar{M}H}{M^2} \sim 10^{-30}$$

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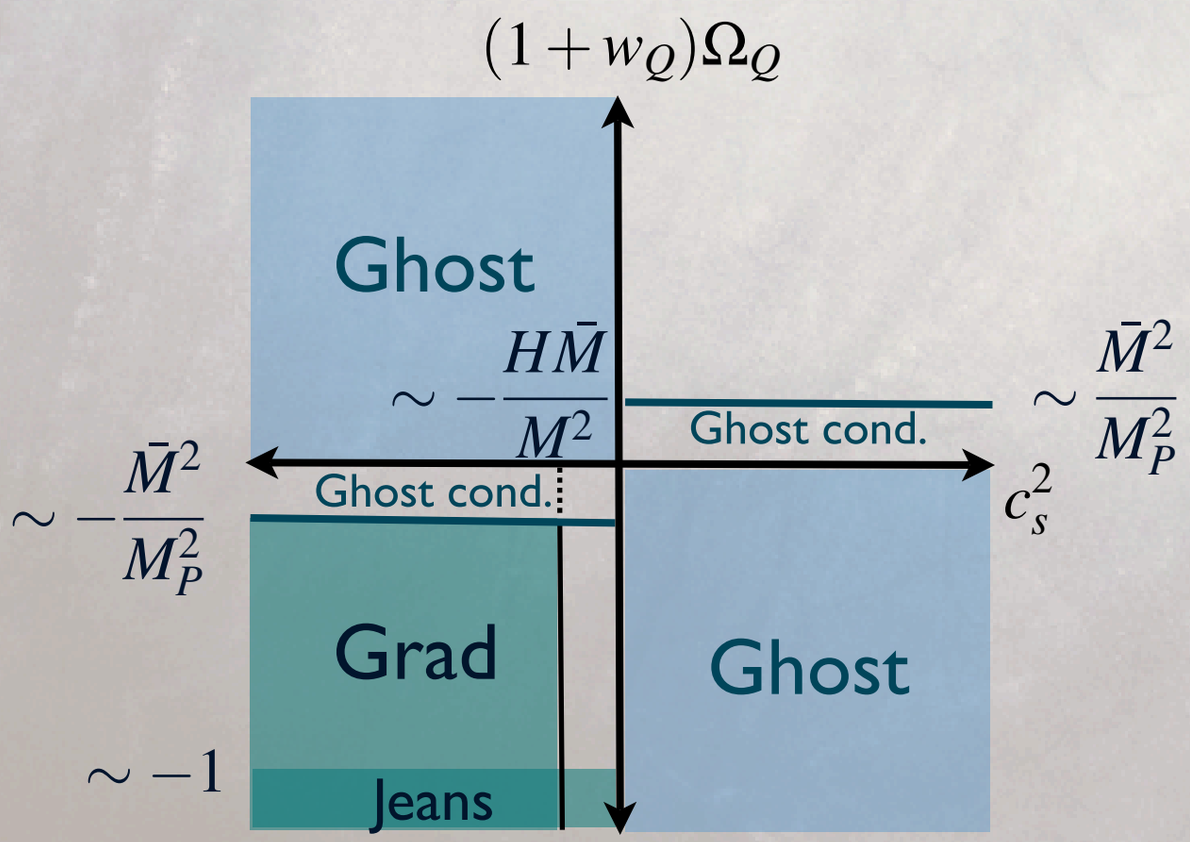

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Gravitational inst. rate

$$\omega_{Jeans}^2 \approx -\left(\frac{\bar{M}M^2}{M_P^2}\right)^2$$

Phenomenology

When $-\frac{\bar{M}^2}{M_P^2} < (1+w_Q)\Omega_Q < \frac{\bar{M}^2}{M_P^2}$ it's like a ghost condensate

In particular you can show that the perturbations turn off.

Phenomenology

When $-\frac{\bar{M}^2}{M_P^2} < (1+w_Q)\Omega_Q < \frac{\bar{M}^2}{M_P^2}$ it's like a ghost condensate

In particular you can show that the perturbations turn off.

Outside this region everything behaves like a k-essence

$$\delta'_Q + 3\mathcal{H}(c_s^2 - w_Q)\delta_Q = -(1+w_Q)\left[k^2 + 9\mathcal{H}^2(c_s^2 - c_a^2)\right]\frac{\theta_Q}{k^2} - (1+w_Q)\frac{h'}{2}$$

$$\frac{\theta'_Q}{k^2} + \mathcal{H}(1-3c_s^2)\frac{\theta_Q}{k^2} = \frac{c_s^2}{1+w_Q}\delta_Q$$

Bottomline: The simplest quintessence models make sense with w < -1 if the sound speed is nearly zero. And you can treat it as usual with the fluid equations.

The Phantom Divide

The w = -1 line cannot be crossed dynamically??

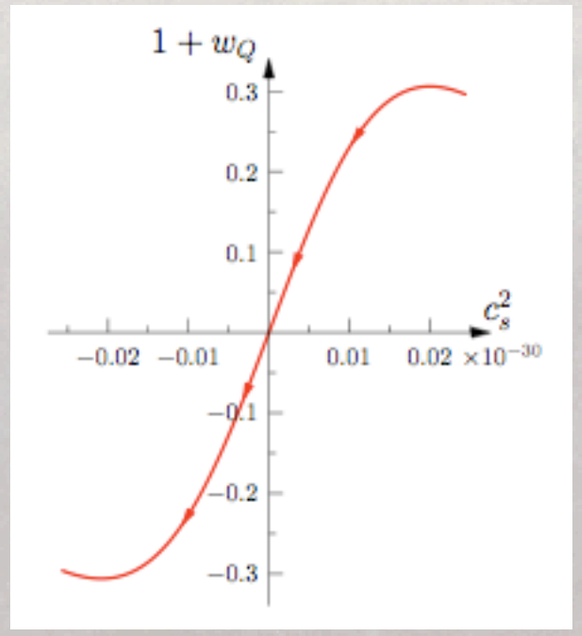
Caldwell and Doran, 2005
Hu, 2004
Vikman, 2005

Actually, a sensible theory of quintessence can cross the "Phantom Divide" with a single scalar dof.

Background $\dot{H}(t) = \frac{\mu^4}{M_P^2}(mt-1)$ $H(t) = \frac{\mu^4}{M_P^2}\left(\frac{m}{2}t^2 - t\right) + H_*$

A big enough M guarantees a small speed of sound:

$$c_s^2 = \frac{\mu^4(1-mt)}{\mu^4(1-mt)+2M^4}$$



Large Scale Structure

There are presently no constraints on the sound speed of dark energy.

A strong case has been made for a quintessence with $w < -1$ and zero sound speed.

We expect it to have interesting effects on the formation of large scale structures.

$$\delta_Q = \frac{1 + w_Q}{1 - 3w_Q}\delta_m$$

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We can estimate possible effects by doing a Press-Schechter type of calculation.

You begin by computing the spherical collapse.

You can do it because zero sound speed also means zero pressure gradients.

Conclusions

- Effective theory is a useful way to treat quintessence particularly in the $w < -1$ region
- It is possible to formulate a single scalar field model with $w < -1$ and without the instabilities of the ghost requiring the soundspeed to be small.
- You can cross the phantom divide.
- It is interesting to study how this affects the formation of large scale structures.

THE END