Enrico Trincherini
(Scuola Normale Superiore, Pisa)

The galileon as a local modification of gravity

with Alberto Nicolis
and Riccardo Rattazzi

arXiv:0811.2197 [hep-th]

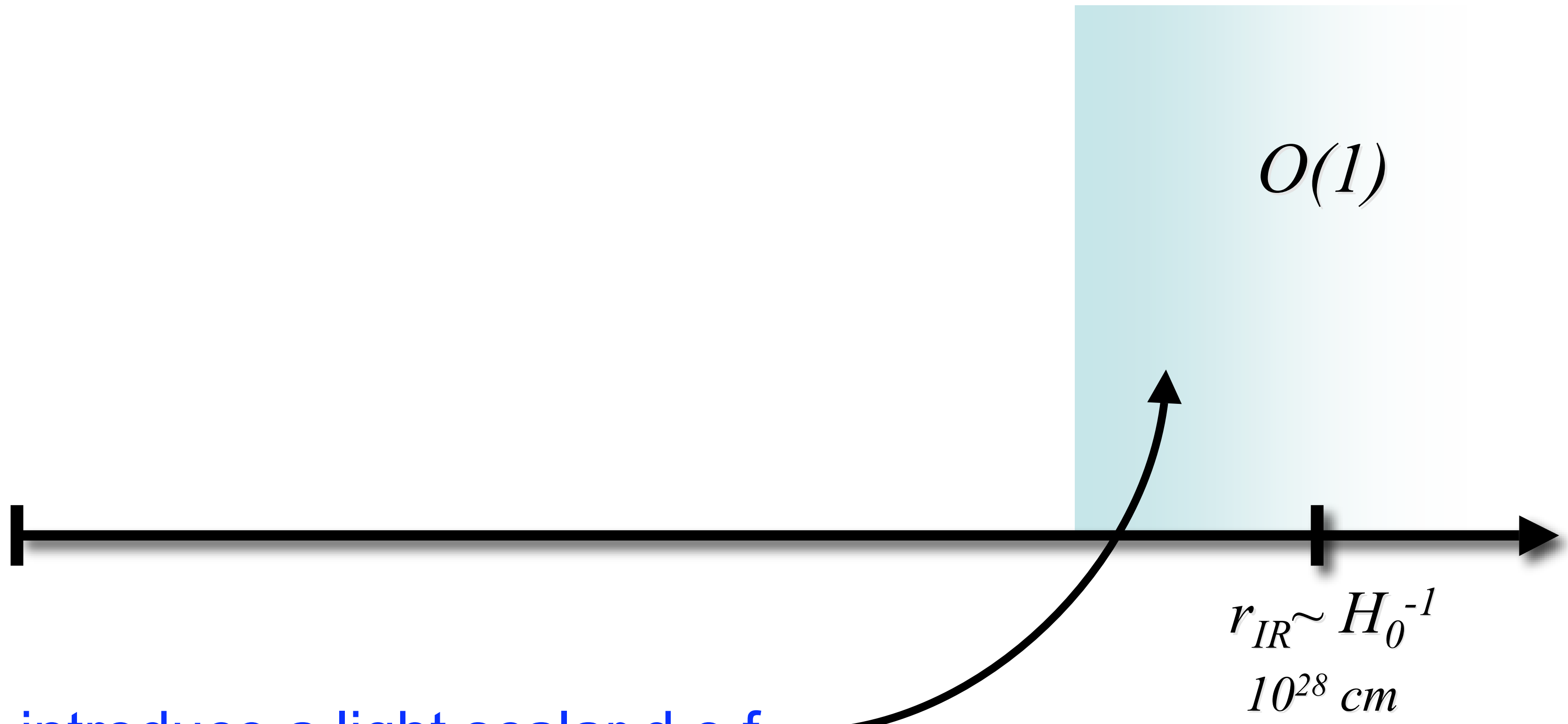
Motivations

Why are we trying to modify GR at large distances?

Experimental finding: the universe is undergoing a phase of accelerated expansion;

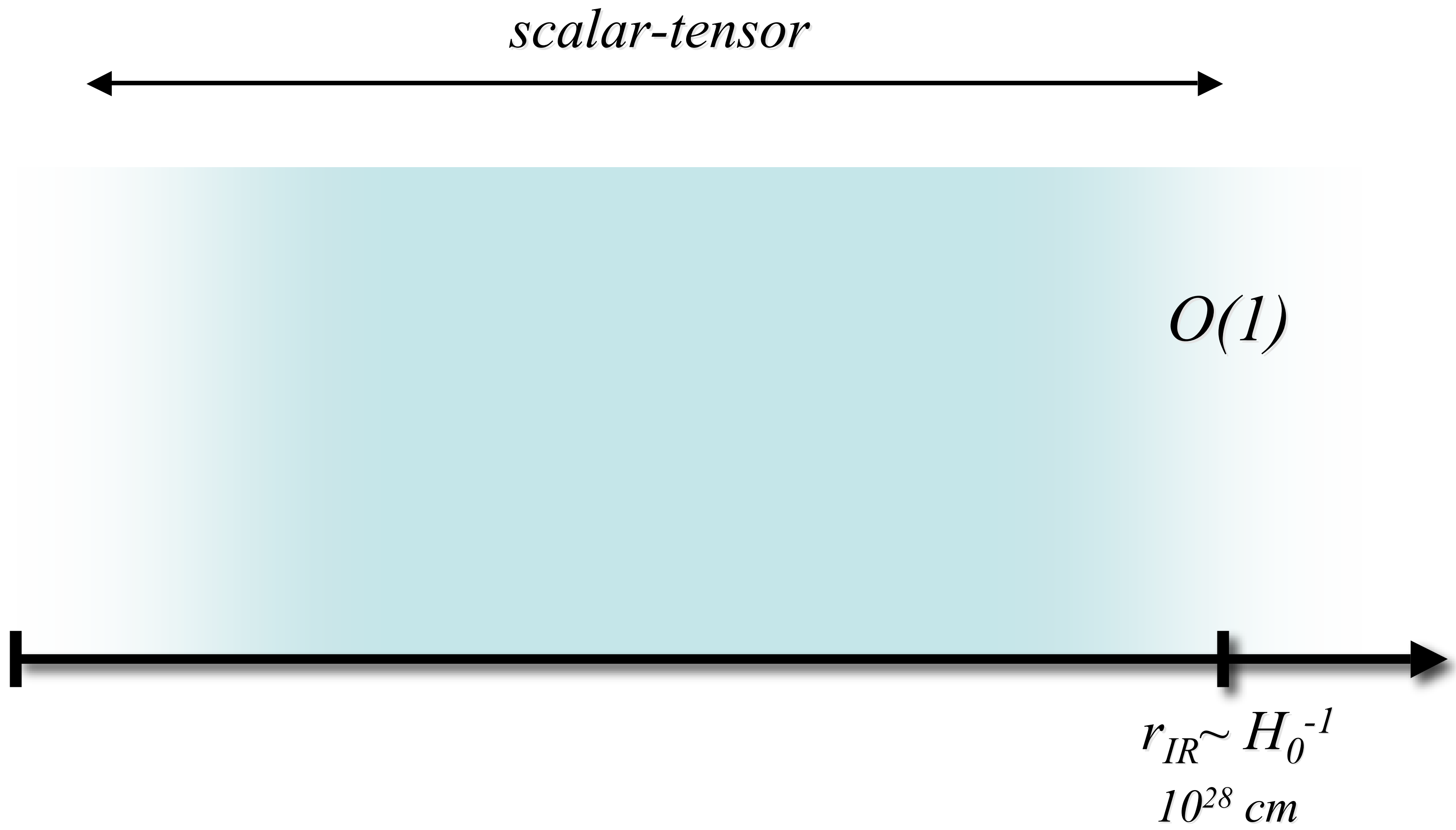
Theoretical challenge: can GR be modified at large distances?
If “yes”, what are the consequences?


O(1)
$r_{IR} \sim H_0^{-1}$
10^{28} cm



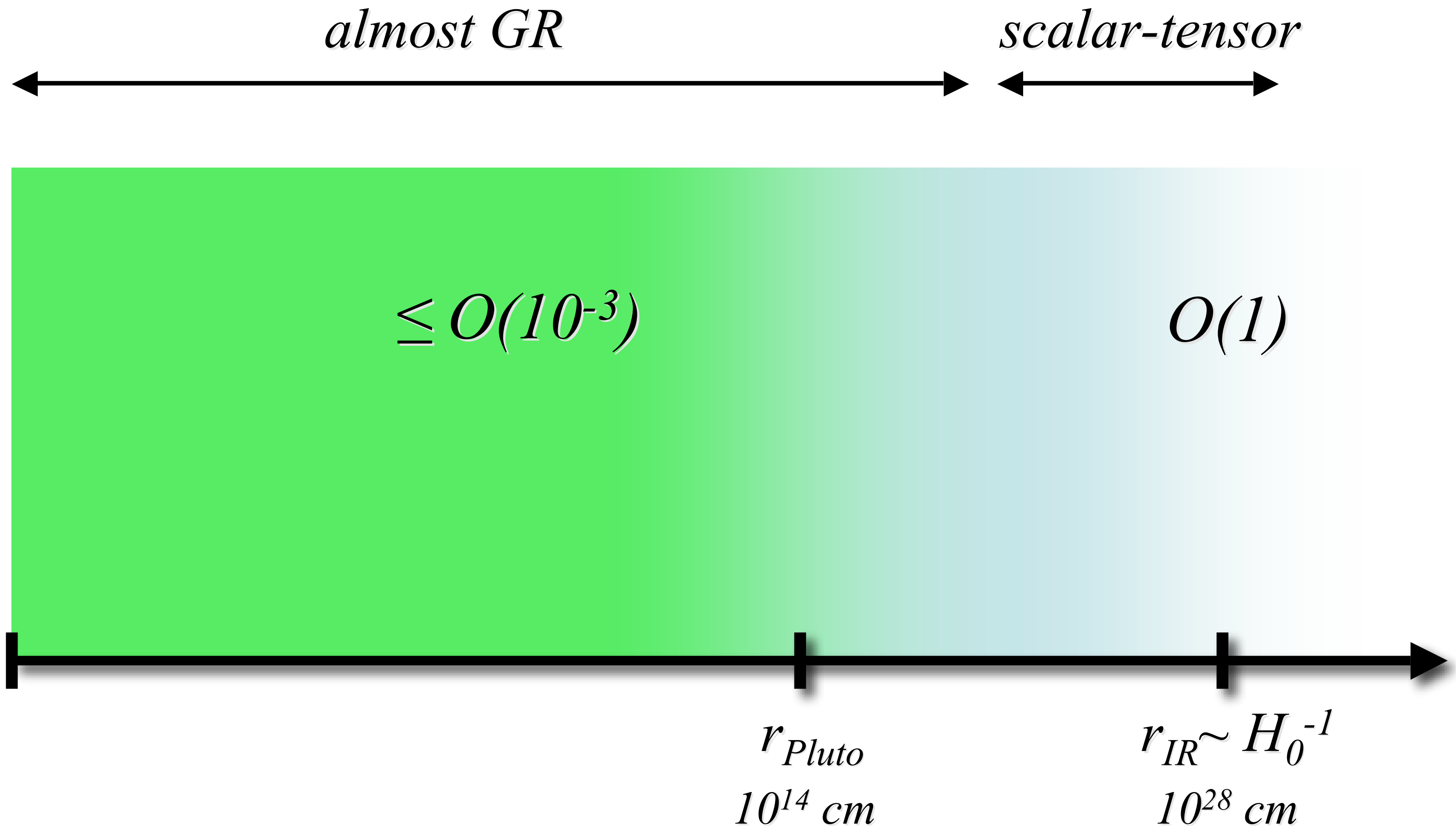
introduce a light scalar d.o.f

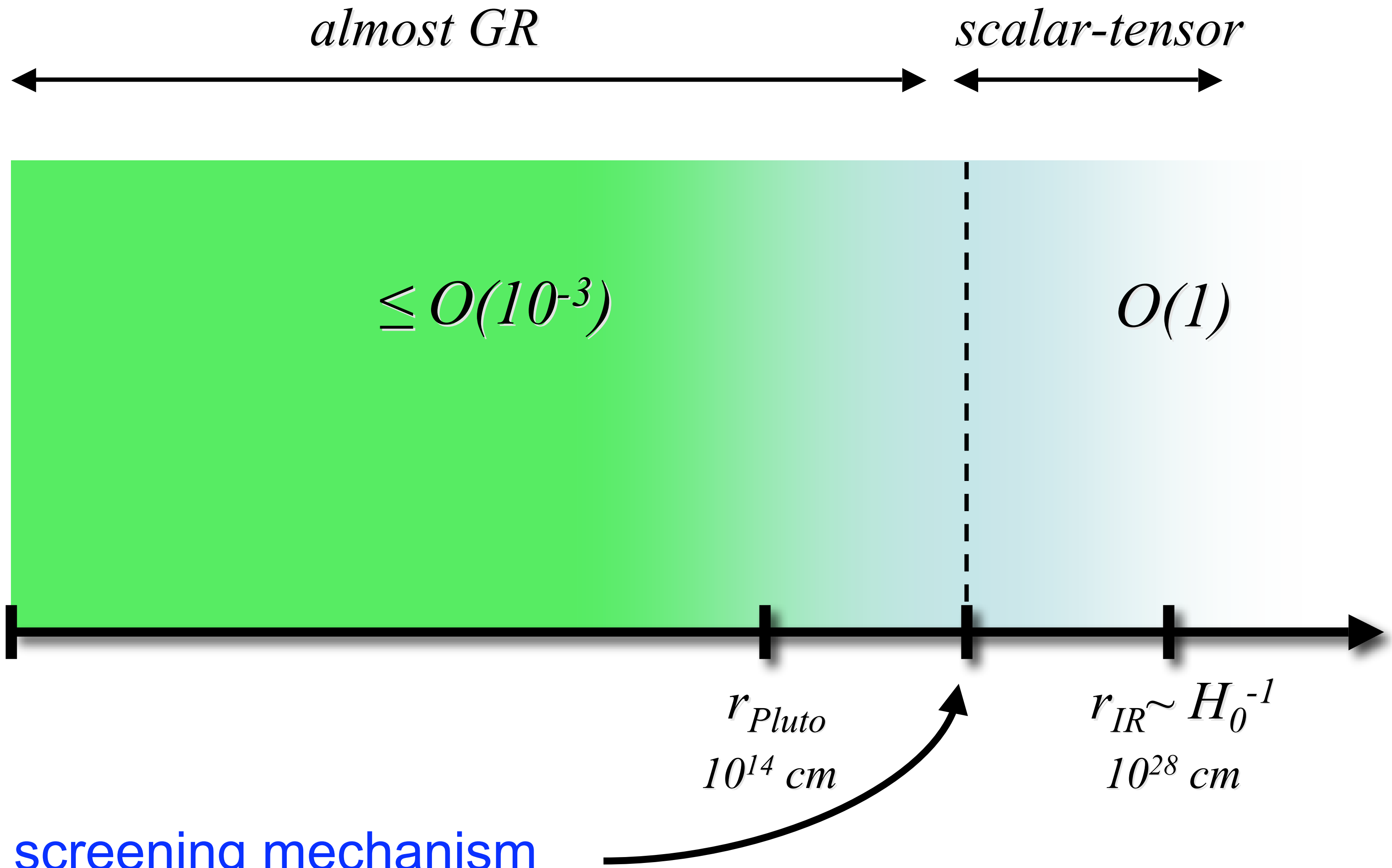
f(R), Brans-Dicke, Fierz-Pauli massive gravity,
DGP, Lorentz-violating massive gravity, ...



vDVZ discontinuity

Van Dam, Veltman, Zacharov 70

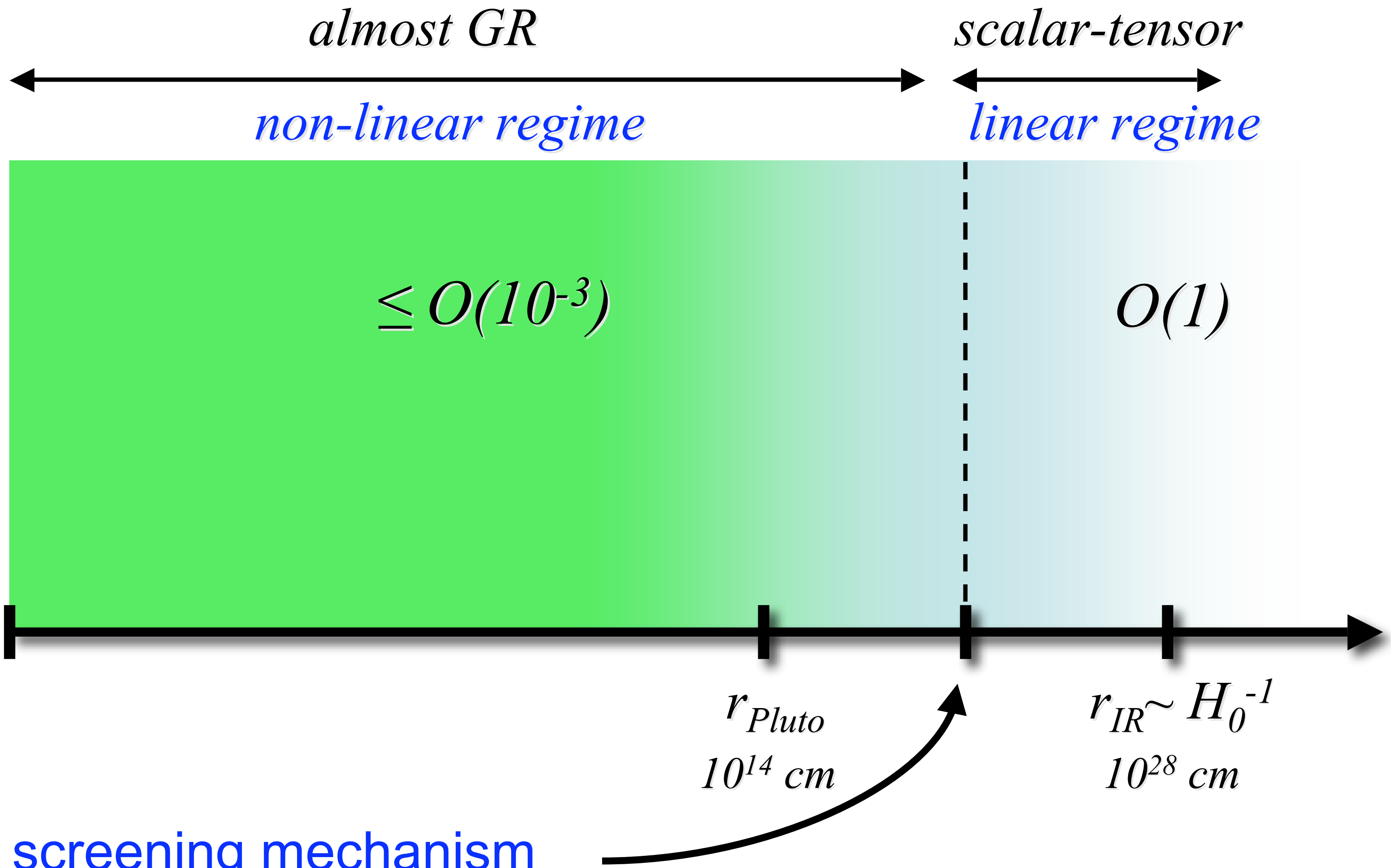

almost GR
scalar-tensor
$\leq O(10^{-3})$
$O(1)$
r_{Pluto}
10^{14} cm
$r_{IR} \sim H_0^{-1}$
10^{28} cm



screening mechanism

f(R)-like: chameleon mechanism   Khoury, Weltman 03

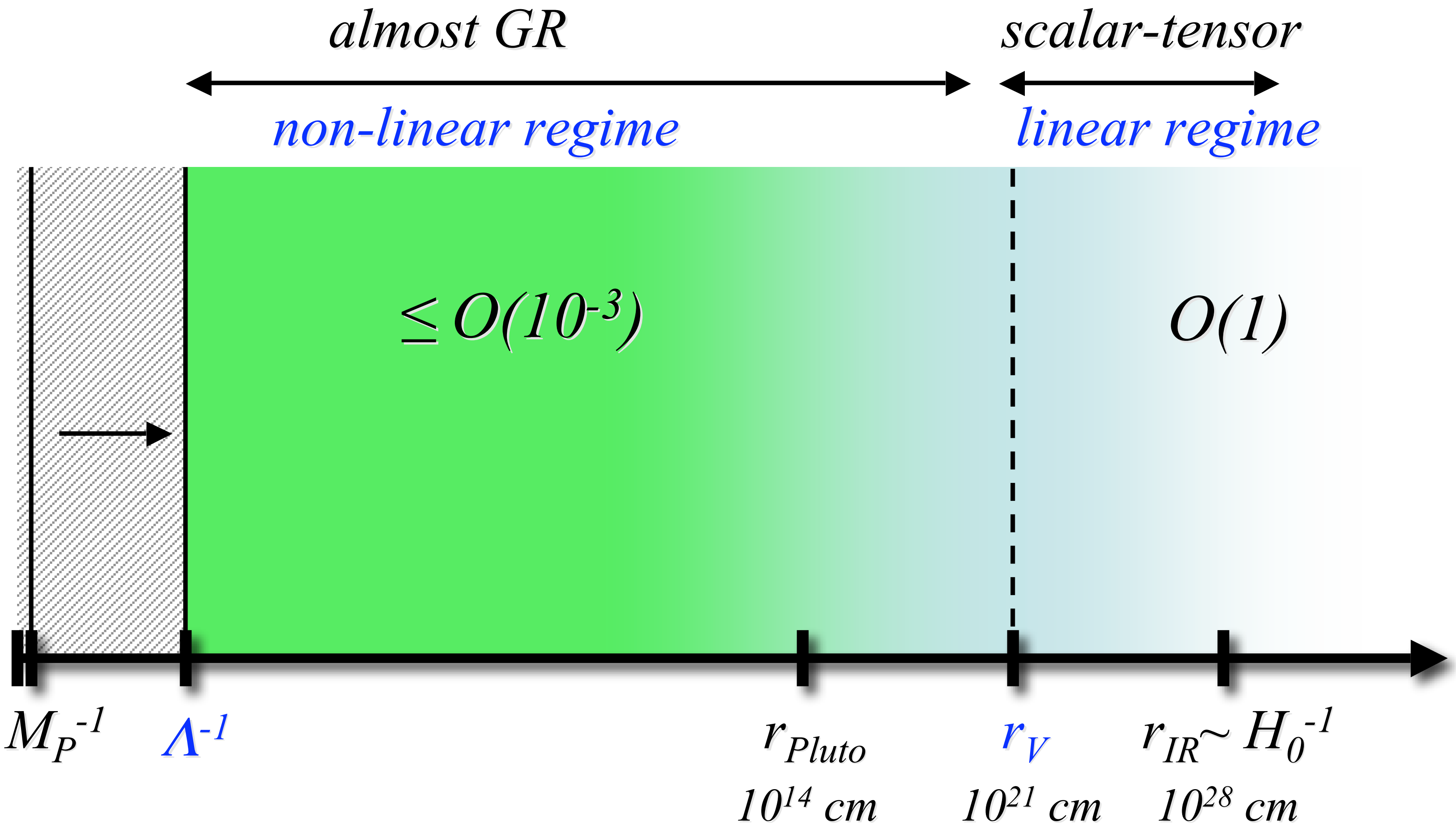
the scalar mass depends on the local density



screening mechanism

Vainshtein effect: non-linear dynamics suppresses the scalar contribution

Vainshtein 72



almost GR
scalar-tensor
non-linear regime
linear regime
$\leq O(10^{-3})$
$O(1)$
M_P^{-1}
Λ^{-1}
r_{Pluto}
10^{14} cm
r_V
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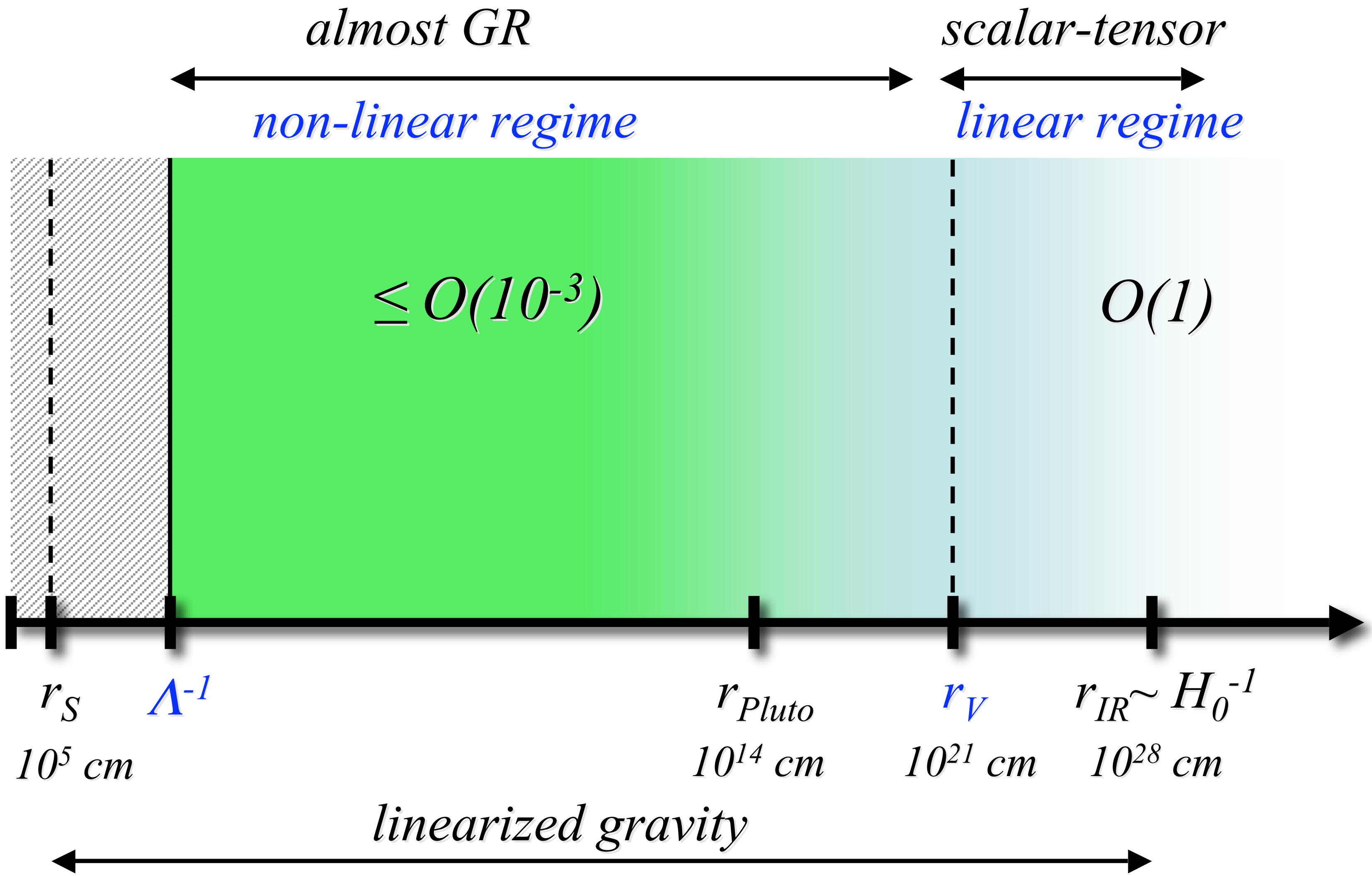
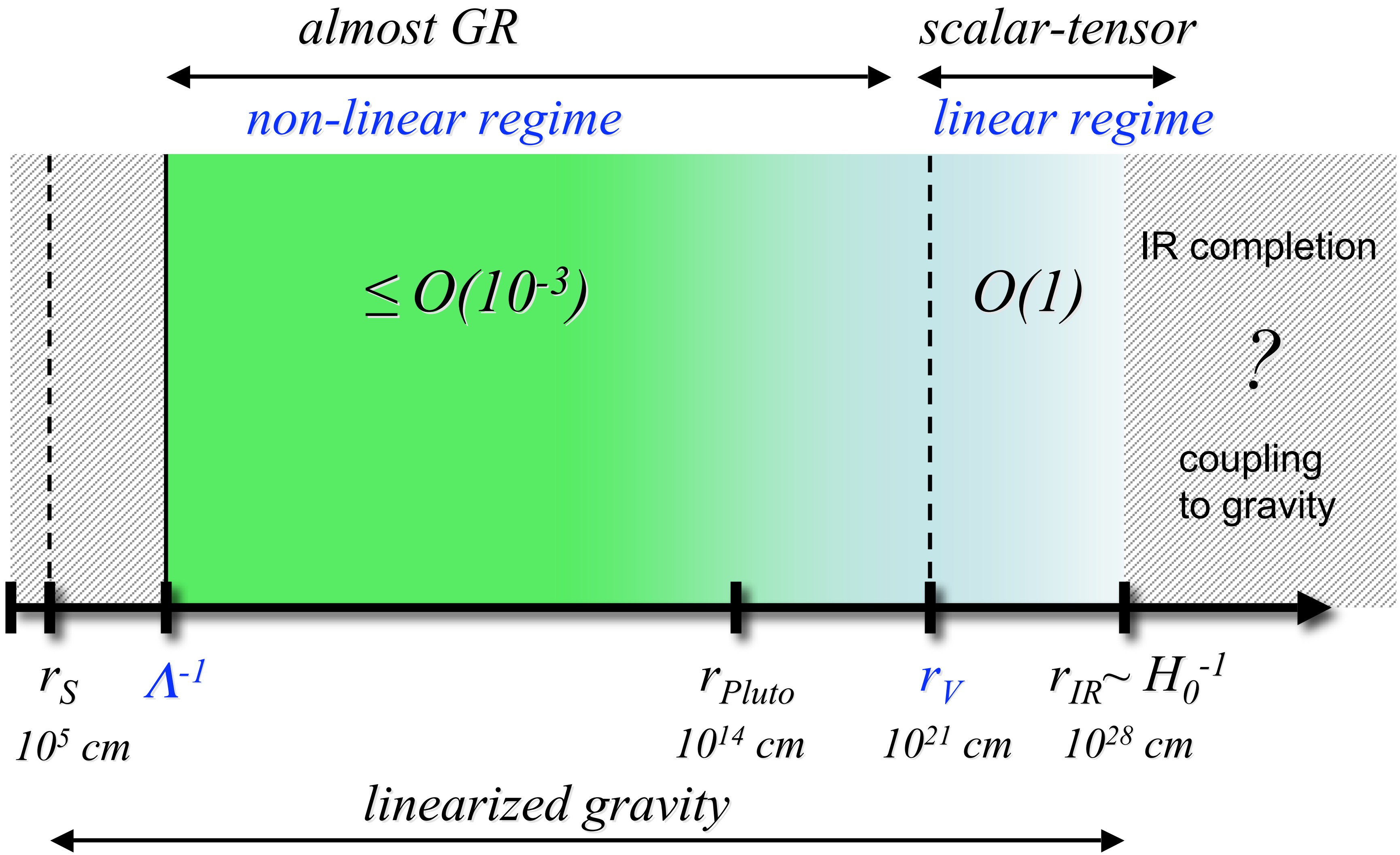


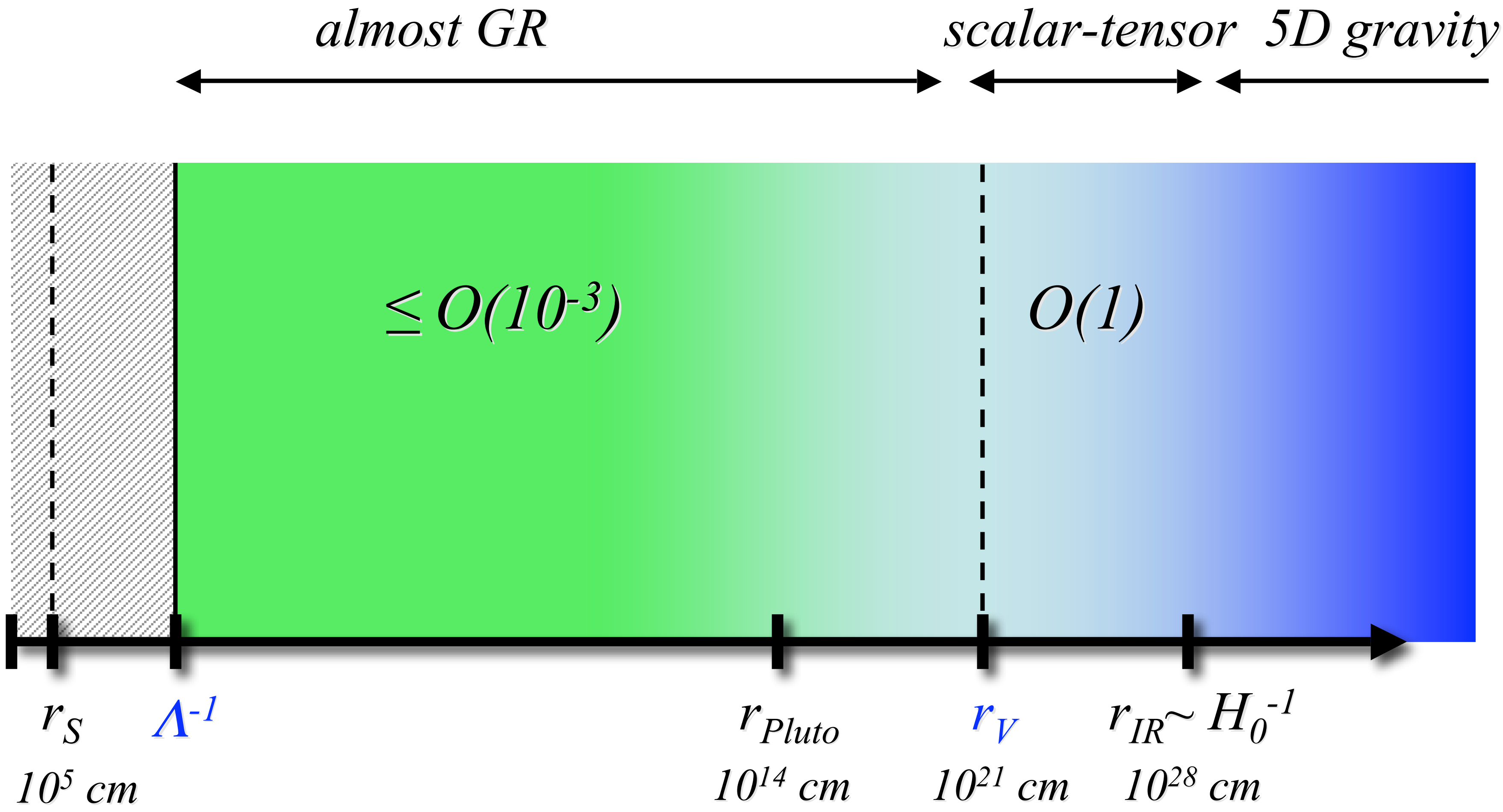
$$ds^2 \simeq \left[1 - \frac{1}{2}H^2|\vec{x}|^2 + \frac{1}{2}(2\dot{H} + H^2)t^2\right]\left(-dt^2 + d\vec{x}^2\right)$$

FRW universe: conformal deformation of Minkowski *the galileon: a local modification of gravity*



$$ds^2 \simeq [1 - \frac{1}{2} H_0^2 x_\mu x^\mu](-dt^2 + d\vec{x}^2)$$ locally de Sitter

DGP Dvali, Gabadadze, Porrati 00



full non-linear 5D “self-accelerating” solution
plagued by a ghost instability

Long-distance modification of GR

- locally and for weak gravitational fields due to a light scalar d.o.f. π

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- π kinetically mixed with the metric:

  replacing $\sqrt{-g}\,R$ with $\sqrt{-g}\,(1-2\pi)R$ plus whatever dynamics π may have on its own;

We allow for generic non-linarities in the π sector
(while we treat the gravitational field and π's contribution to it at linear order)

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Demix π and the metric by performing the Weyl rescaling: $h_{\mu\nu} = \hat{h}_{\mu\nu} + 2\pi\,\eta_{\mu\nu}$

$$S = \int d^4x \left[\frac{1}{2} M_{\rm Pl}^2 \sqrt{-\hat{g}} \hat{R} + \frac{1}{2} \hat{h}_{\mu\nu} T^{\mu\nu} + \mathcal{L}_\pi + \pi\, T^\mu{}_\mu \right]$$

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$$\pi_{\rm dS} = -\frac{1}{4}H_0^2 x_\mu x^\mu + \mathcal{O}(H_0^3 x^3)$$

Towards a realistic modification of gravity, the structure of $\mathcal{L}_\pi$

Give rise to on $O(1)$ modification of the Hubble flow but
no deviation larger than $O(10^{-3})$ at solar system distances:

1) non-linearities in the π dynamics must play a crucial role

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the quadratic approximation at subhorizon distances is invariant under the combined action of a spacetime translation and a shift

$$\boxed{\pi \to \pi + c + b_\mu x^\mu}$$

2) the equations of motion should be invariant under this “galilean” symmetry

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this is equivalent as demanding at least 2 derivatives per π in the e.o.m. $\Rightarrow$ generically ghost degrees of freedom appear (ex. Fierz Pauli massive gravity)

3) π e.o.m. must be of second order

$$\boxed{\frac{\delta \mathcal{L}_\pi}{\delta \pi} = F(\partial_\mu \partial_\nu \pi)}$$

F is a non-linear Lorentz invariant function of $\partial_\mu \partial_\nu \pi$

The galileon Lagrangian

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classify all the operators in the Lagrangian:

- *n* powers of π and 2*n*-2 derivatives
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$$\mathcal{L}_1 \quad = \quad \pi \qquad\qquad \delta \mathcal{L}_1 = b_\rho x^\rho + c = \partial_\mu (b_\rho x^\rho x^\mu / 5 + c x^\mu / 4)$$

The galileon Lagrangian

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$$\begin{aligned}
\mathcal{L}_1 &= \pi \\
\mathcal{L}_2 &= -\frac{1}{2}\,\partial\pi\cdot\partial\pi \\
\mathcal{L}_3 &= -\frac{1}{2}\,[\Pi]\,\partial\pi\cdot\partial\pi \\
\mathcal{L}_4 &= -\frac{1}{4}\left([\Pi]^2\,\partial\pi\cdot\partial\pi - 2\,[\Pi]\,\partial\pi\cdot\Pi\cdot\partial\pi - [\Pi^2]\,\partial\pi\cdot\partial\pi + 2\,\partial\pi\cdot\Pi^2\cdot\partial\pi\right) \\
\mathcal{L}_5 &= -\frac{1}{5}\left([\Pi]^3\,\partial\pi\cdot\partial\pi - 3[\Pi]^2\,\partial\pi\cdot\Pi\cdot\partial\pi - 3[\Pi][\Pi^2]\,\partial\pi\cdot\partial\pi + 6[\Pi]\,\partial\pi\cdot\Pi^2\cdot\partial\pi\right.
\end{aligned}$$

$\Pi^\mu{}_\nu \equiv \partial^\mu\partial_\nu\pi$

$[\ldots]$ stand for the trace operator

There are only 4+1 Lagrangian terms in 4D!

each derivative term is associated with one invariant of the matrix Π
number of invariant = rank of Π = number of spacetime dimensions

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$$\boxed{\mathcal{L}_\pi = \sum_{i=1}^{5} c_i \mathcal{L}_i}$$

$$c_2 = M_{Pl}^2 \quad c_3 = M_{Pl}^2 L_{DGP}^2 \quad c_1 = c_4 = c_5 = 0$$

is the DGP π Lagrangian

Luty, Porrati, Rattazzi 03

The coupling to matter

π is dimensionless $\Rightarrow c_n = [m]^{6-2n} \sim \dfrac{f^2}{H_0^{2n-4}}$

$$\mathcal{L} = f^2 \partial\pi\partial\pi \, F(\frac{\partial\partial\pi}{H_0^2}) + \pi T \quad \Rightarrow \quad \mathcal{L} = \partial\pi_c\partial\pi_c \, F(\frac{\partial\partial\pi_c}{f H_0^2}) + \frac{\pi_c T}{f}$$

The scale controlling higher order terms must be H_0 so that the π contribution affects at $O(1)$ the present expansion

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$\Delta T_{\mu\nu}$ does not lead to a curvature $\gg H_0^2$ at $r \lesssim H_0^{-1} \Rightarrow f \lesssim M_{Pl}$

Lower bound: direct contribution of π to the geometry percieved by matter

$$\frac{\pi}{h_N} \sim \left(\frac{M_P}{f}\right)^{2/3} \frac{H_0^{4/3} r^2}{R_S^{2/3}} = \left(\frac{M_P}{f}\frac{H_0^2}{\mathcal{R}}\right)^{2/3}$$

$\mathcal{R} \sim R_S/r^3$ is the Riemann curvature

Strongest bound on π/h_N comes from the solar system but $\mathcal{R} >> H_0^2$

In galaxy superclusters $\mathcal{R} \gtrsim H_0^2$ and GR is valid at *O(1)* $\Rightarrow f > \dfrac{M_{Pl}}{\text{a few}}$

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$$\Lambda \sim (f H_0^2)^{1/3} \sim (1000\,\mathrm{km})^{-1}$$

Results

The existence of only 5 possible terms makes possible a thorough analysis of very symmetric solutions. The sign of the coefficients c_n can be chosen such that:

1. the galileon admits a deSitter solution in the absence of a cosmological constant;

2. it is ghost free;

3. about this dS configuration there exist spherically symmetric solutions that describe the π field generated by compact sources;

4. these configurations are also be stable against small perturbations;

5. the Vainshtein effect is implemented.

Spherically symmetric solution and the Vainshtein effect

localized source $\rho = M\delta^3(\vec{r})$

spherical symm $\pi = \pi_0(r)$ + shift and galilean symm $\Rightarrow$ drastical simplification

e.o.m. is an algebraic equation for $\pi_0'(r)$

$$\frac{\delta \mathcal{L}_\pi}{\delta \pi} = \sum_i d_i\,\mathcal{E}_i = \frac{1}{r^2}\frac{d}{dr}r^3\left[d_2(\pi_0'/r) + 2d_3(\pi_0'/r)^2 + 2d_4(\pi_0'/r)^3\right] = M\delta^3(\vec{r})$$

$$\Rightarrow P\left(\frac{\pi_0'}{r}\right) \equiv d_2\left(\pi_0'/r\right) + 2d_3\left(\pi_0'/r\right)^2 + 2d_4\left(\pi_0'/r\right)^3 = \frac{M}{4\pi r^3}$$

Spherically symmetric solution and the Vainshtein effect

$$d_n = \frac{M_{Pl}^2}{H_0^{2n-4}} \tilde{d}_n$$

rewrite the equation of motion as

$$M_{Pl}^2 \tilde{d}_2 \left(\pi_0'/r\right) + 2M_{Pl}^2 \tilde{d}_3 \left(\pi_0'/r\right)\left(\pi_0'/H_0^2 r\right) + 2M_{Pl}^2 \tilde{d}_4 \left(\pi_0'/r\right)\left(\pi_0'/H_0^2 r\right)^2 = \frac{M}{4\pi r^3}$$

in the linear regime, far from the source, the kinetic term dominates and we have the asymptotic behaviour

$$\pi_0 \sim \frac{M}{M_{Pl}^2} \frac{1}{r} \qquad\qquad \frac{\pi_0}{\Phi_N} \sim \mathcal{O}(1)$$

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the non-linear terms become relevant at the scale

$$\frac{\pi_0'}{H_0^2 r} \sim 1 \Rightarrow r \sim \left(\frac{M}{M_{Pl}^2}\frac{1}{H_0^2}\right)^3 \equiv r_V \quad (\sim 10^{21} \text{ cm for the sun})$$

$$\pi_0 \sim \left(\frac{M}{M_{Pl}^2}H^4\right)^{1/3} r \quad \text{for } r << r_V \qquad\qquad \frac{\pi_0}{\Phi_N} \sim \frac{r^2}{r_V^2}$$

Problems and open questions

1. What about the existence and stability of more complicated (less symmetric) solutions, e.g. with several compact sources in generic positions?

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2. Common to DGP:

i) Small excitations are forced to be superluminal at large distances from compact sources, even if we do not require the existence of dS solutions;

On a non-Lorentz invariant background is not necessarily an inconsistency, but UV completion cannot be a Lorentz invariant, local QFT

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi 06

ii) Very low strong-coupling scale for $\pi\pi$ scattering;

In the subclass of Lagrangians $c_4=c_5=0$, the large kinetic term supresses quantum fluct and the effective cutoff increases above the naive esitmate $\Lambda \sim (1000\ \mathrm{km})^{-1}$

Nicolis, Rattazzi 04

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3. What is the fate of the galileon in the IR? How do we couple it to gravity?

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First neglecting gravity, we promoted the Poincaré group and our galilean invariance to the conformal group (spontaneously broken): the existence of deSitter-like solutions may be expected by symmetry considerations,

our local galilean invariant terms are short-distance limit of globally defined conformally invariant 4D Lagrangian terms.

Coupling the system to gravity breaks conformal invariance and spoils the deSitter geometry at times of order Hubble

not necessarily a phenomenological problem, but the symmetry motivation for this “conformal completion” is somewhat lost.

Nicolis, ET, Rattazzi 08

Conclusions

We have considered IR modifications of gravity that, at least in a local patch smaller than the cosmological horizon, are due to a relativistic scalar d.o.f. kinetically mixed with the metric;

the dynamics of π enjoys an internal galilean symmetry (only 5 possible Lagrangian terms) and the e.o.m. is of second order;

π decouples from matter at short scales thanks to derivative self-interactions.

We found stable deSitter solutions in the absence of a c.c. and stable spherically symmetric Vainshtein-like background around compact sources

Small excitations are forced to be superluminal at large distances from compact sources

The UV cutoff around flat space is $\Lambda \sim (1000\ \mathrm{km})^{-1}$

Conclusions

We considered if the galileon can be promoted to a global scalar d.o.f. coupled someway to 4D GR;

neglecting gravity, we promoted the Poincaré group and our galilean invariance to the conformal group, then the existence of deSitter-like Solutions may be expected by symmetry considerations,

our local galilean Invariant terms are short-distance limit of globally defined conformally invariant Lagrangian terms.

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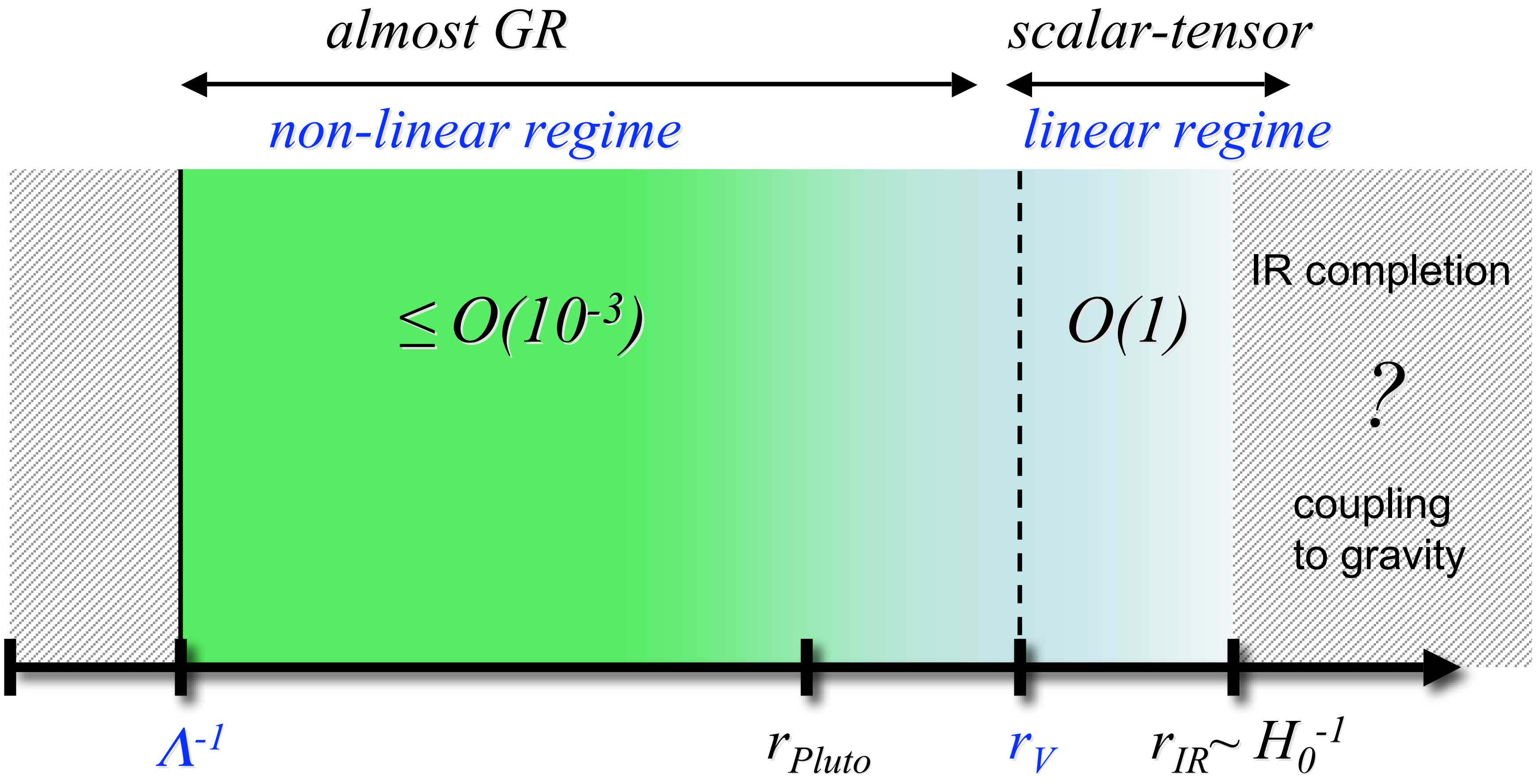
The galileon

$$\sqrt{-g}M_{\rm Pl}(1-2\pi)R + h_{\mu\nu}T^{\mu\nu} + \sum_{n=1}^{5} c_n\, \partial\pi\partial\pi(\partial\partial\pi)^{n-2}$$



Stable “self-accelerating” solutions

Spherically symm Vainshtein-like solutions around compact sources

Superluminal excitations at $r > r_V$

Very low strong-interaction scale for $\pi\pi$ scattering