The hypothesis of a real Cabibbo Kobayashi Maskawa matrix [a]

- **General framework:**

  Try to extract information independently from K experiments

- **Input data**

- **Methods and Results**

- **Conclusions**

[a] E. Piotto F. Simonetto and P.C.

General framework

- The Cabibbo-Kobayashi-Maskawa matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- expressed in terms of 4 Wolfenstein parameters:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

where

$$\lambda = |V_{us}| = 0.2196 \pm 0.0023,\ \ A = \frac{|V_{cb}|}{\lambda^2} = 0.819 \pm 0.035.$$

A real CKM matrix?

many contributions to extract ρ and η [a] from measurements

- η is the complex phase accounting for CP violation

- CP violation from neutral Kaon decay $|\epsilon_k|$

- neglected on porpose in the following:

- Is CP violation visible independently?

- Is η compatible with 0 or

CKM compatible to be real?

[a] $\rho(\eta) \rightarrow \bar{\rho}(\bar{\eta}) = \rho(\eta)(1 - \lambda^2/2)$ for higher orders in λ

Input data

B_d^0 oscillations

In SM Δm_d related to CKM:

$$\left[(1-\rho)^2+\eta^2\right]=$$

$$=\frac{\Delta m_d}{\frac{G_F^2}{6\pi^2}m_t^2 m_{B_d^0}\left(f_{B_d}\sqrt{B_{B_d}}\right)^2\eta_B F(\frac{m_t^2}{m_W^2})A^2\lambda^6}$$

- Δm_d measured (LEP+SLD+CDF) with good precision
- but $f_{Bd}\sqrt{B_{B_d}}$ (from lattice QCD) known with a 20% order uncertainty

B_s^0 oscillations

- Now the ratio $\frac{\Delta m_d}{\Delta m_d}$ related to CKM:

$$\left[(1-\rho)^2+\eta^2\right]=\frac{\Delta m_d}{\Delta m_s}\frac{1}{\lambda^2}\frac{m_{B_s^0}}{m_{B_d^0}}\xi^2$$

- the ratio $\xi=\frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$ known with a better precision

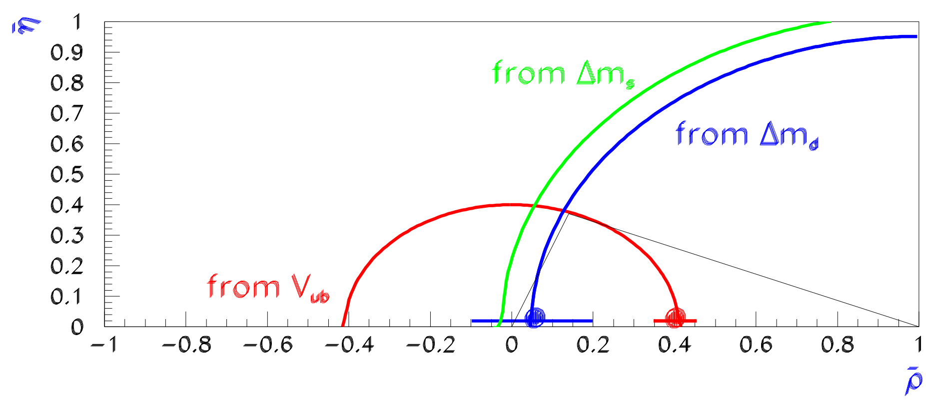
$|V_{ub}|$ from semileptonic b decay

- The ratio $\frac{|V_{ub}|}{|V_{cb}|}$ directly related to:

$$\frac{|V_{ub}|}{|V_{cb}|} = \lambda\sqrt{\rho^2 + \eta^2}$$

- $\frac{|V_{ub}|}{|V_{cb}|}$ measured by CLEO and LEP

The equations correspond to



If $\eta \to 0$:

- Two independent measurements of (+ one limit on) ρ

Input data

parameter	new input	source
Δm_d	0.471 ± 0.016 ps^{-1}	LEPBOSC 99/1 [a]
Δm_s	> 12.3 ps^{-1}(95% C.L.)	LEPBOSC 99/1 [a]
$\|V_{ub}\| / \|V_{cb}\|$	0.090 ± 0.012	CLEO + LEPVUB 99/1
$f_{B_d}\sqrt{B_{B_d}}$	$0.215^{+0.040}_{-0.030}$ GeV	LATTICE QCD 98
	$0.210^{+0.039}_{-0.032}$ GeV	LATTICE QCD 98+ $\frac{f_{B_d}}{f_{D_s}}$ [b]
ξ	$1.14^{+0.07}_{-0.06}$	LATTICE QCD 98

[a] LEP+SLD+CDF

[b] F. Parodi et al. , LAL 99-03, DELPHI 99-27 CONF 226.

Methods and Results

BLUE Method

-Compatibility of $\bar{\rho}_i = \bar{\rho}^{V_{ub}}, \bar{\rho}^{\Delta m_d}$ **to estimate the hypothesis**

real CKM matrix

- to include all the correlations use Best Linear Unbiased Estimator :

$$\bar{\rho}^{BLUE} = \frac{\Sigma_{i=1}^{2}\Sigma_{j=1}^{2}\bar{\rho}_i(\mathbf{M}^{-1})_{ij}}{\Sigma_{i=1}^{2}\Sigma_{j=1}^{2}(\mathbf{M}^{-1})_{ij}}$$

- with the variance

$$\sigma_{\bar{\rho}}^2 = \frac{1}{\Sigma_{i=1}^{2}\Sigma_{j=1}^{2}(\mathbf{M}^{-1})_{ij}}.$$

- where M includes correlated and uncorrelated contributions and non diagonal terms Δ **:**

$$M_{ij} = \delta_{ij}\sigma_i^{uncorr}\sigma_j^{uncorr} + \Sigma_{\alpha=1}^{m}\Delta_{\alpha i}\Delta_{\alpha j}$$

- and the χ^2

$$\chi^2 = \Sigma_i\Sigma_j\left(\left[\bar{\rho}_i - \bar{\rho}^{BLUE}\right](\mathbf{M}^{-1})_{ij}\left[\bar{\rho}_j - \bar{\rho}^{BLUE}\right]\right).$$

BLUE Results

- **We have:**

$$\bar{\rho}^{\Delta m_d} = 0.05^{+0.16}_{-0.17}$$

$$\bar{\rho}^{V_{ub}} = \pm(0.40 \pm 0.05)$$

$$\bar{\rho}^{\Delta m_s} > -0.05$$

- **The negative solution for $\bar{\rho}^{V_{ub}}$ discarded**

$$\bar{\rho}^{BLUE} = \mathbf{0.35\ (0.36) \pm 0.05}$$

and $\chi^2 = 4.3\ (3.9)$ (1 d.o.f.)

or $\mathcal{P}(\chi^2) = 3.7\ (4.9)\%$ (L. QCD 98)

The hypothesis real CKM is disfavoured by present B Physisc data [a]

[a] contradicts statements in: *Phys. Rev.* **D 59** (1999) 113011 (even with the same data)

A method for $\bar{\rho}$ and $\bar{\eta}$

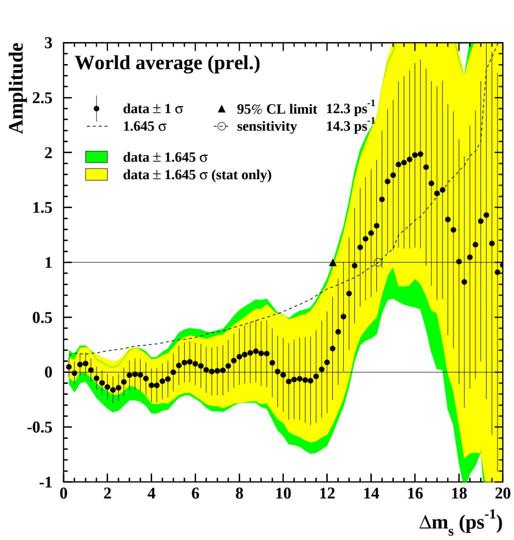
- The equations (circles) from Δm_d and $\frac{|V_{ub}|}{|V_{cb}|}$
- give often a solution for $\bar{\rho}$ and $\bar{\eta}$:

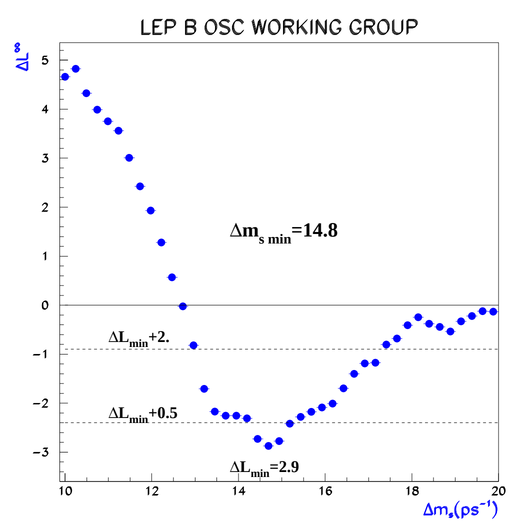
$$\bar{\rho} = 0.13^{+0.13}_{-0.23} \text{ and } \bar{\eta} = 0.38^{+0.07}_{-0.09}$$

Include Δm_s information

- Average amplitude $A(\Delta m_s)$ and σ_A related[a] to log-likelihood ($A = 1$) referenced to $\Delta m_s \to \infty$:

$$\Delta\mathcal{L}^{\infty}(\Delta m_s) \equiv \mathcal{L}(\Delta m_s) - \mathcal{L}(\infty) = \frac{1/2 - A(\Delta m_s)}{\sigma^2_{A(\Delta m_s)}}$$





[a] H.G. Moser and A. Roussarie, *Nucl. Instr. and Meth.* **A 384** (1997) 491.

- A global log-likelihood function:

$$\mathcal{L} = \Delta\mathcal{L}^{\infty}(\bar{\rho}, \bar{\eta}, \xi) + \mathcal{L}^{\Delta m_d, V_{ub}}(\bar{\rho}, \bar{\eta}, f_{B_d}\sqrt{B_{B_d}}, ..)$$

- where $\mathcal{L}^{\Delta m_d, V_{ub}}$ obtained from eq. (circle Δm_d , circle V_{ub}) as function of the input parameters and their errors

- The method (result) is much more significant:

$$\bar{\rho} = 0.14^{+0.05}_{-0.06} \text{ and } \bar{\eta} = 0.37 \pm 0.05$$
$$sin2\beta = 0.73^{+0.07}_{-0.08} \text{ and } \gamma = 69.2^{+8.7}_{-6.9}\,^{\circ}$$

- Caution : ghost minima on $\Delta\mathcal{L}^{\infty}(\Delta m_s)$ cannot be excluded ($\mathcal{O}$(few%))

Conclusions

- **Real CKM matrix hypothesis tested on the basis of B Physics and Lattice QCD calculations**

- **Best Linear Unbiased Estimator used:**

- **Excluded at more than 95% C. L.**

- **Pure superweak theories disfavored**

- **Method including the Δm_s information**

- **Good precision on $\bar{\rho}$ and $\bar{\eta}$ is achievable**

$$\bar{\rho} = 0.14^{+0.05}_{-0.06} \text{ and } \bar{\eta} = 0.37 \pm 0.05$$
$$sin2\beta = 0.73^{+0.07}_{-0.08} \text{ and } \gamma = 69.2^{+8.7}_{-6.9}{}^{\circ}$$

IF SM THEN CP VIOLATION

CAVEAT

- Up to now pure superweak disfavoured

- Clearly if new physics modifies $\rho(\Delta m_d)$ and $\rho(\Delta m_s)$ nothing can be said

- Suggested [a]: the variations on ρ, $\Delta\rho^{\Delta m_d} \sim 0.5 F_d$ and $\Delta\rho^{\Delta m_s} \sim 0.5(F_d - F_s)$ depend on the fractional contributions $F_{d,s} = \delta_{d,s}/\Delta m_{d,s}$ where $\delta_{d,s}$ is the new physics contribution to $\Delta m_{d,s}$

- With the present data small positive values of F_d are excluded :

$$F_d > 0.04 \text{ at the } 95\% \text{ Confidence Level}$$

- while negative values are excluded except for the case of large positive F_s ($\mathcal{O}(30\%)$) which would allow for negative values of ρ.

[a] R. Barbieri et al., *Phys. Lett.* **B 425** (1998) 119.

REMARK

- The dominant error in $\rho^{\Delta m_d}$ is on $f_{B_d}\sqrt{B_{B_d}}$ from Lattice QCD.
- Take into account a flat error
- Several experimental results for $\rho^{\Delta m_d}$ and $\rho^{V_{ub}}$ with central values equal to ρ^{BLUE}.
- For $\rho^{\Delta m_d}$ shift the value of $f_{B_d}\sqrt{B_{B_d}}$ and allow it to vary within a flat distribution with the R.M.S. of the quoted error
- All the other parameters vary with a gaussian distribution
- Fraction of simulated experiments with a $\chi^2 > \chi^2_{Data} < 5\%$