**Planck 2009** – Padova, Italy – 25-29 May 2009

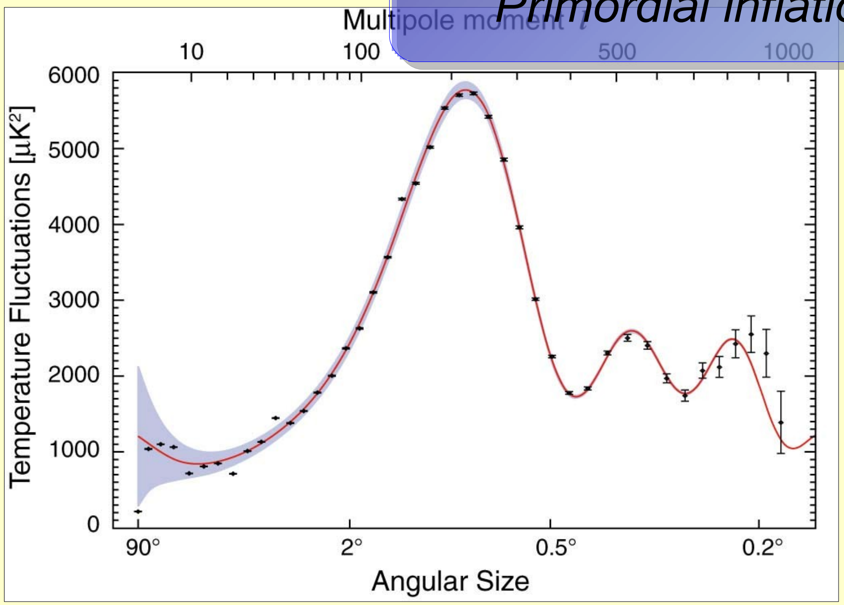
# de Sitter Entropy and the Volume of the Universe after Inflation

*Giovanni Villadoro*
CERN

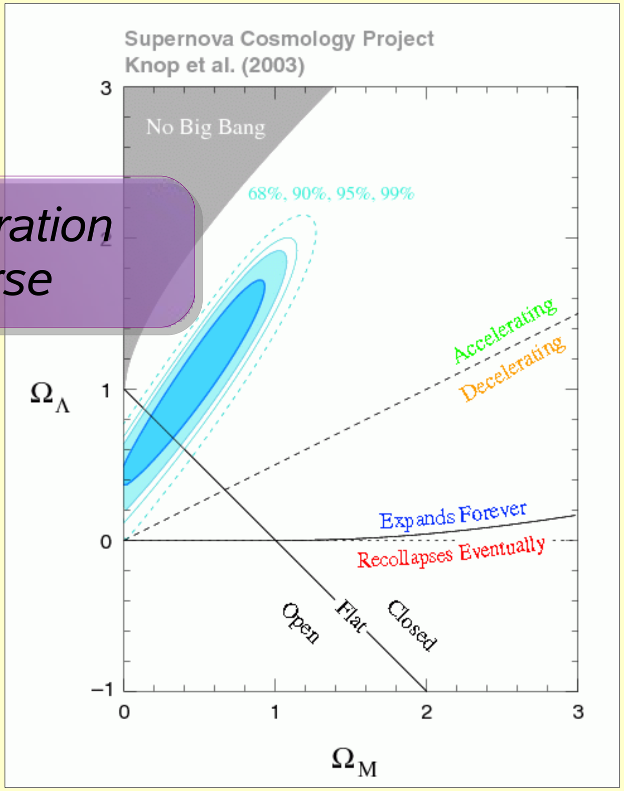
*work in collaboration with*
***S.Dubovsky and L.Senatore** – arXiv:**0812.2246** – JHEP 0904 (2009), 118*

## Cosmological data point to the existence of at least 2 phases of accellerated expansion of the Universe

*Primordial inflation*




*Current accelleration of the Universe*



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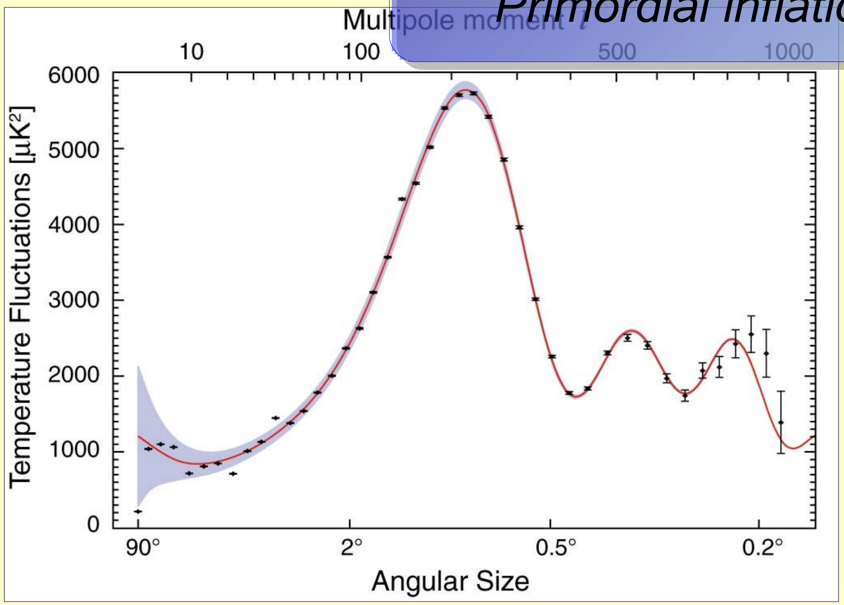


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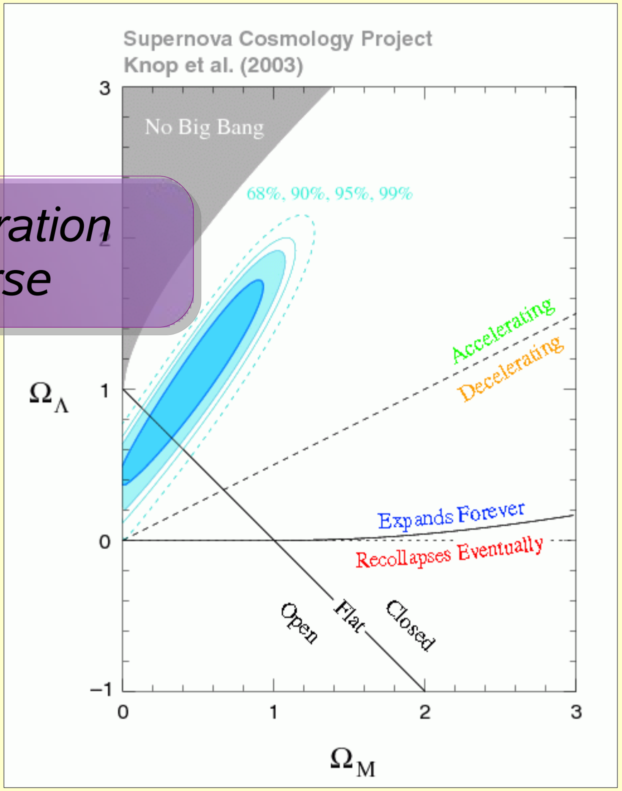
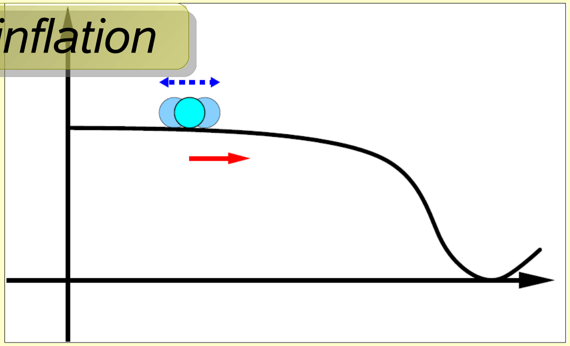



*Standard picture of primordial inflation*

*slow roll inflation*



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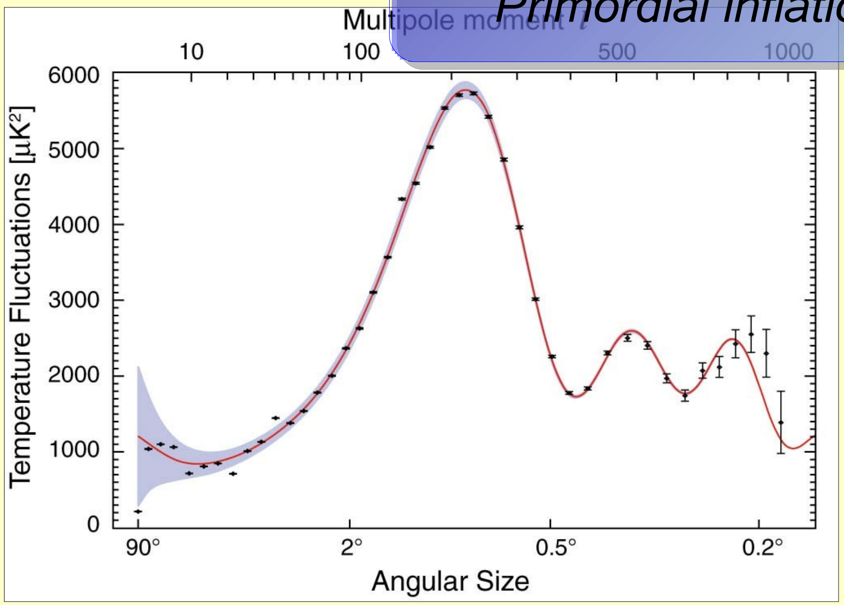


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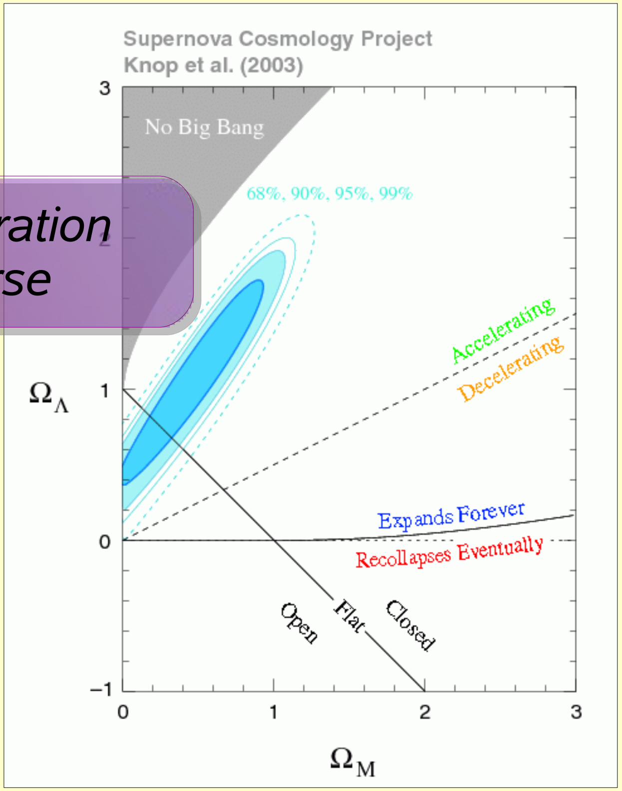


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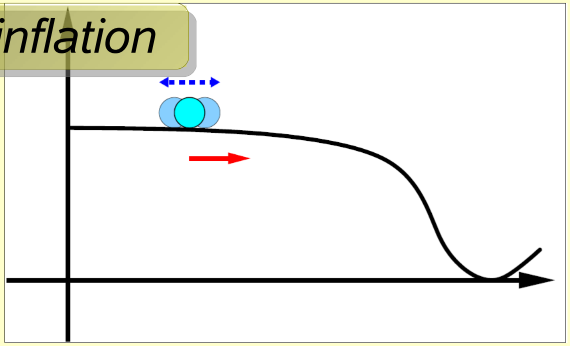
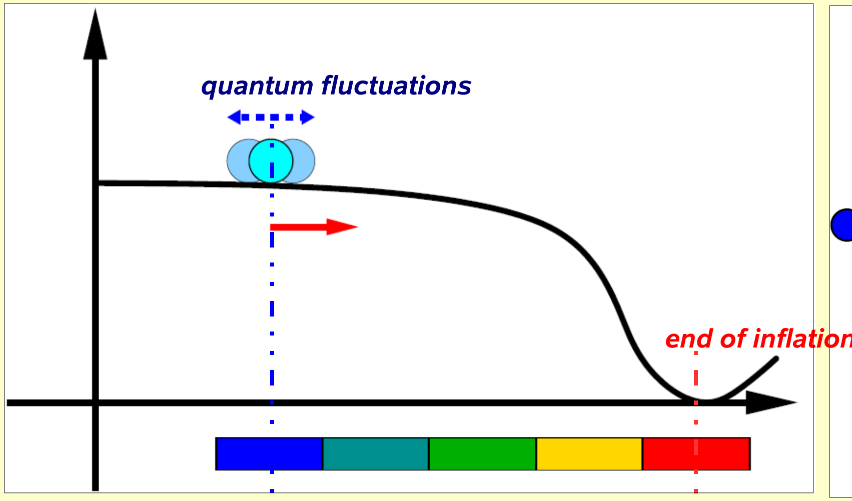


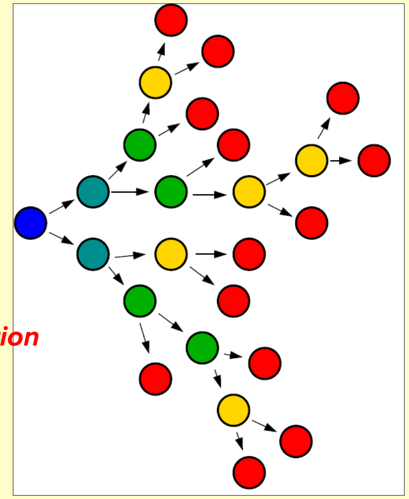
*Picture for the current accelleration?*

*- Cosmological constant?*

*- Light dof ?*

## *The two phases of inflation:*
## Non Eternal *vs* Eternal Inflation



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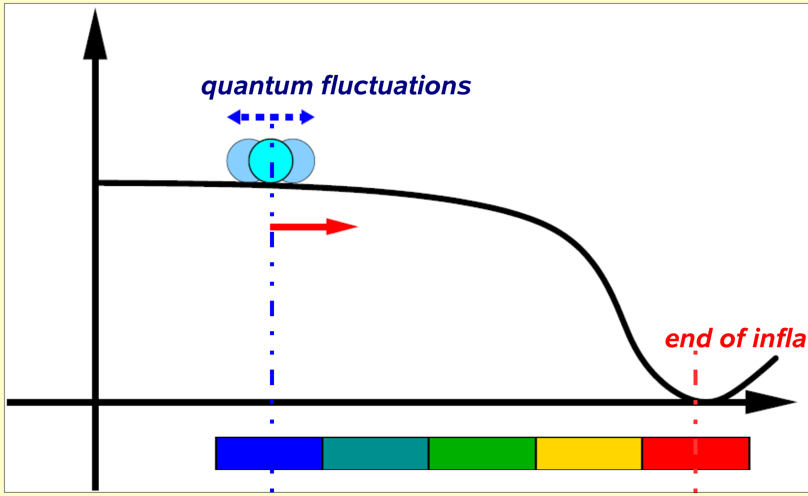
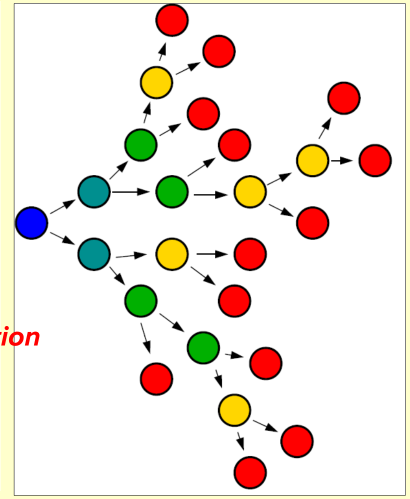
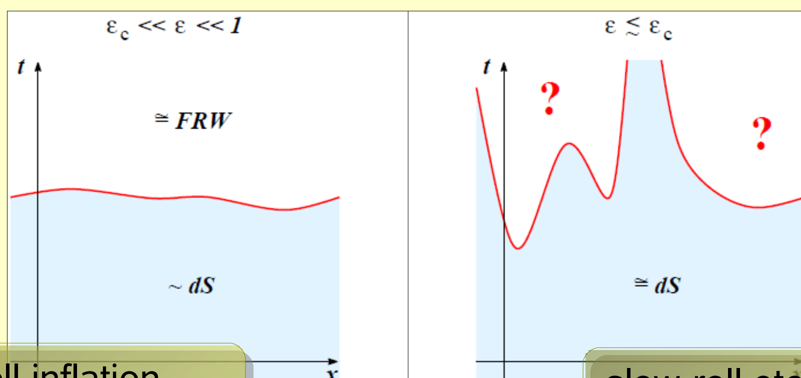


when $\delta\phi_q > \delta\phi_{cl} \sim \partial\phi / \partial N_e$

*eternal inflation regime*




slow-roll inflation

slow-roll eternal inflation

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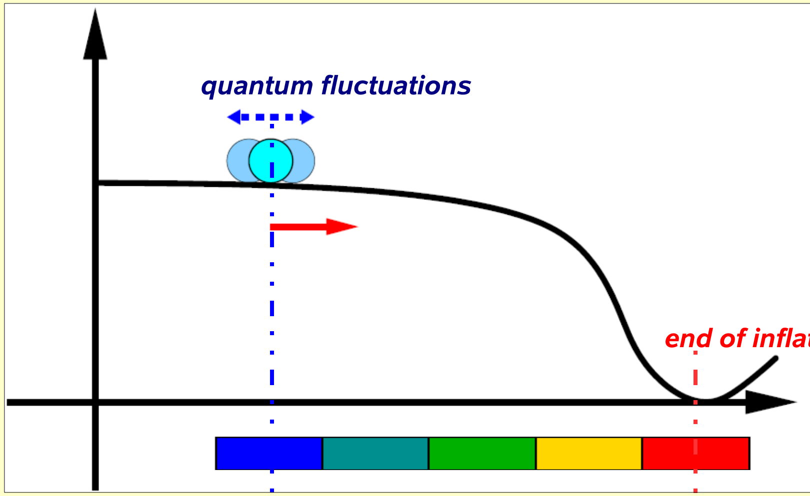
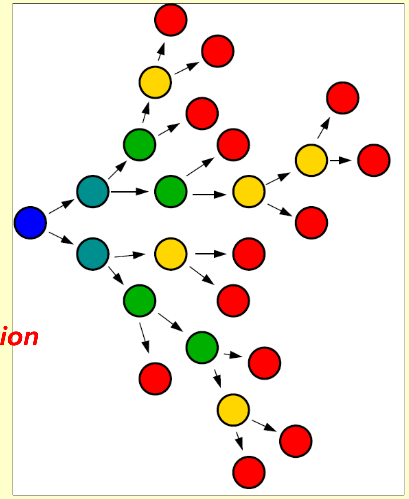
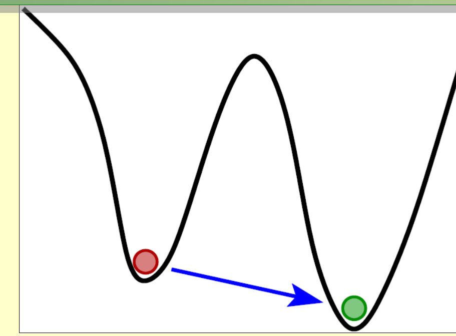


Analogously for false vacuum inflation



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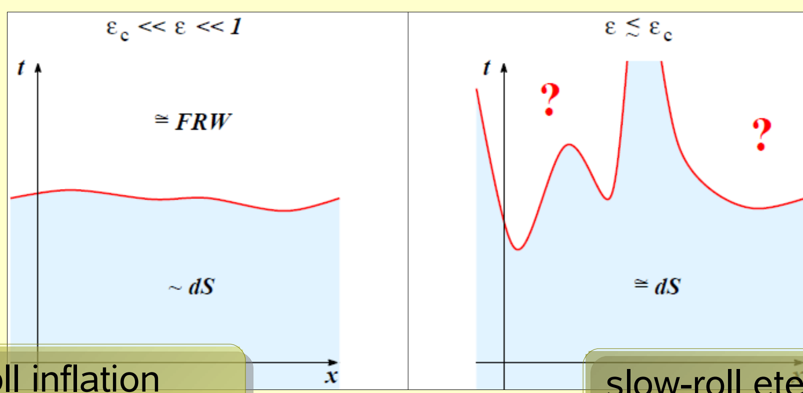
*eternal inflation regime*

$\Gamma > H^4 \Rightarrow$ non eternal inflation




slow-roll inflation

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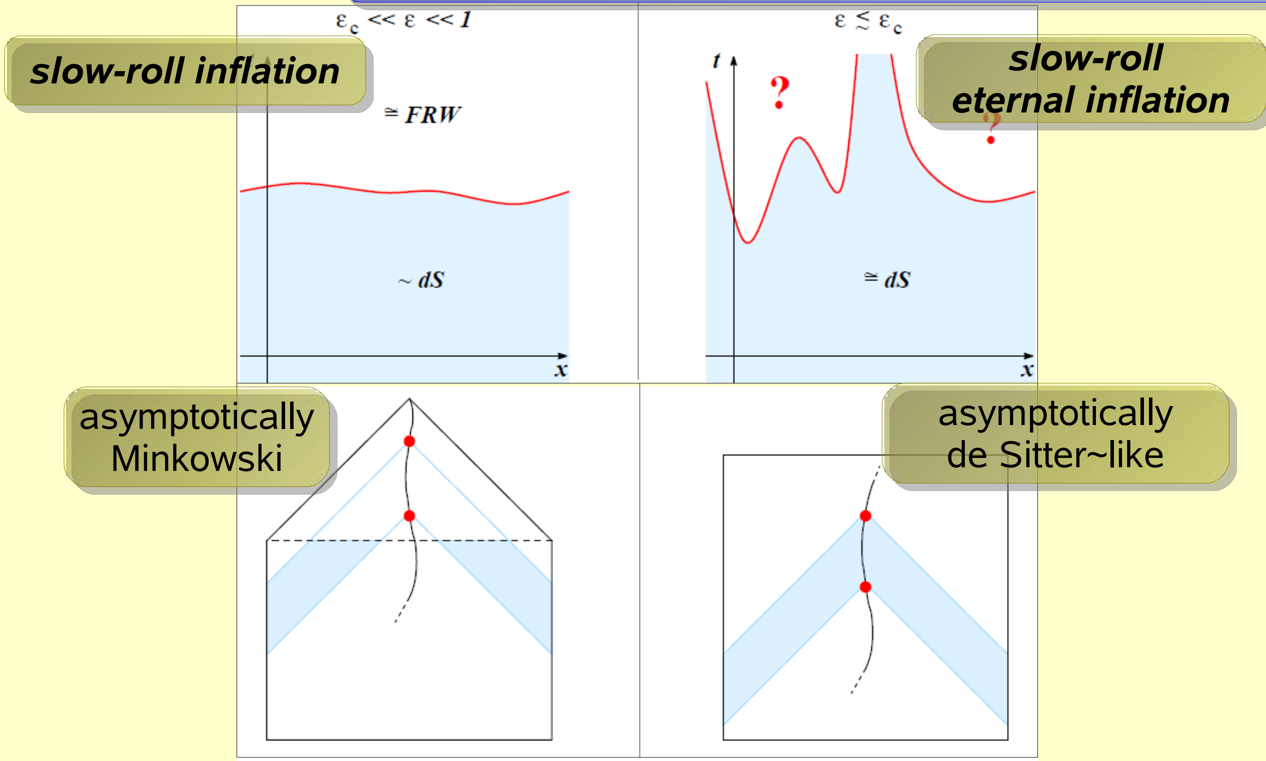
*an example*

SM Higgs-potential for light Higgs

# ...two deeply different regimes



# ...two deeply different regimes

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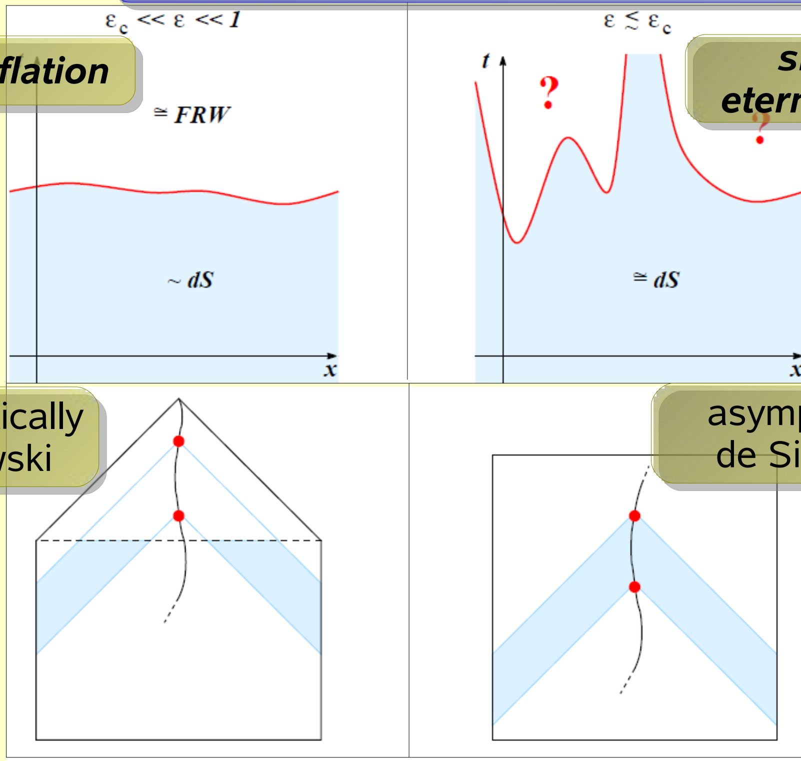
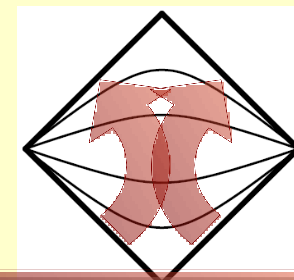



asymptotically Minkowski

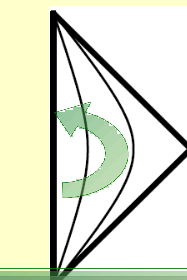
asymptotically de Sitter~like

**no local observables in Quantum Gravity**

*(local correlators are not gauge invariant)*



$S$-matrix in Minkowski



boundary CFT in AdS

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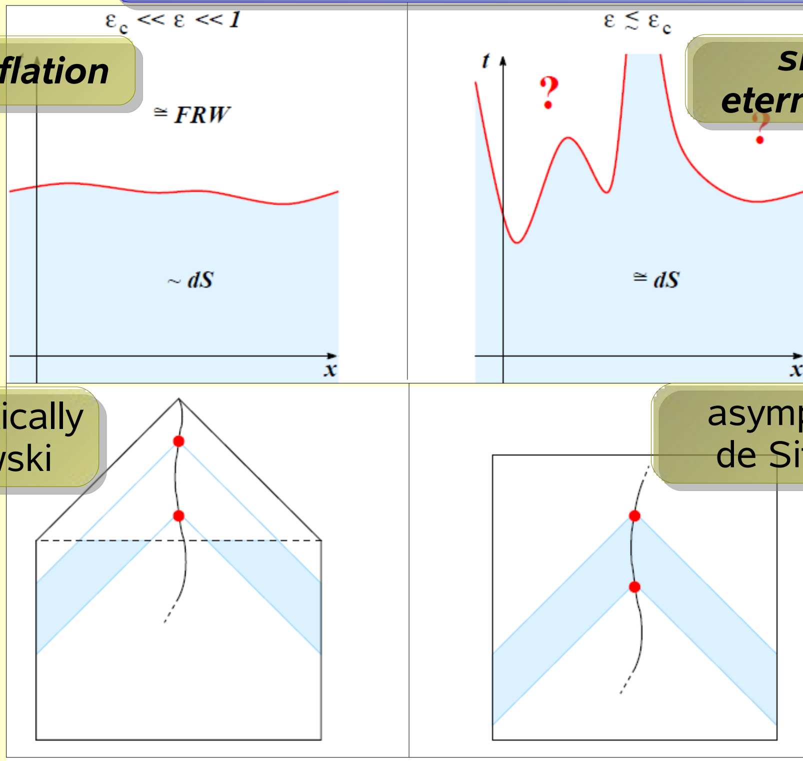


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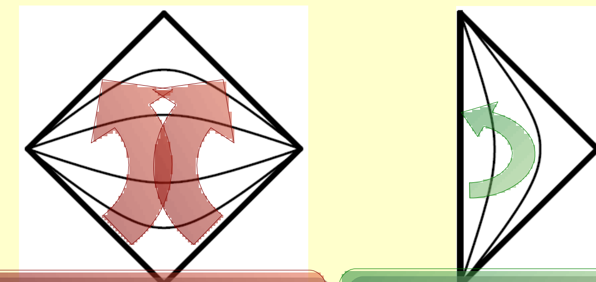


$S$-matrix in Minkowski

boundary CFT in AdS

Bekenstein bound

$$S \lesssim ER$$

In QG a finite density of states produces an irreducible uncertainty on local observables

$$\langle \phi(x)\phi(y) \rangle$$

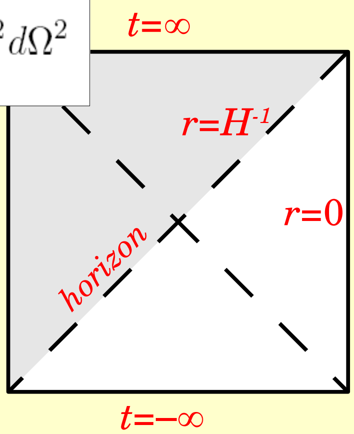
$$\#_{dof} \lesssim (x-y)^2/G \quad \Rightarrow \quad N \lesssim e^{\frac{(x-y)^2}{G}} \quad \Rightarrow \quad \delta_{\langle \phi(x)\phi(y) \rangle} \gtrsim e^{-\frac{(x-y)^2}{G}}$$

# Quantum Gravity and de Sitter space

$$ds^2 = -dt^2 + e^{2Ht}d\vec{x}^2$$

Exponentially expanding Universe

$$ds^2 = -\left(1 - \frac{1}{H^2r^2}\right)dt^2 + \left(1 - \frac{1}{H^2r^2}\right)^{-1}dr^2 + r^2d\Omega^2$$



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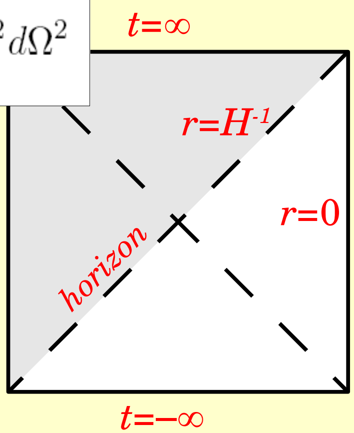


*Horizon ⇒ Hawking temperature* $T_{dS} \sim H$

de Sitter Entropy $S_{dS} = A_{dS} / 4G = \pi / GH^2$

de Sitter - infinite space-time with finite entropy, finite # of states

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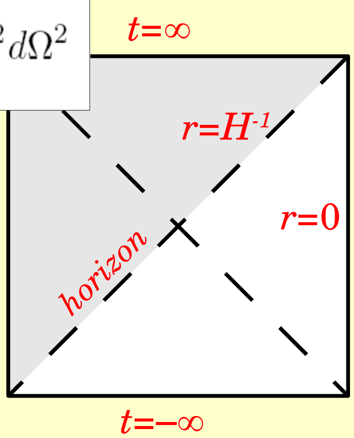


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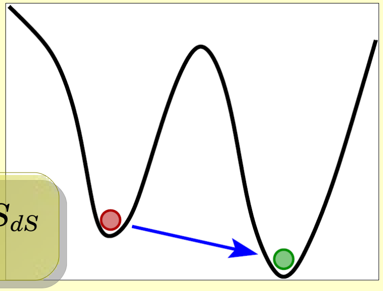
Poincaré recurrence $t \sim e^{S_{dS}}$

Decay rate (CdL/HM instantons) $\Gamma > e^{-S_{dS}}$



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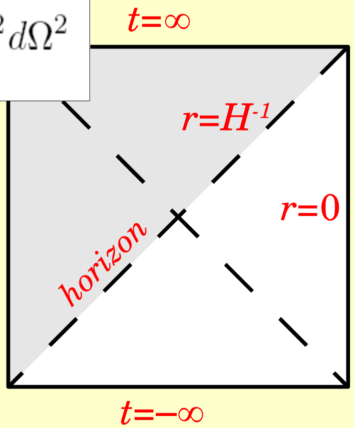


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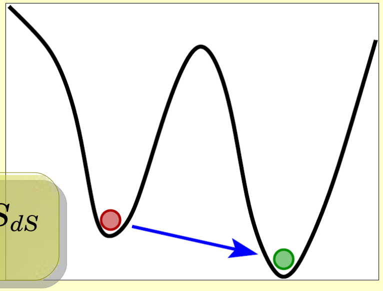

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*In Black Hole physics, EFT breaks down after a time* $t \sim S_{BH} R_S$,
from non-pert. non-local effects

analogously for de Sitter we may expect to EFT to break at $t \sim S_{dS} H^{-1}$, *indeed...*

# A bound on slow-roll inflation

*Arkani-Hamed, Dubovsky, Nicolis, Trincherini, GV - 2007*

Regularize de-Sitter via slow-roll inflation

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**upper bound before entering eternal inflation phase**

$$N_e < S_{dS}$$

$$\Leftrightarrow \quad t < S_{dS} H^{-1}$$

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**upper bound before entering eternal inflation phase**

$$N_e < S_{dS} \qquad \Leftrightarrow \qquad t < S_{dS} H^{-1}$$

*slow roll example:*

$$\langle \delta\phi^2 \rangle \sim H^3 t \quad \Rightarrow \quad \delta\phi_q \sim H$$

$$\dot{\phi}^2 \sim \dot{H}/G \quad \Rightarrow \quad \delta\phi_{cl} \sim \dot{\phi}/H \sim \left( \frac{\dot{H}}{GH^2} \right)^{1/2}$$

$$\left( \frac{\delta\phi_{cl}}{\delta\phi_q} \right)^2 \sim \frac{\dot{H}}{GH^4} \gtrsim 1 \quad \Rightarrow \quad \frac{dS_{dS}}{dN_e} \gtrsim 1 \quad \Rightarrow \quad N_e < S_{dS}$$

***Valid in general for theories not violating the NEC***

***holographic interpretation?***

***$V \sim e^{3N} \sim$ # of patches (indep. measurements?)***

***$e^S \sim$ # of states***

# defining a 'meta'-observable

## 1) sharpening the bound

$$\left(\frac{\delta\phi_{cl}}{\delta\phi_q}\right)^2 \sim \frac{\dot{H}}{GH^4} \gtrsim 1$$

# defining a 'meta'-observable

## 1) sharpening the bound

*Creminelli, Dubovsky, Nicolis, Senatore, Zaldarriaga '08*

$$\left(\frac{\delta\phi_{cl}}{\delta\phi_q}\right)^2 \sim \frac{\dot{H}}{GH^4} \gtrsim 1 \quad \Rightarrow \quad \frac{\dot{H}}{GH^4} > \frac{6}{\pi} \qquad \frac{dS_{dS}}{dN_e} > 12$$

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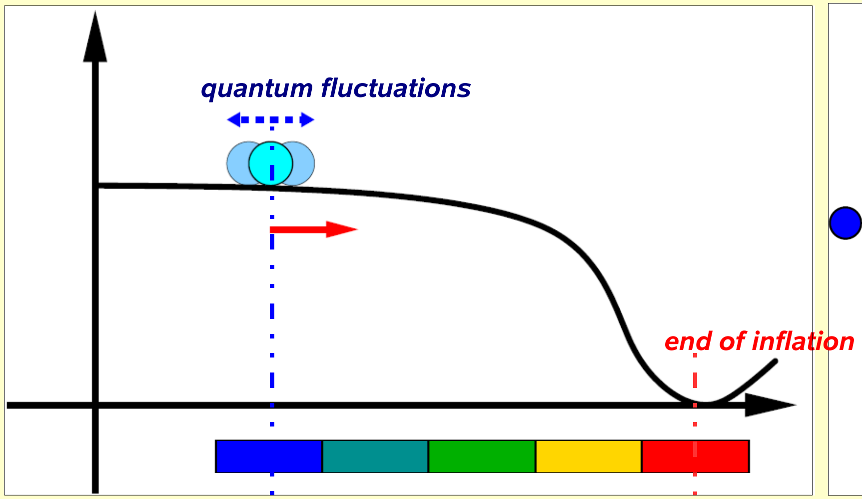
2) Because of quantum fluctuation, whatever $N_e$ is, there is always a finite probability to produce a volume $V > e^{3N_e}$

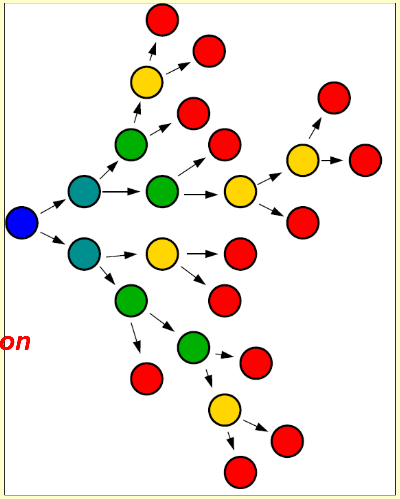
**To properly define the bound**
**we need to look at the probability distribution**
**of the Volume of the Universe after inflation – "$\rho(V,\phi)$"**

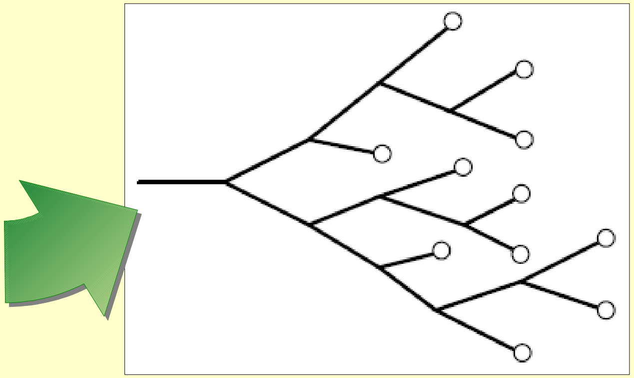
**<u>Defining the 'meta'-observable</u> $\rho(V,\phi)$ :**

***Probability of creating, at the end of inflation,***
***a volume V (as measured on the reheating surface)***
***starting with an inflaton value $\phi$***

# From the inflaton to a path integral





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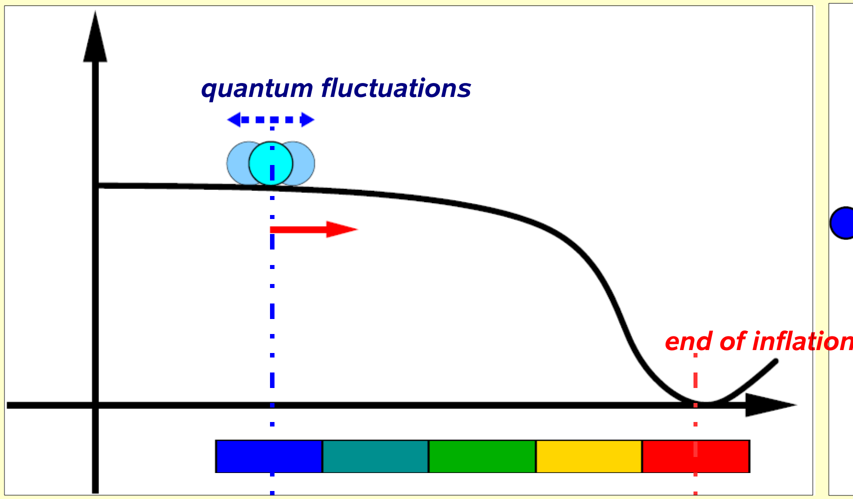
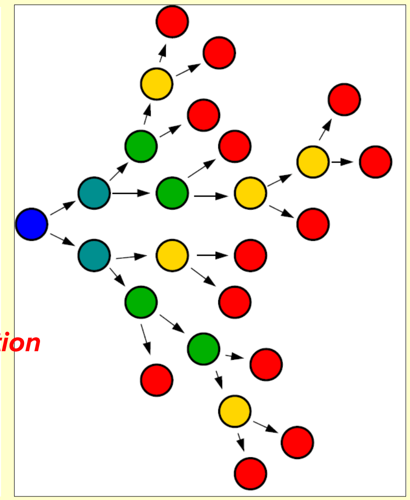
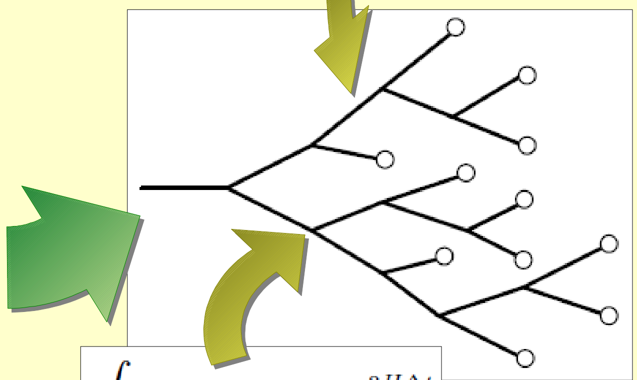


$$\langle \overline{\psi}(\phi_1)\psi(\phi_2)\rangle \propto P(\Delta t; \phi_1, \phi_2)$$



$$\int d\phi \psi(\phi)\overline{\psi}(\phi)^{e^{3H\Delta t}}$$

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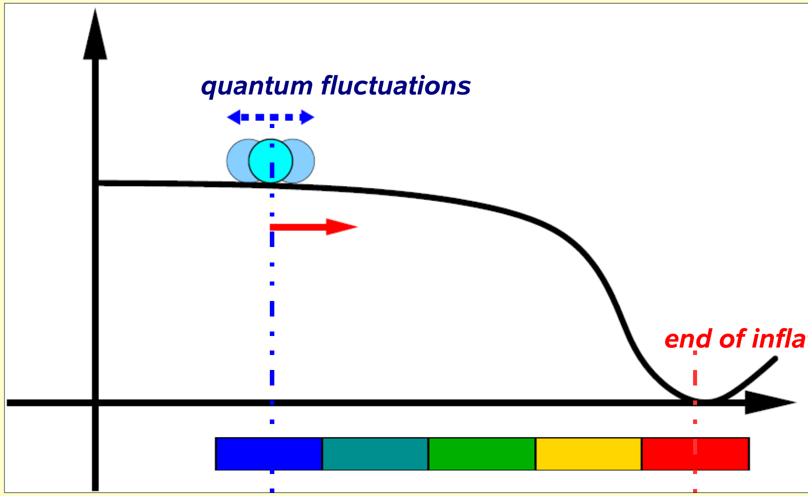


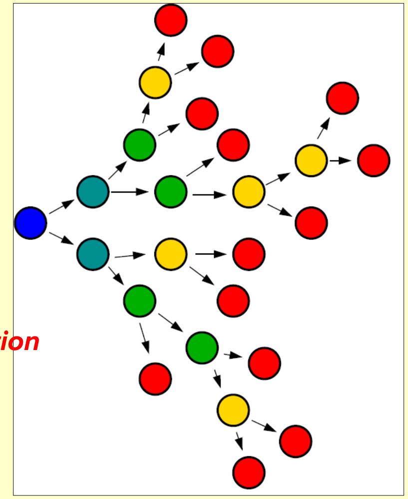

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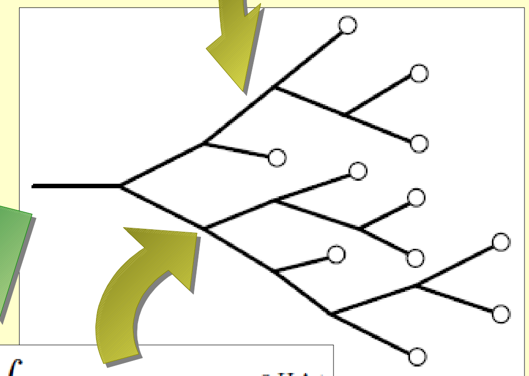

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## Fokker-Planck equation

$$\frac{4\pi^2}{H^3}\partial_t P(t,\phi_1,\phi_2) = \frac{1}{2}\partial^2_{\phi_2} P(t,\phi_1,\phi_2) + \frac{2\sqrt{6\pi^2\Omega}}{H}\partial_{\phi_2} P(t,\phi_1,\phi_2)$$

$$\Omega \equiv \frac{2\pi^2}{3}\frac{\dot{\phi}^2}{H^4}$$

$$\tau = 2\pi\sqrt{6}\phi/H$$

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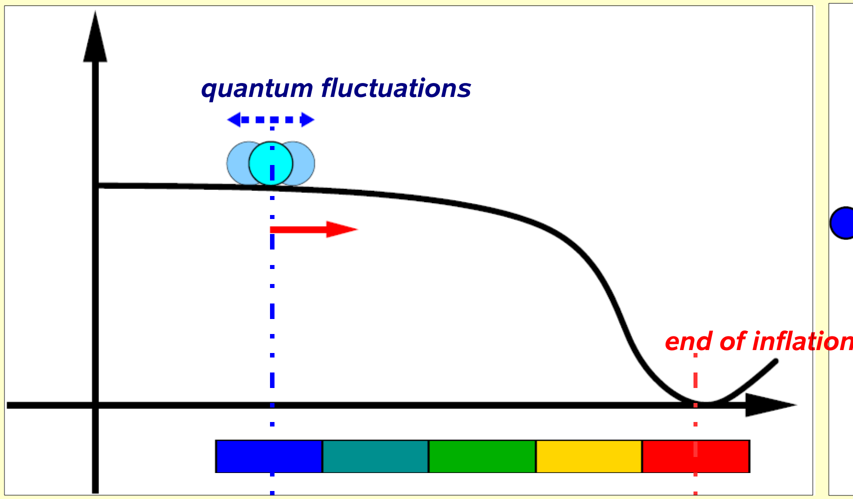


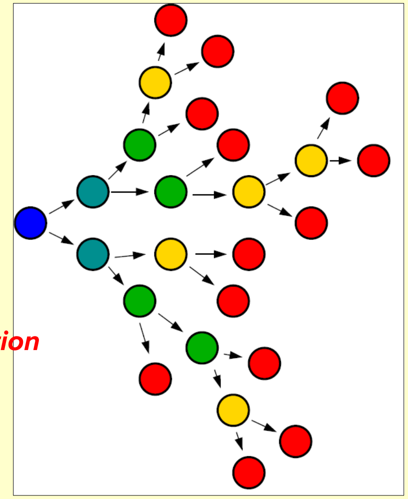

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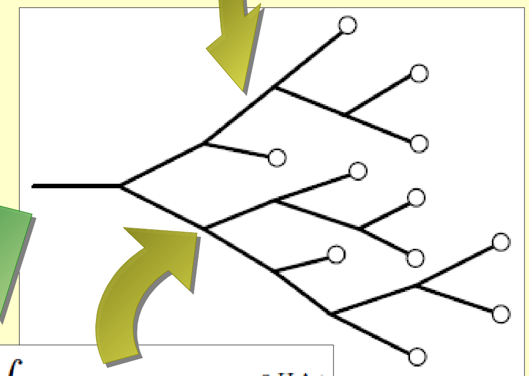

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## Path Integral representation

$$\rho(k;\phi) \quad \sim \quad \frac{1}{k!}\int \mathcal{D}\psi\mathcal{D}\overline{\psi}\ e^{iS(\psi,\overline{\psi})}\ \overline{\psi}(\phi)\psi(0)^k$$

$$S(\psi,\overline{\psi}) \quad = \quad -\int d\phi_1 d\phi_2\ \psi(\phi_1)P^{-1}(\Delta t;\phi_1,\phi_2)\overline{\psi}(\phi_2) + \int d\phi\psi(\phi)\overline{\psi}(\phi)^{e^{3H\Delta t}}$$

# from the path integral to a differential equation

## Continuum limit

$$Z(j,\bar{\jmath}) = \int \mathcal{D}\psi(\phi)\mathcal{D}\overline{\psi}(\phi)\; e^{iS(\psi,\overline{\psi})+i\int d\phi\,(j\psi+\bar{\jmath}\overline{\psi})}$$

$$S = \psi(0)\overline{\psi}(0) + 3H\Delta t \int d\tau\psi(\tau)\left(\overline{\psi}''(\tau) - 2\sqrt{\Omega}\overline{\psi}'(\tau) + \overline{\psi}(\tau)\log[\overline{\psi}(\tau)]\right)$$

$$\rho(V) = \frac{1}{V!}\langle\overline{\psi}(\tau)[\psi(0)]^V\rangle$$

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$$\rho(V) = \frac{1}{V!}\langle\overline{\psi}(\tau)[\psi(0)]^V\rangle \quad\Longrightarrow\quad \rho(V,\tau) \;=\; \int_{-i\infty}^{i\infty} dz\, e^{-Vz}\,\langle\overline{\psi}(\tau)\rangle_{j(z)}$$

$$j(z) = -e^{-z}\delta(\tau)$$

$$\begin{aligned}
\frac{\partial\Gamma}{\partial\psi(\tau)} &= \overline{\psi}''(\tau) - 2\sqrt{\Omega}\overline{\psi}'(\tau) + \overline{\psi}(\tau)\log[\overline{\psi}(\tau)] = 0\\
\frac{\partial\Gamma}{\partial\psi(0)} &= -\overline{\psi}(0) = -e^{-z}
\end{aligned}$$

# from the path integral to a differential equation

## Continuum limit

$$Z(j,\bar{j}) = \int \mathcal{D}\psi(\phi)\mathcal{D}\overline{\psi}(\phi)\; e^{iS(\psi,\overline{\psi})+i\int d\phi\,(j\psi+\bar{j}\overline{\psi})}$$

$$S = \psi(0)\overline{\psi}(0) + 3H\Delta t \int d\tau\,\psi(\tau)\left(\overline{\psi}''(\tau) - 2\sqrt{\Omega}\overline{\psi}'(\tau) + \overline{\psi}(\tau)\log[\overline{\psi}(\tau)]\right)$$

$$\rho(V) = \frac{1}{V!}\langle\overline{\psi}(\tau)[\psi(0)]^V\rangle \quad \Rightarrow \quad \rho(V,\tau) \;=\; \int_{-i\infty}^{i\infty} dz\, e^{-Vz}\,\langle\overline{\psi}(\tau)\rangle_{j(z)}$$

$$j(z) = -e^{-z}\delta(\tau)$$

$$\begin{aligned}\frac{\partial\Gamma}{\partial\psi(\tau)} &= \overline{\psi}''(\tau) - 2\sqrt{\Omega}\overline{\psi}'(\tau) + \overline{\psi}(\tau)\log[\overline{\psi}(\tau)] = 0\\ \frac{\partial\Gamma}{\partial\psi(0)} &= -\overline{\psi}(0) = -e^{-z}\end{aligned}$$

## Summarizing

$$\rho(V,\tau) = \frac{1}{2\pi i}\int_{0^+-i\infty}^{0^++i\infty} dz\, f(\tau;z)e^{zV}\,,$$

$$\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log[f(\tau;z)] = 0\,,$$

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$$\dot{f}(\tau_b;z) = 0\,,$$

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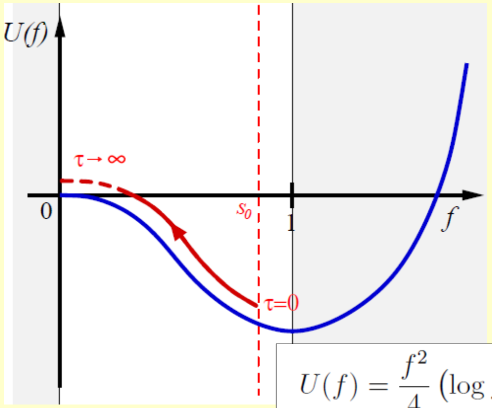


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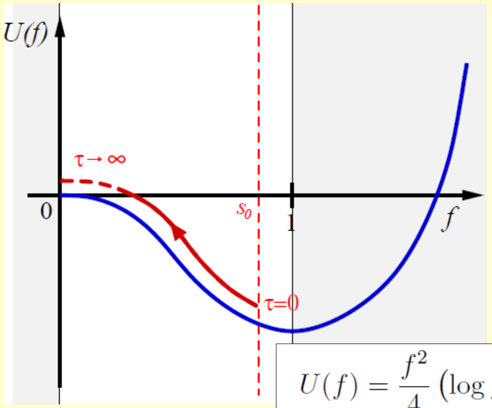


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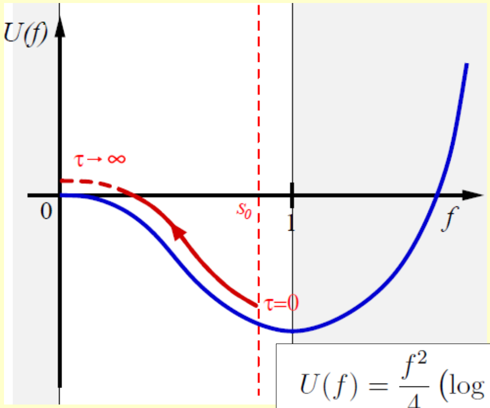


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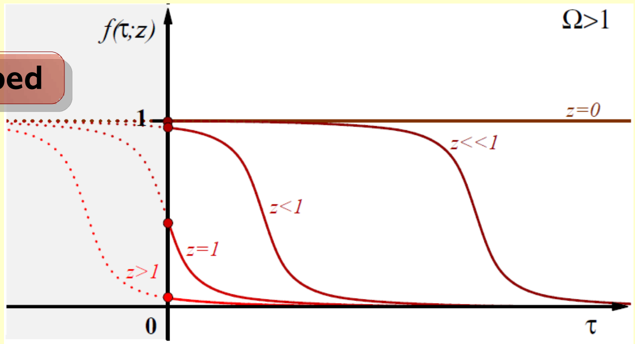
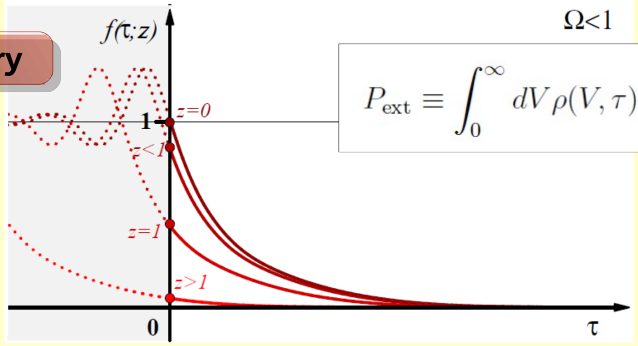
**overdumped**




**oscillatory**

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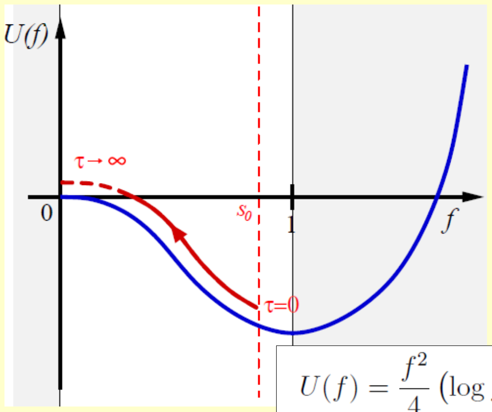


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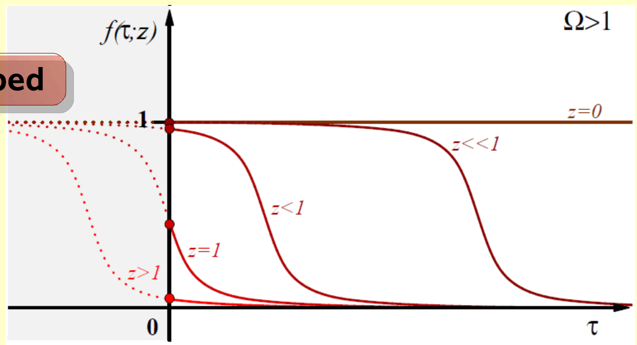


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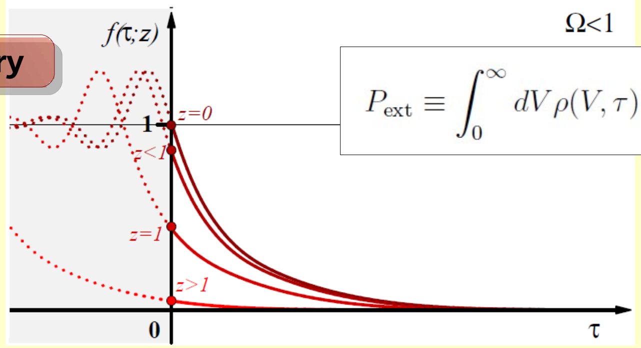


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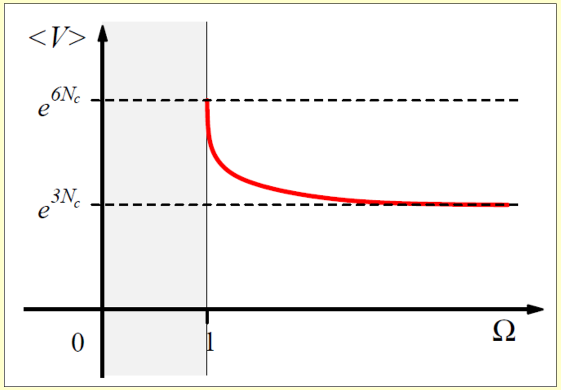
## e.g. Average

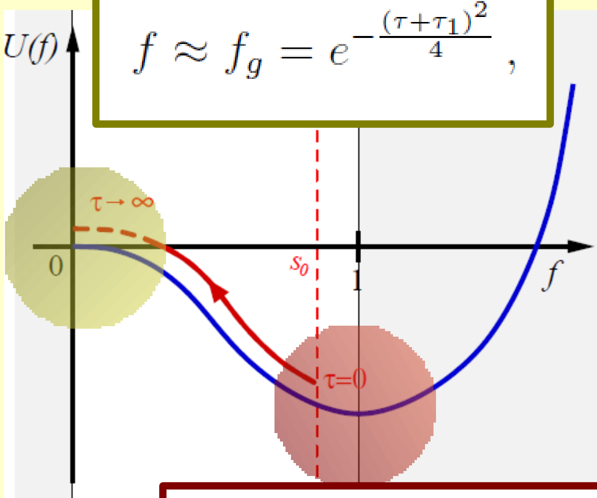
$$\langle V\rangle = -f_0'(\tau) = \frac{e^{\omega_+\tau+\omega_-\tau_b} - \omega_+^2 e^{\omega_-\tau+\omega_+\tau_b}}{e^{\omega_-\tau_b} - \omega_+^2 e^{\omega_+\tau_b}}$$

$$\lim_{\tau_b\to\infty}\langle V\rangle = e^{\omega_-\tau} = e^{3N_c\frac{2}{1+\sqrt{1-1/\Omega}}}\,.$$



# extimating the probability distribution

$$f \approx f_g = e^{-\frac{(\tau+\tau_1)^2}{4}} \,,$$

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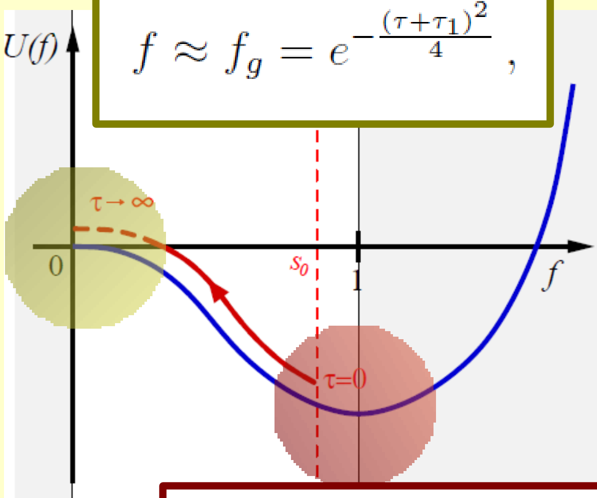


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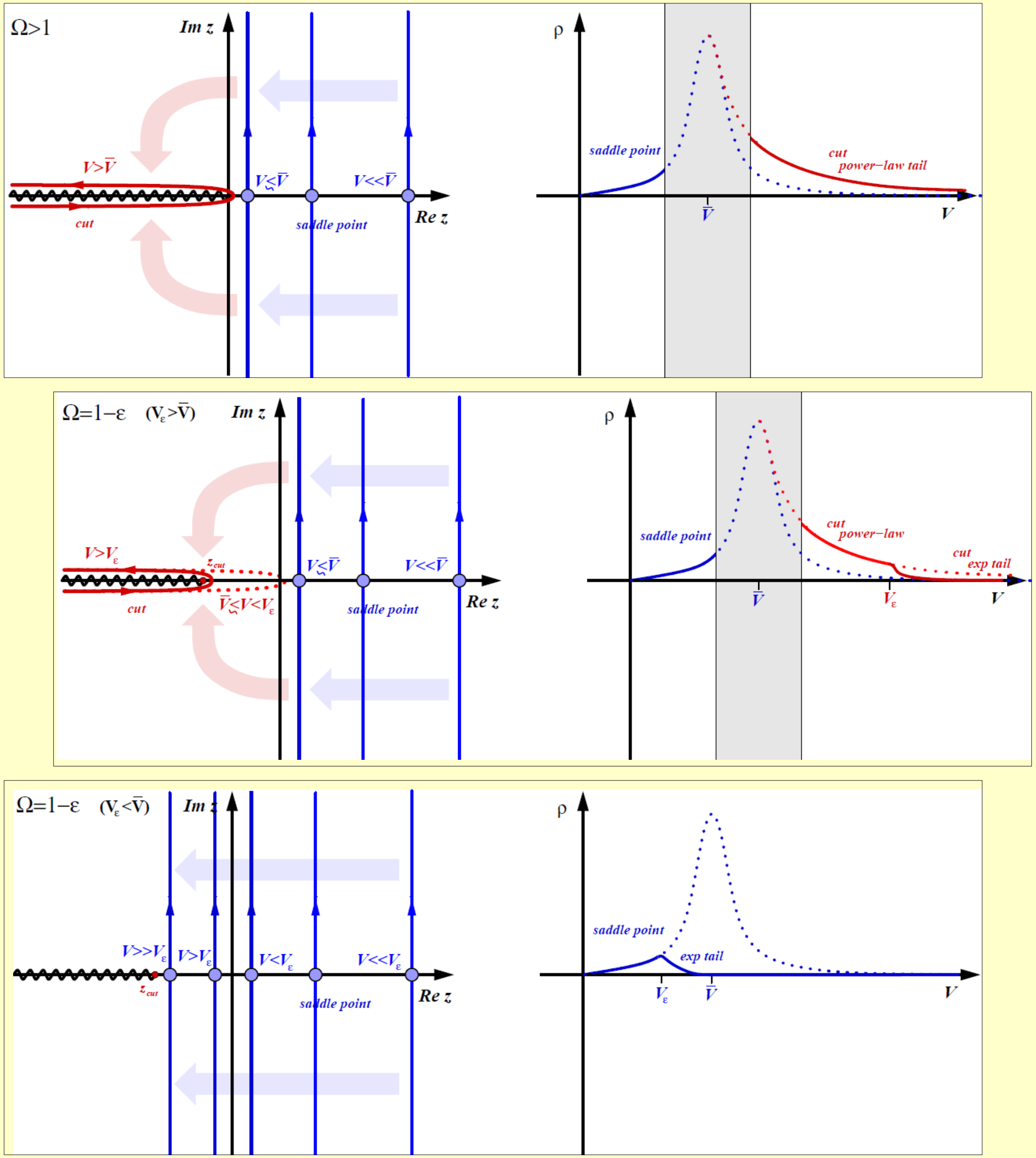

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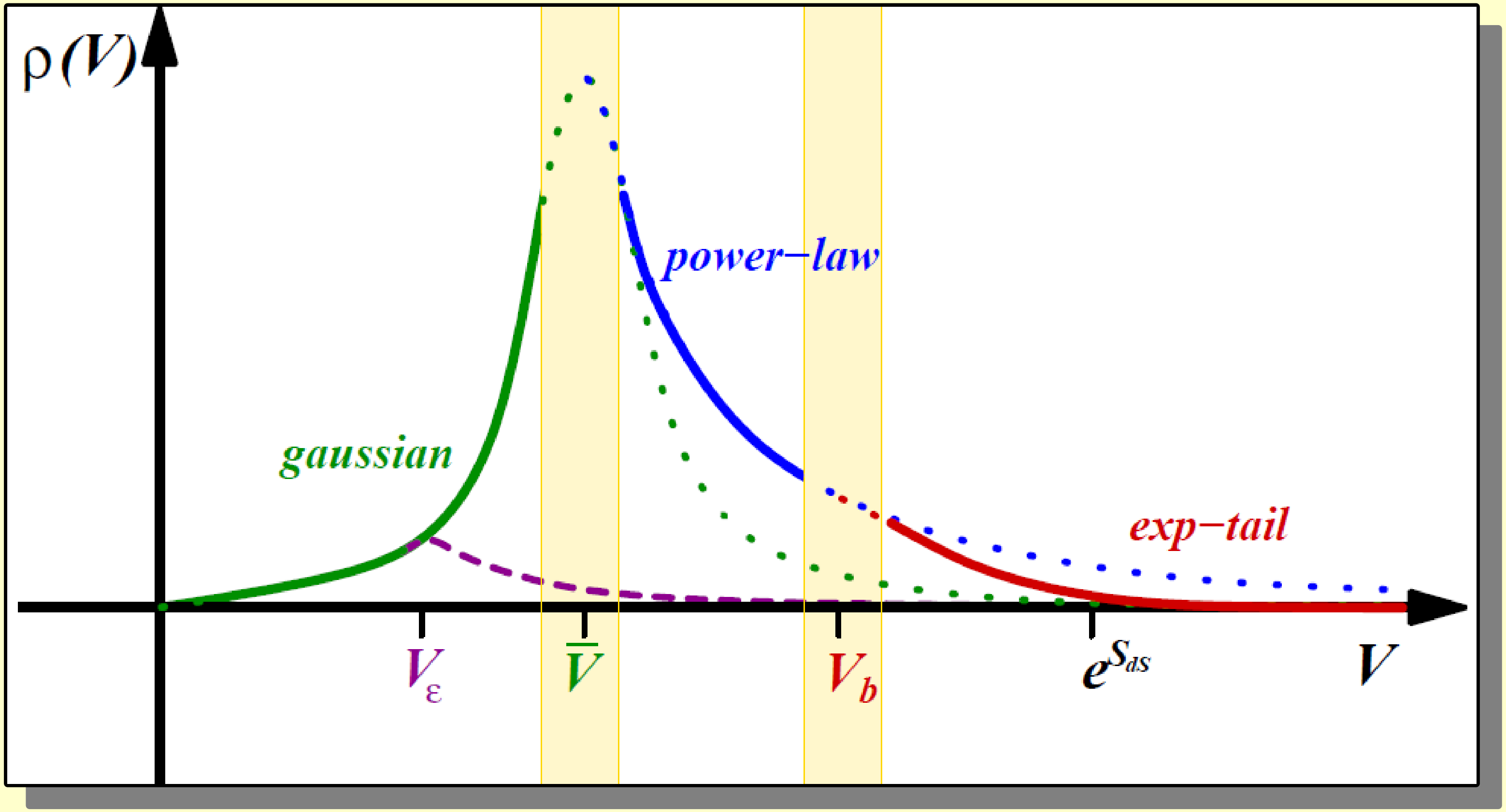

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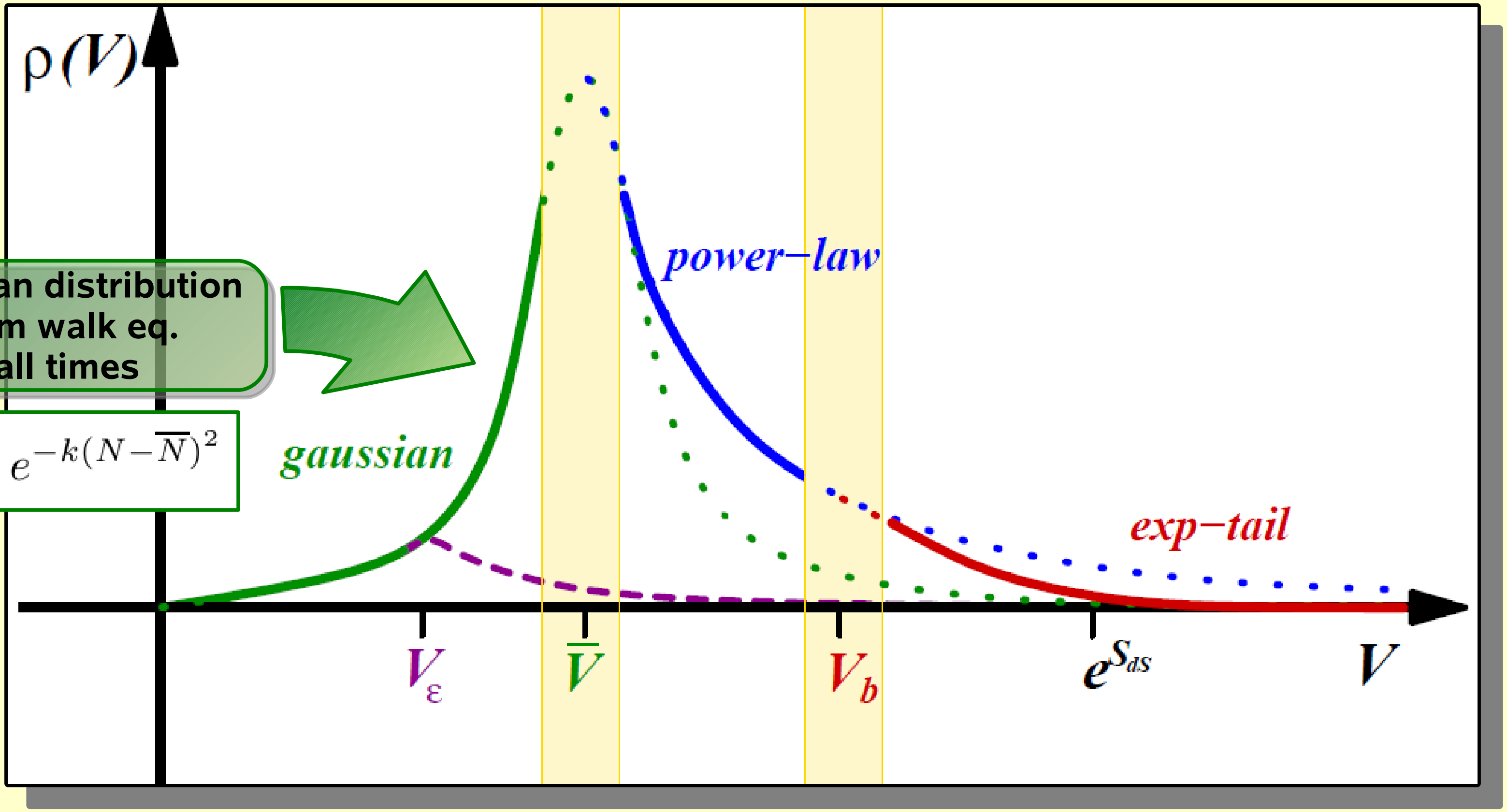


**from gaussian distribution of random walk eq. at small times**

$$e^{-\frac{(\phi-\overline{\phi})^2}{2\sigma^2}} \sim e^{-k(N-\overline{N})^2}$$

# the result

$$e^{-k\frac{(\phi-\overline{\phi})^2}{\Delta\phi}} \sim e^{-k(N-\overline{N})}$$

from exp. distribution
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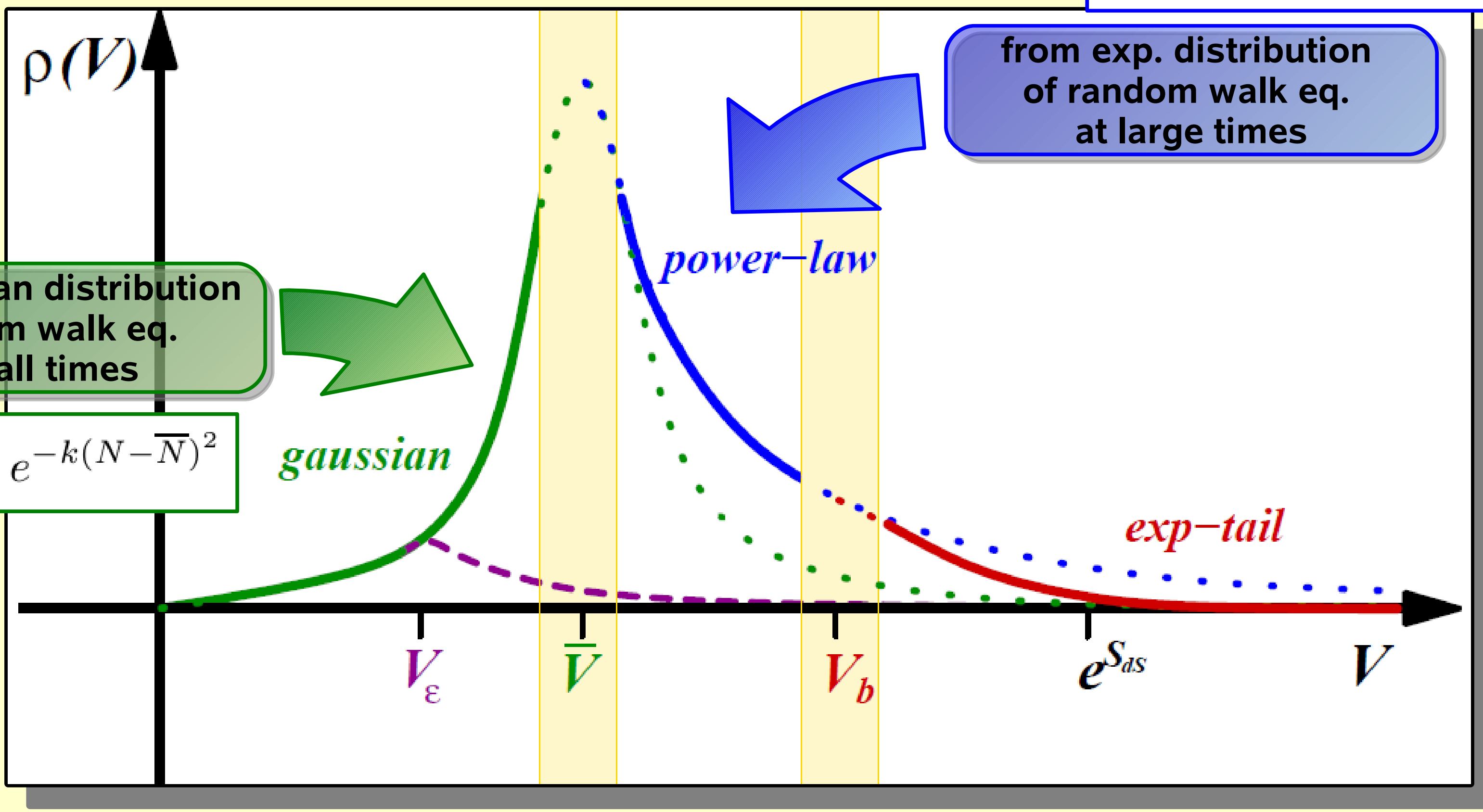

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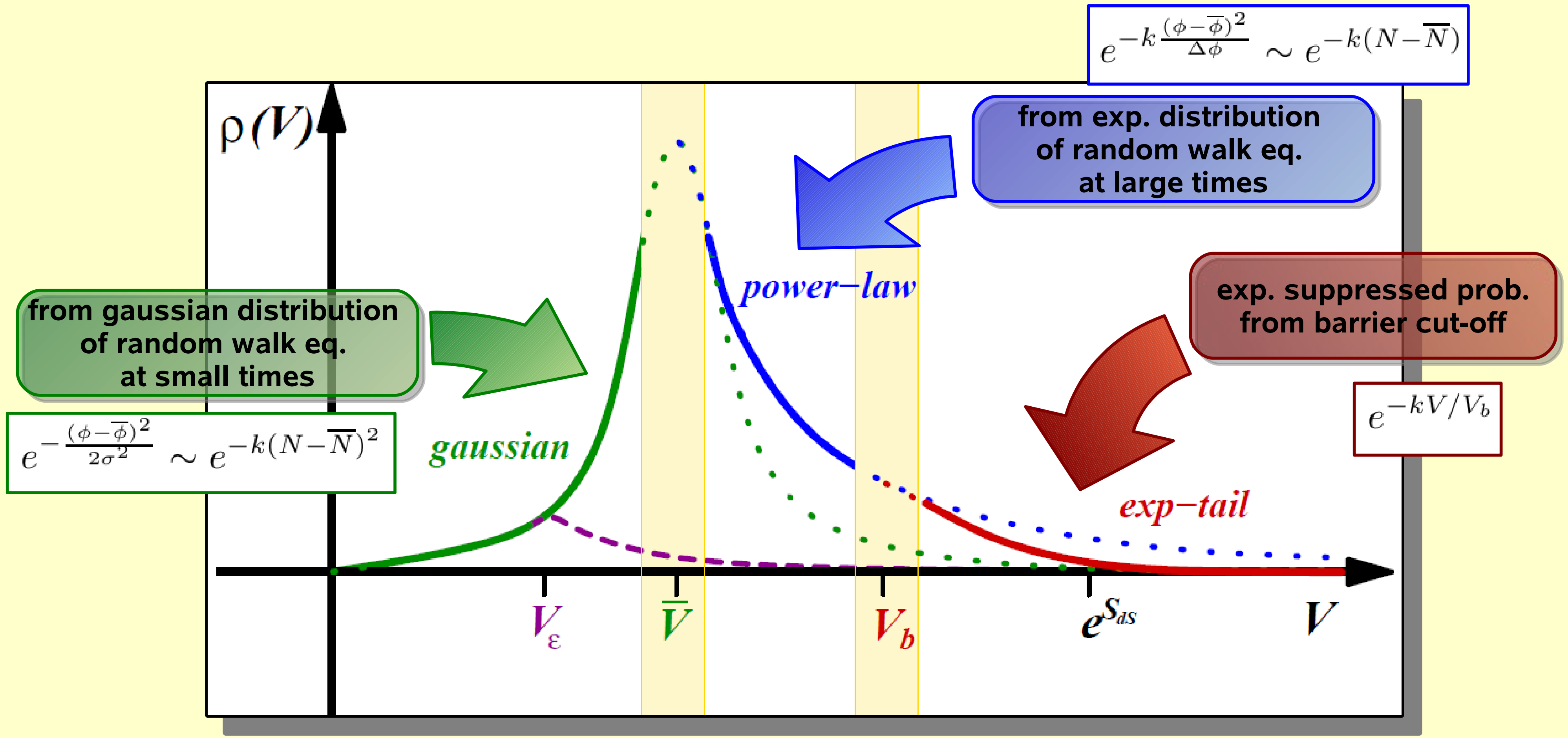

$e^{-k\frac{(\phi-\overline{\phi})^2}{\Delta\phi}} \sim e^{-k(N-\overline{N})}$
from exp. distribution
of random walk eq.
at large times
exp. suppressed prob.
from barrier cut-off
$e^{-kV/V_b}$
from gaussian distribution
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ρ (V)
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power−law
exp−tail
$V_\varepsilon$
$\overline{V}$
$V_b$
$e^{S_{ds}}$
V

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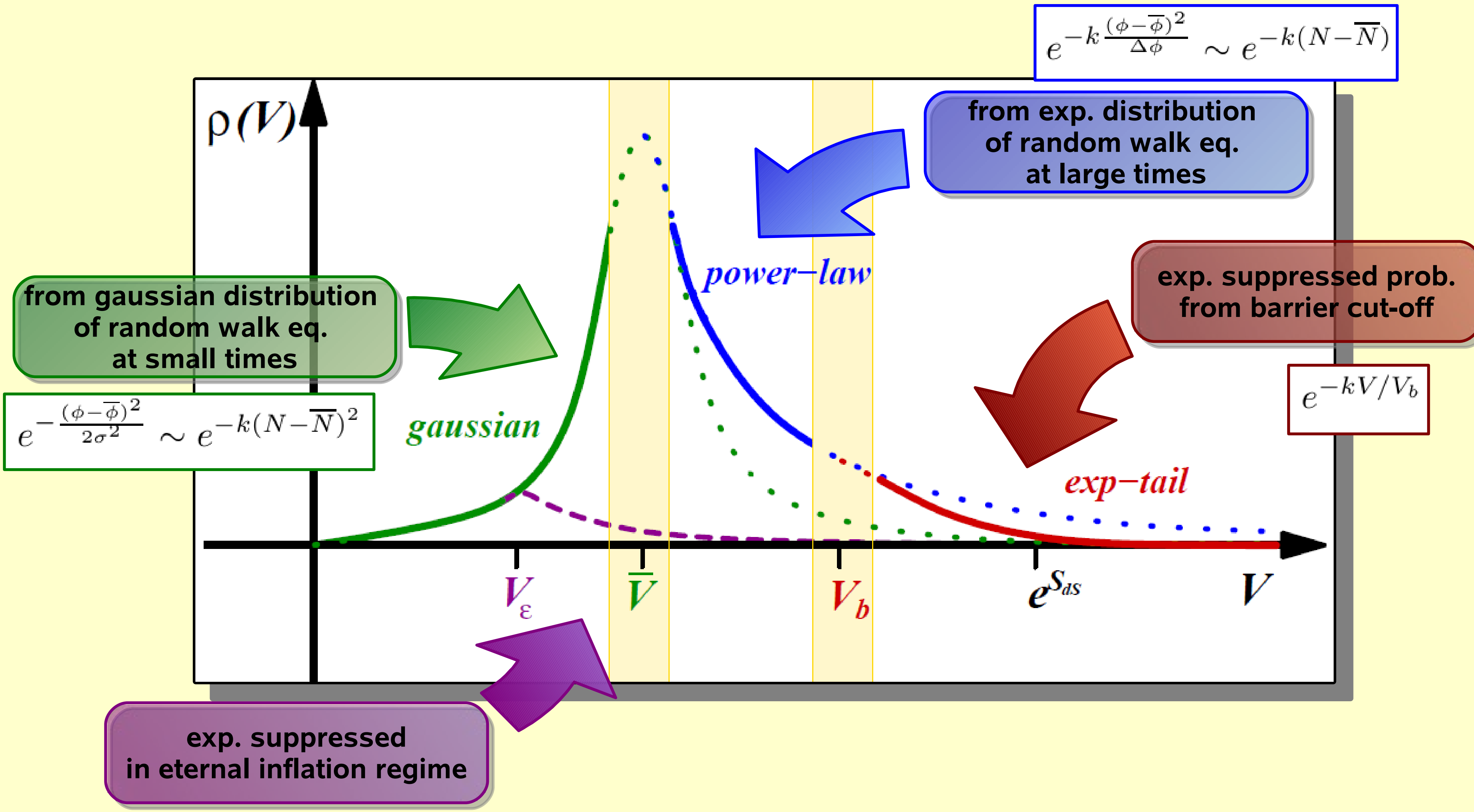

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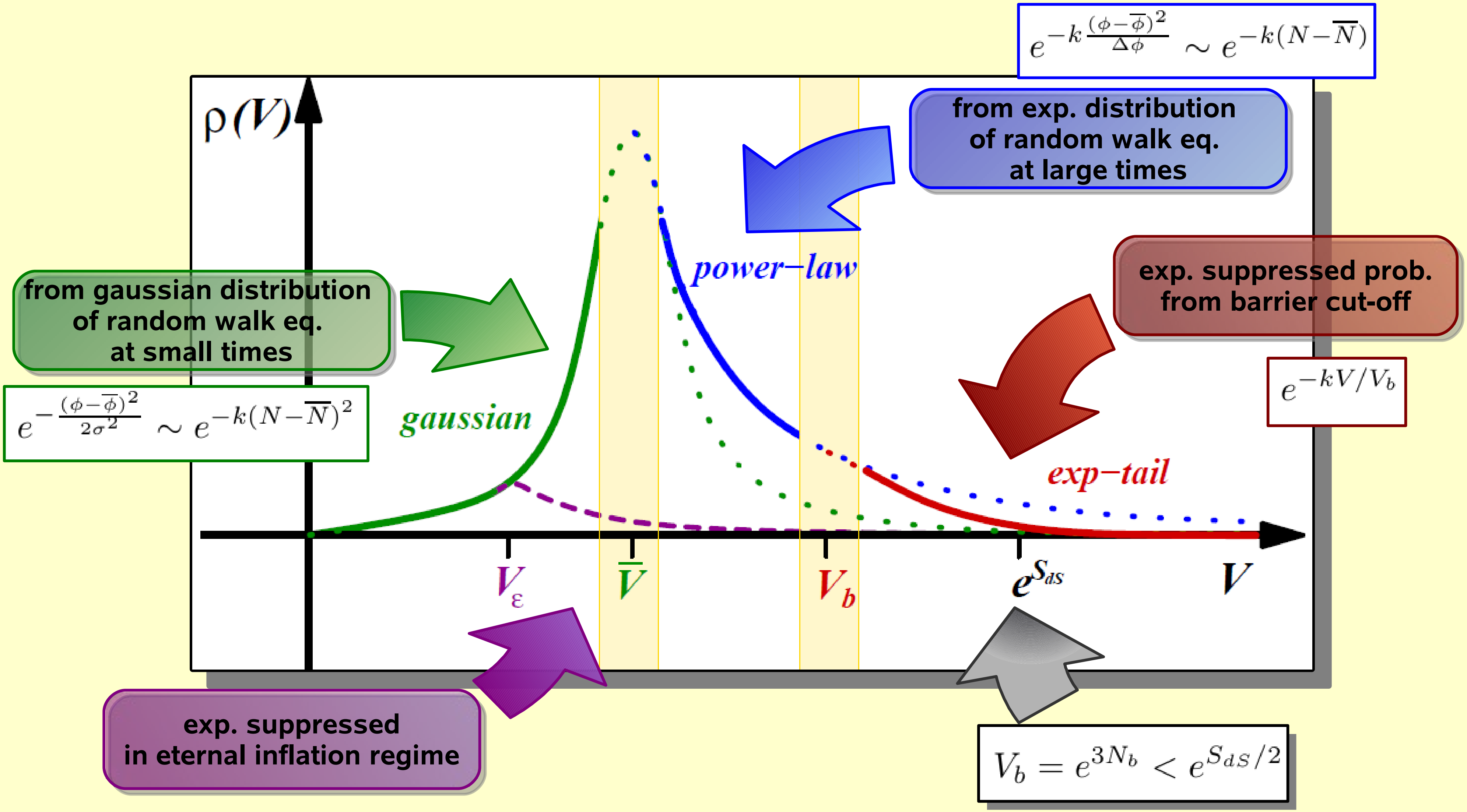

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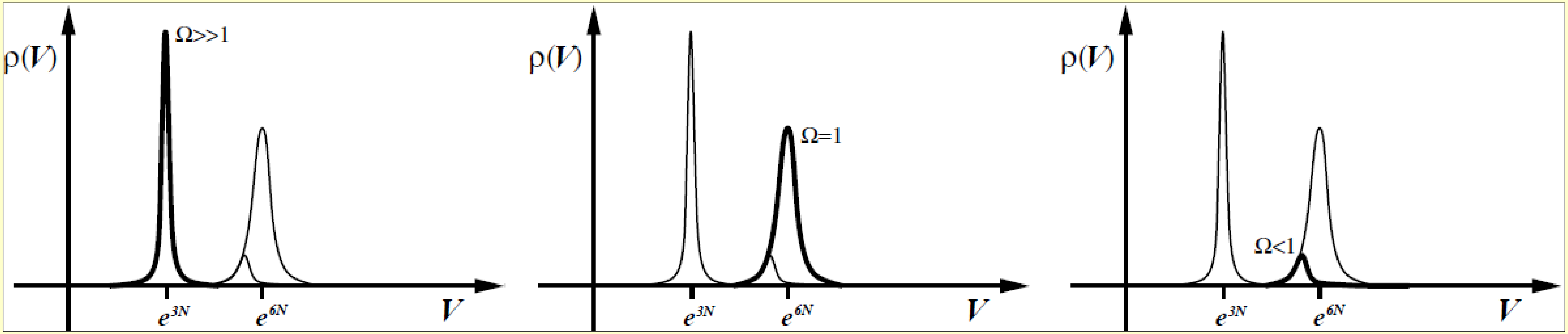
exp. suppressed in eternal inflation regime
$V_b = e^{3N_b} < e^{S_{dS}/2}$

# $\rho(V)$ and the bound



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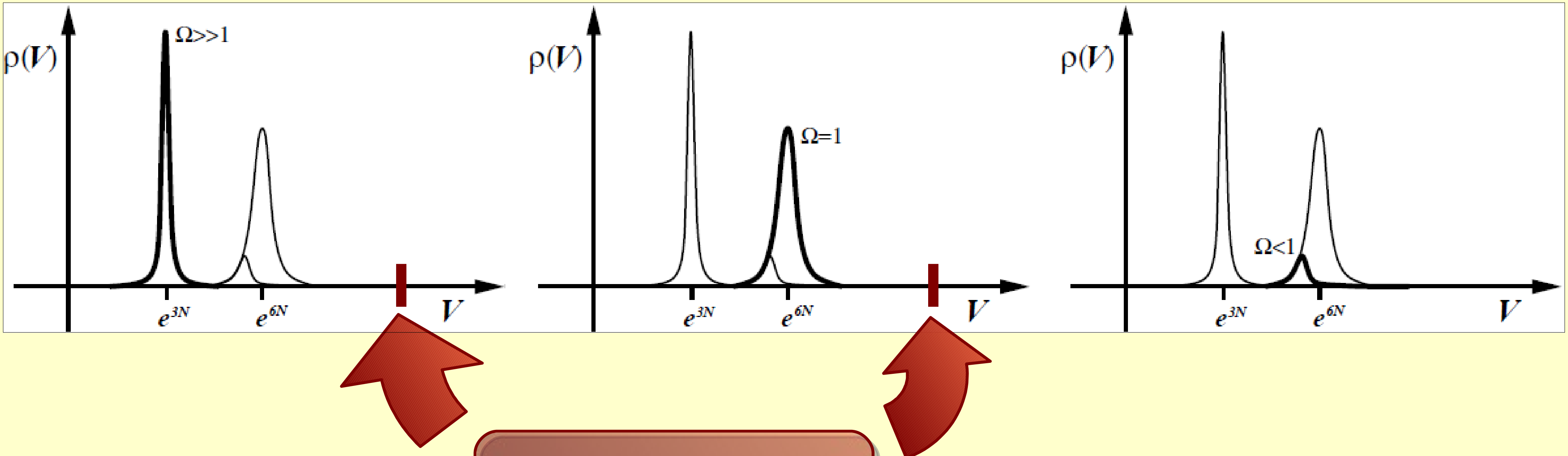


$$\Omega \geq 1 \quad \Rightarrow \quad e^{S_{dS}} > e^{12N}$$

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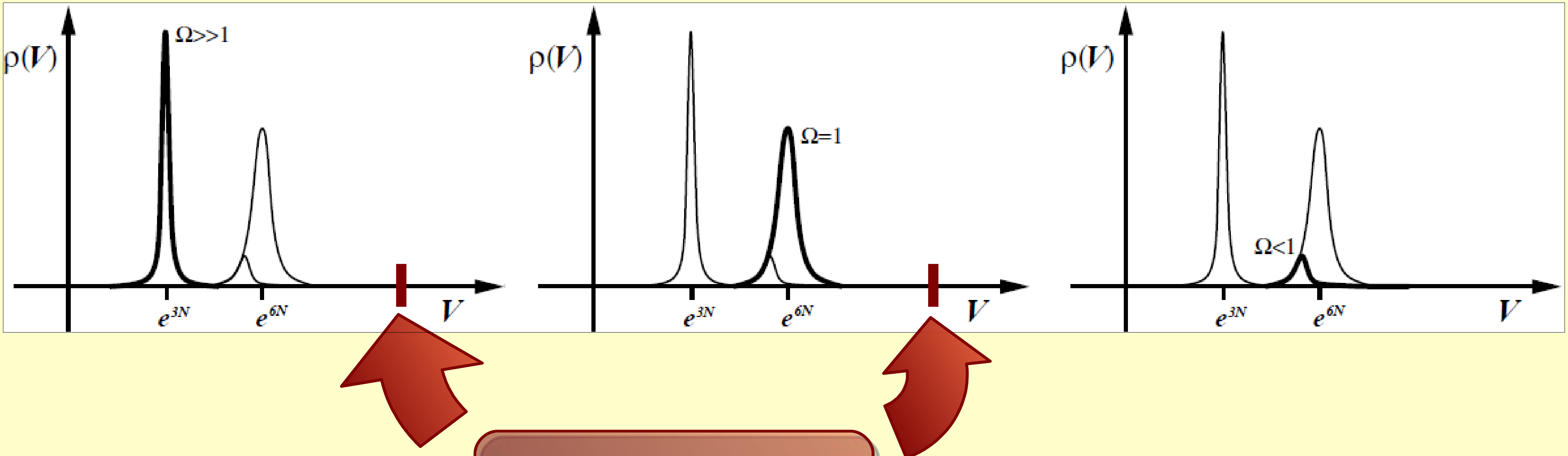


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**The bound is confirmed at the quantum level in the following form:**

*The probability for slow-roll inflation to produce a finite volume larger than* $e^{S_{dS}/2}$, *where* $S_{dS}$ *is de Sitter entropy at the end of the inflationary stage, is suppressed below the uncertainty due to non-perturbative quantum gravity effects.*

# Conclusions

**Two main results:**

1) The probability distribution itself; non trivial informations on the phase transition to eternal inflation.

2) Confirmation of a sharp bound at the quantum level.

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2) **Confirmation of a sharp bound at the quantum level.**

**Open problems and further extensions:**

- **Does the value "½" posses a deeper meaning (e.g. as the ½ in the Page argument for black holes), is it universal?**
- **Is the result robust against modification of the setting:**
  - **-multi field inflation (more species)**
  - **-non slow-roll inflation**
  - **-different number of dimensions**
  - **-etc...**
- **Is the bound associated with complementarity? How?**