TEST OF ADVANCED TOPICS IN THE THEORY OF THE FUNDAMENTAL INTERACTIONS

12.2.2021

NAME: FAMILY NAME:
registration number:

Problem I

1. Consider the mass operator for scalar φ, Dirac fermion ψ and vector boson A_μ in d dimensions:

$$m^2\varphi^2 \quad , \qquad m\bar{\psi}\psi \quad , \qquad m^2 A_\mu A^\mu \quad .$$

Can the operators be marginal in some dimension d? **No**

2. Find d such that $(\bar{\psi}\psi)^2$ is marginal. $d = 2$

3. Find d such that $(\varphi)^6$ is marginal. $d = 3$

4. Classify the operator $(\varphi\, \Box\varphi^2)$ for integer d.

$d < 2$ **relevant,** $d = 2$ **marginal,** $d < 2$ **irrelevant**

Problem II

5. Determine the correct power counting, in terms of the electric charge e, the electron mass m and the light velocity c of the NRQED operators:

$$\bar{\chi}\vec{\sigma}\cdot\vec{B}\chi \quad , \quad \bar{\chi}(\vec{D}\cdot\vec{E} - \vec{E}\cdot\vec{D})\chi \quad , \quad i\bar{\chi}\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})\chi \quad , \quad \bar{\chi}(\vec{D}\cdot\vec{D})^2\chi$$

where $\vec{D} \equiv \vec{\partial} + i\frac{e}{c}\vec{A}$ is the spatial covariant derivative and χ is the two-component electron field.

$$e/mc \quad , \quad e/m^2c^2 \quad , \quad e/m^2c^2 \quad , \quad 1/m^3c^2 \quad .$$

6. Find the transformation properties of the operators

$$\bar{\chi}(\vec{\sigma}\cdot\vec{D}\ \vec{B}\cdot\vec{D} + \vec{D}\cdot\vec{B}\ \vec{\sigma}\cdot\vec{D})\chi \quad , \quad i\bar{\chi}(\vec{B}\cdot\vec{B} - \vec{E}\cdot\vec{E})\chi \quad , \quad \bar{\chi}(\vec{B}\cdot\vec{B} + \vec{E}\cdot\vec{E})\chi$$

under parity and time-reversal and determine their power counting in terms of e, m and c.

The second operator is not T-invariant.

$$e/m^3c^3 \quad , \quad e^2/m^3c^4 \quad , \quad e^2/m^3c^4.$$

Problem III

The particle content of a U(1) gauge theory includes a gauge vector boson, N generations of four-component spinors $\psi_k^{(1)}$ $(k = 1, ..., N)$, N generations of four-component spinors $\psi_k^{(2)}$ $(k = 1, ..., N)$, one four-component spinor χ and one complex scalar φ. The scalar carries a positive unit of U(1) charge. The U(1) charges Q of the fermion sectors are: $Q\left(\psi_{kL}^{(1)}, \psi_{kR}^{(1)}, \psi_{kL}^{(2)}, \psi_{kR}^{(2)}, \chi_L, \chi_R\right) = (+1, 0, 0, -1, x, y)$.

7. Determine the gauge current j^μ conserved at the classical level.

$$j^\mu = \sum_{k=1}^{N} \overline{\psi_{kL}^{(1)}}\gamma^\mu\psi_{kL}^{(1)} - \sum_{k=1}^{N} \overline{\psi_{kR}^{(2)}}\gamma^\mu\psi_{kR}^{(2)} + x\bar{\chi}_L\gamma^\mu\chi_L + y\bar{\chi}_R\gamma^\mu\chi_R - i(D^\mu\varphi^\dagger\cdot\varphi - \varphi^\dagger D_\mu\varphi)$$

8. Determine the group of global symmetries $G_{\texttt{gl}}$ of the theory, assuming generic values of x and y and vanishing Yukawa interactions. Justify your answer.

With generic values of x and y there is a U(1) acting on the scalar field, a U(2N) acting on $\left(\psi_{kL}^{(1)}, \psi_{kL}^{c(2)}\right)$, a U(2N) acting on $\left(\psi_{kR}^{(2)}, \psi_{kR}^{c(2)}\right)$, and a U(1)$\times$ U(1) acting on χ_L and χ_R: $G_{\texttt{gl}} = U(1) \times U(2N) \times U(2N) \times U(1) \times U(1)$. For specific values of x and y, $G_{\texttt{gl}}$ can be enhanced.

9. Assuming that the theory is in the U(1) spontaneously broken phase, and the gauge vector boson acquire a mass M, determine the low-energy effective theory $\mathcal{L}_{\texttt{eff}}$ in the regime $E \ll M$, at the tree-level and in the static approximation. Analyze only the contribution to $\mathcal{L}_{\texttt{eff}}$ that depend on fermions.

$$\mathcal{L}_{\texttt{eff}} = -\frac{g^2}{2M^2} j^\mu j_\mu$$

10. Choose $N = 4$. Determine x and y in such a way that the theory is anomaly free and $\mathcal{L}_{\texttt{eff}}$ does not depend on χ_R.

$\mathcal{L}_{\texttt{eff}}$ does not depend on χ_R for $y = 0$. Then, the gauge anomaly cancellation requires $4 \times 1^3 + 4 \times 1^3 + x^3 = 0$, solved by $x = -2$.

11. Assuming $N = 4$ and the values of x and y determined at the previous point, give an example of a global, anomaly free, symmetry $\mathrm{U}(1)_{\texttt{gl}}$ of the theory, in the limit of vanishing Yukawa interactions. In your answer, specify the charges $Q_{\texttt{gl}}\left(\psi_{kL}^{(1)}, \psi_{kR}^{(1)}, \psi_{kL}^{(2)}, \psi_{kR}^{(2)}, \chi_L, \chi_R\right)$.

One example is $Q_{\texttt{gl}}\left(\psi_{kL}^{(1)}, \psi_{kR}^{(1)}, \psi_{kL}^{(2)}, \psi_{kR}^{(2)}, \chi_L, \chi_R\right) = (+1, 0, 0, 0, -1, 0)$