Self-Organized-Criticality Model Consistent with Statistical Properties of Edge Turbulence in a Fusion Plasma

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The statistical properties of the intermittent signal generated by a recent model for self-organized criticality are examined. A successful comparison is made with previously published results of the equivalent quantities measured in the electrostatic turbulence at the edge of a fusion plasma. This result reestablishes self-organized criticality as a potential paradigm for transport in magnetic fusion devices, overriding shortcomings pointed out in earlier works [E. Spada *et al.*, Phys. Rev. Lett. **86**, 3032 (2001); V. Antoni *et al.*, Phys. Rev. Lett. **87**, 045001 (2001)].

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Self-organized criticality (SOC) is believed to account for the behavior of several extended nonequilibrium systems exhibiting bursty activity with long-range correlations in space and time [1]. In past years, SOC was candidated as a paradigm for the understanding of anomalous transport of energy and matter in magnetically confined fusion plasmas [2]. Within the huge complexity of transport in fusion devices, in fact, a number of features were identified, which could easily be cast into the framework of SOC systems: from the existence of critical average gradients and profile resilience to the power-law power spectra of plasma parameters fluctuations (density, temperature, magnetic field, ...). Many properties of SOC numerical models are satisfactorily compared against experimental data [3–7]: e.g., transport barriers have been recently reproduced using SOC models [5]; nonlocality, which is intrinsic in SOC, appears to be a possible ingredient for core transport [6]; numerical simulations of edge turbulence gave a SOC-like phenomenology [7].

However, recently further analysis showed some irreducible discrepancies between numerical and experimental time series [8–10]: These claims have by now been confirmed in almost all devices. Essentially, numerical time series [normally coming from the Bak-Tang-Wiesenfeld (BTW) model [1], the sandpile in the manner of Kadanoff [11], or the Hwa-Kardar "running" sandpile [12]] were not found to display realistic temporal correlations. It was pointed out that experimental data display distribution of waiting times with power-law tails and that intermittency appears as an ubiquitous property of plasma time series: a behavior not registered in the analyzed SOC models. The first paper addressing the former point in the context of laboratory plasma was Ref. [9], carrying on an analysis of the probability distribution function (PDF) of waiting times between bursts in density fluctuations measured on the RFX reversed field pinch experiment [13]. Experimentally, there was found a power-law curve with an exponent about -2, while the considered SOC models had exponential-like PDFs. Later, it became clear that the statistics of waiting times is not truly a stringent test about the existence of SOC, because several SOC models also have nonexponential PDFs of waiting times (e.g., see [14,15], and references therein).

Intermittency in turbulence implies the lack of self-similarity between time scales [16]. It has been characterized in plasma experiments by looking at the PDF of signal differences (or with the most sophisticated tool of the continuous wavelet transform [17]) at different time scales [10,18]. It turns out that the shape of the PDFs does not collapse to a single curve, irrespective of the time scale. There was found the presence of non-Gaussian PDFs in the RFX signal, approximately stretched exponentials, at the smaller τ's (higher frequency), gradually recovering the Gaussian shape as τ increased. Again, this is not the case in the analyzed SOC models.

SOC alone is a paradigm, which needs to be implemented into specific models in order to provide verifiable predictions. Part of the extracted predictions, hence, will be intrinsic to SOC, while others will depend on the model. Normally, as long as a specific model fails to account for some empirical evidence, one may devise a variant of it that is able to cure the specific shortcoming, but the improved accuracy of the final model is obtained at the expense of some loss of generality. In our case, the simplicity of the SOC paradigm would be spoiled and obscured by model-specific features. A good option would be one where the added features reflect a property that the true physical system is likely to possess anyway.

Previously, turbulent features were already found in time series of "waves" in the BTW model [19]. The same features were later found in experimental time series of solar flares [20]. Waves are decompositions of avalaches, and by definition, within an avalanche, they start from the same site. Thus, a message from this fact is that spatial features of the drive are important, affecting the temporal correlations in the activity.

Sánchez, Newman, and Carreras advanced several suggestions, to make compatible the findings of Refs. [9,10]

within a SOC-like scenario. The first proposal [21] was that of a mixed avalanche-diffusive transport, where the self-similarity of time scales would be naturally broken by the second contribution. A later suggestion [22] hinted to the possibility of inducing modifications to the waiting times distribution through correlations built into the drive, hence remaining within a pure SOC framework. However, the authors pointed out that introducing this sort of explicit correlations into a SOC model could be not trivial, since the generating mechanism is likely to be specific to any physical system, and therefore scarcely useful. We show that, indeed, it turns out to be a pessimistic view: A very simple and general correlation mechanism can still generate a very complex time series and output.

We are going to present in this work a SOC model that is able to reproduce the findings of Refs. [9,10]. This is obtained by postulating the existence of a correlation between successive locations of the fueling. This requisite appears rather plausible in a model that attempts to simulate the physics in a plasma device: Particle and heat sources in these environments are more or less localized, in any case not uniformly randomly distributed.

The SOC system is visualized as a standard 1-dim lattice automaton of length L, very similar to the sandpile studied by Kadanoff *et al.* [11]. The choice of the model and of the boundary conditions will allow a correspondence between its profile h and ordinary profiles in laboratory plasmas. Each site i $(1 \leq i \leq L)$ of the lattice holds h_i "grains." Different choices for the boundary conditions can be taken: In Ref. [14] periodic ones $(h_{L+1} = h_1,\ h_0 = h_L)$ were used. In this work, we will use one open boundary $(h_{L+1} = 0)$, while the other one is closed $(h_0 = h_1)$. Stable pairs of adjacent sites (j, ℓ) fulfill the local stability condition if $|h_j - h_\ell| < H$, where the constant H is a threshold. A local instability is resolved by a toppling, which consists in moving α grains from the upper to the lower site. This lower site in turn can become unstable, and chain reactions of transport are possible (avalanches). All instabilities in the systems are updated in parallel, and this procedure is iterated until the avalanche ends because all sites returned stable. The whole process takes place in one "time step." Instabilities arise because the system is driven out of equilibrium: A new grain is added at each time step at a position i: $h_i \rightarrow h_i + 1$. Correlations are introduced at the stage of choosing a new site i' for deposition at time step t, provided that at the previous step $(t - 1)$ the grain had been deposited on site i. In earlier models [1,11,12], the choice is completely random and uncorrelated from the value i. Here, on the contrary, we make the opposite hypothesis of a strongly correlated diffusing dynamics: With equal probability, $i' = i + 1$ or $i' = i - 1$. Reflecting boundary conditions for the drive are used to avoid i' falling outside the lattice.

We do not expect qualitatively different results by varying slope-related quantities such as H and α. Hence, we will keep them fixed from now on: $H = 4$, $\alpha = 2$. We shall see, instead, that some results are not L-invariant. For reference, we will use $L = 512$. In what follows, we will monitor for diagnostic purposes the total activity s, i.e., the total number of topplings within each avalanche. This activity is naturally identifiable with the quantity of matter or heat delivered to the device's walls, hence close to experimental investigation. All the simulations were carried out by starting with an empty system and letting it load until a stationary critical phase is reached. Then recording phases of several 10^6 time steps were performed. One sample of the typical time series is given in Fig. 1.

The power spectrum $S(f)$ is a statistical measure that can be easily computed from data sets and gives fundamental information about time correlations built into the signal. Hence, an important issue is that about its scaling properties. The signature of SOC models is a power-law spectrum. Most experiments (also including "numerical experiments") claim to find a power law, or possibly more than one over different frequency ranges, with a variance of the exponent. Its absolute value, in several works, is higher than unity over most of the sampled frequency range, usually ranging between 2 and 3 [23]. There are, however, some exceptions [4,7,24,25] where a $1/f$ scaling appears to be recovered. It is possible that, in the first set of papers, the $1/f$ region exists, too, but its width is too small to be detectable, and what was measured was instead just the high-frequency part of the spectrum (see, in this regard, Fig. 3 in Ref. [24]).

In Fig. 2, we show the power spectrum from the model, for two choices of boundary conditions (we study both because the choice of boundary conditions does affect the spectrum). In the periodic boundaries case, a $1/f$ slope is clearly discernible. In the other case, the trend still appears but less clearly. This is quite remarkable a result: It is well known that the power spectrum of a random-walk-like signal is a $1/f^2$ curve. Hence, the random-walk dynamics of the driver couples to the rules for the stability of the sandpile to produce an highly nontrivial output.

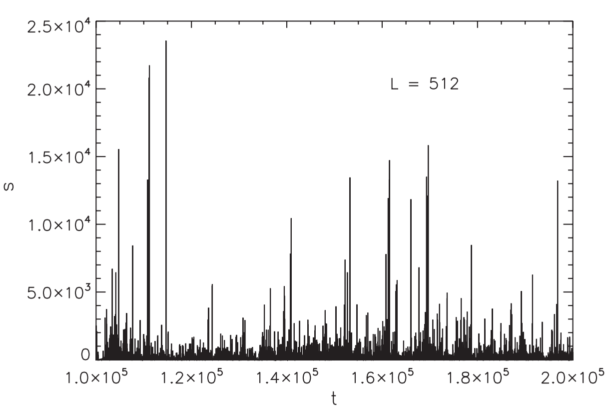

FIG. 1. A sample of the time trace of the output of the model, showing its bursty behavior.

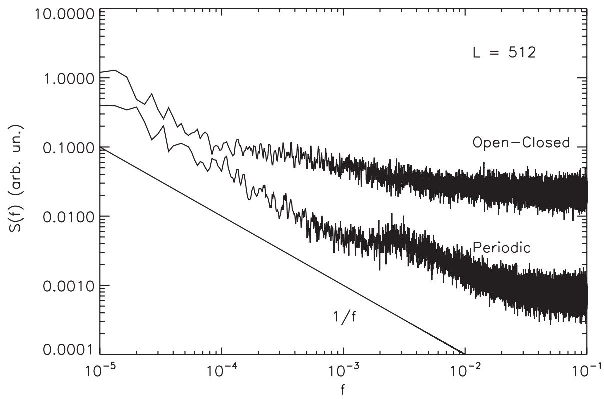

FIG. 2. Power spectrum of the synthetic signal for both periodic and open-closed boundary conditions. Overplotted for comparison is a $1/f$ curve.

The first quantity of interest is the PDF of waiting times between bursts, $P(t_w)$, shown in Fig. 3. We define as bursts the points $s(t)$ for which $s(t) > 3\langle s \rangle$, with $\langle s \rangle$ the average value over the sample. Most of the $P(t_w)$ curve is accurately fitted by a power law with exponent ≈ -2; interestingly, it is quite close to experimental values from RFX. The appearance of a non-Poissonian PDF is related to the existence of time correlations inherited by the driving [14].

So far, we have shown that the present model fulfills the questions raised in Ref. [9] as long as the scaling of waiting times is concerned. The issue of the departure from self-similarity is positively addressed in Fig. 4. There we plot the PDF of the wavelet transform of the signal at three time scales. The PDF has fat tails at the smaller time scales, gradually approaching a Gaussian shape at the longest time scales. The explanation we give for the breaking of the self-similarity is this: The seeding has an effective "diffusivity" $D = (\text{length step})^2/(\text{time step}) = 1$. After a time of order $T = L^2/D = L^2$, the seeding has sampled the whole lattice, irrespective of the starting position. Thus, T plays the role of a correlation time: The system keeps some memory of its past for as long as this time. Semiquantitatively, some confirmations of this statement may be found: (I) In Fig. 4, we show that the convergence to Gaussian shape is faster with decreasing L. (II) By sampling the system at a frequency $f < 1/T$, one is actually sampling from independent realizations of the same system (Gibbsian ensemble). Hence, the scalar signal picked out must be equivalent to a random number from within a uniform finite distribution, whose power spectrum is white noise f^0. Indeed, the power spectrum in Fig. 2 begins to flatten towards an f^0 spectrum for f lesser than about 10^{-5}, consistent with the $1/L^2 \approx 4 \times 10^{-6}$ estimate.

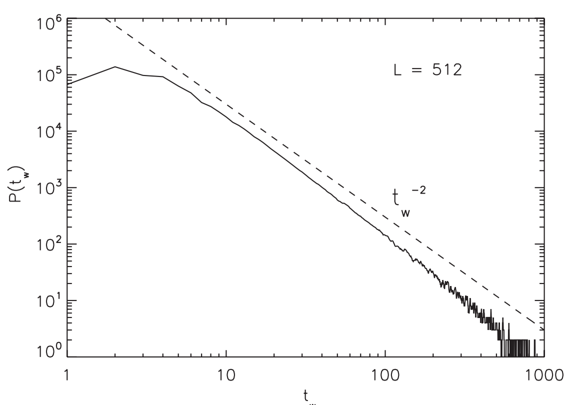

FIG. 3. PDF of waiting times. Overplotted for comparison is a $1/t_w^2$ curve.

Although successful when benchmarked against all the sought statistical tests, the model here proposed is still quite minimal, and add-ons may be envisaged that can enrich the built-in physics without appreciably spoiling its simplicity. An option we are pursuing is that of implementing into the model the running sandpile version of Hwa and Kardar [12]. Indeed, a time scale common to both the drive and the system dynamics appears a suitable feature for models of turbulent phenomena [2,5,15].

Nowadays, the intermittent character of turbulence at the edge of fusion devices is attributed to the existence of long-lived coherent structures moving on top of a background plasma (dubbed blobs), which are responsible for most of the transport to the wall [26]. The precise mechanism of blob generation is self-regulating and involves the triggering of an interchange instability driven by pressure gradients with formation of a radially elongated structure, which ultimately is separated from the core plasma by the differential rotation and expelled towards the edge, with an accompanying relaxation of the mean profile [7,27]. The affinities with SOC dynamics are apparent

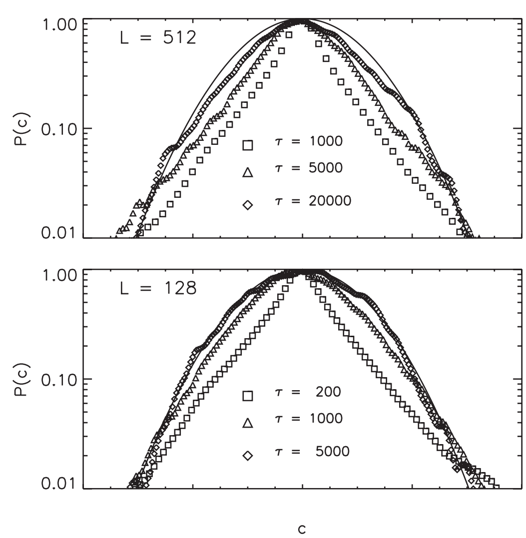

FIG. 4. Distributions of coefficient of wavelets transform at three time scales. The heights of the PDFs have been scaled to their maximum value and the widths to the standard deviation. The solid curve is a Gaussian with the same amplitude and variance, plotted for reference. Upper plot, the lattice length is $L = 512$; lower plot, for comparison, $L = 128$.

and have been noticed by some authors (see, e.g., [28,29]); however, to our knowledge, the understanding of the blobs' physics has not yet advanced enough to allow for quantitative comparisons with SOC models.

Undoubtedly, the overall edge transport comes from an intricate pattern where the physics of creation, destruction, motion, and interaction of the coherent structures between them and with the background needs to be carefully taken into account in order to provide a correct simulation of experimental evidence. It is, therefore, still an open question whether the SOC paradigm would be comprehensive enough to account for all this physics. The evidence presented in this Letter suggests, however, that further analysis is necessary before discarding SOC as a potential transport paradigm. Indeed, in the past there has sometimes been a tendency to see a dichotomy between SOC and magnetohydrodynamics (MHD) turbulence that, perhaps, is not there: Attempts of developing cellular automata consistent with MHD and Maxwell's equations are actually found in the literature [30].

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