AT in QFT

This is a monographic course on Effective field Theories.
EFT is the fundamental tool used today to describe a large number of physical systems. While this course will focus on fundamental interactions in a relativistic regime, the range of applications is much wider, involving for instance

in Cosmology { Post-Newtonian gravity — PNG
Large scale structure formation — LSS
Inflation — I

LSS deals with power spectrum, neutrino masses, ...

in thermal systems (application to e.g. heavy ion collisions)

in Dark Matter (to describe particle interaction at the nucleon level)

in Nuclear Physics

in Condensed Matter
BCS theory of superconductivity
(removing phonons from the large length description)

Example of ET

When an engineer plans a building or a bridge he/she uses the laws of classical Mechanics, not those of Quantum Mechanics even if the latter are more general and more fundamental.

Why?

Simply because Classical Mechanics is more suitable for the problem at hand and using Quantum Mechanics would generate an unnecessary complication in the specific application.

Classical mechanics provide an approximation of the fundamental laws but this is totally justified since quantum effects arise at length scales of atomic order while the building has size of many orders of magnitude larger.

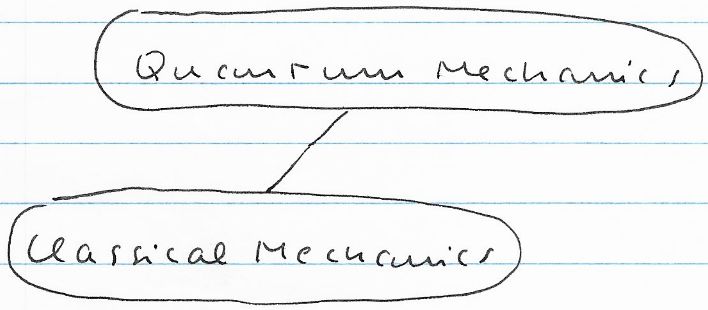
Between the 2 theories there is a large separation of scales, which produces a small expansion parameter $\frac{a_0}{L}$ → Bohr radius, ↘ macroscopic distance

or better $\hbar$.

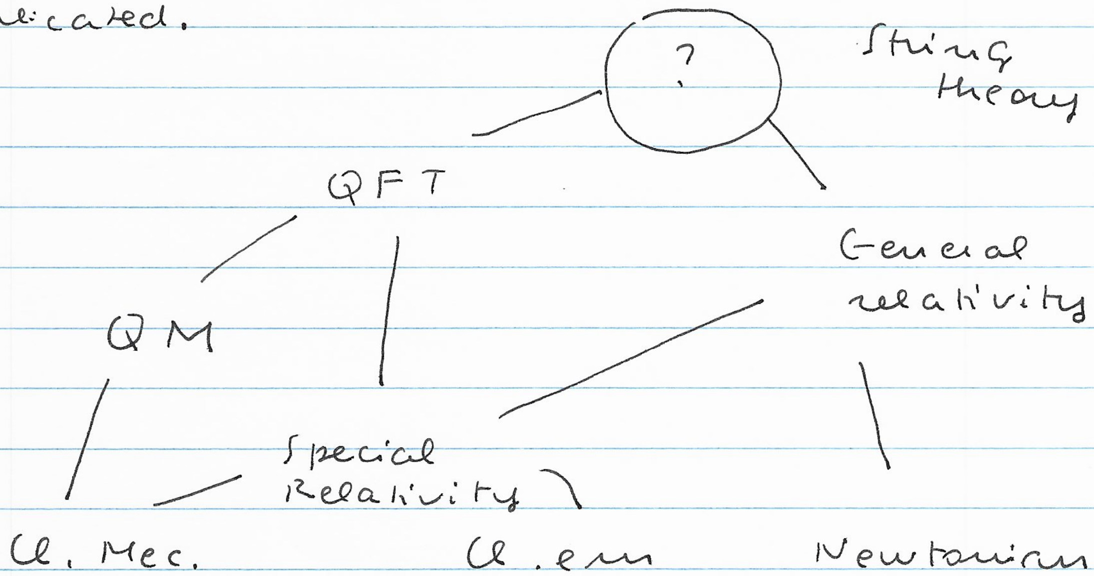
~~If we~~ In this simple case the corrections to the classical picture are totally negligible compared to the precision required in the planning.

Wikipedia definition: EFT is a type of approximation for an underlying physical theory

Les Houches 2017 school: EFT is a general method for describing systems with multiple length scales in a tractable fashion.



The network of theories can be much more complicated.



gravity

Also each link can be travelled in 2 directions: bottom-up (BU) or top-down (TD)

TD: the higher level is more fundamental but its application to the chosen problem is extremely complicated, like in previous example.

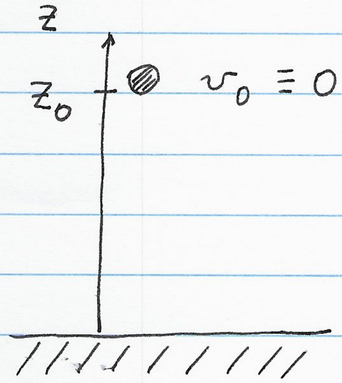
BU: we want to push our knowledge to a more fundamental level. We formulate our lower level in such a way that it can suggest how to improve the theory. Example: Fermi theory of electroweak interactions

In this course we will often travel in both directions.

EFT

Example I : falling body near the Earth surface

Task :



$z_0 \approx 10-100$ m range

set $t=0$ at the beginning

compute the arrival time

$$t = \sqrt{\frac{2 z_0}{g}}$$

$g \approx 9.81 \ m s^{-2}$

gravity acceleration

which physics law did we use ?

Newton's law

But Newton's law tells $F = \frac{GMm}{r^2} \ldots$

For our problem $F = \frac{GMm}{(R+z)^2}$

$$= \underbrace{\frac{GM}{R^2}}_{g} m \left[1 - 2\frac{z}{R} + O\left(\frac{z}{R}\right)^2 \right]$$

$$F = mg\left(1 - 2\frac{z}{R} + \ldots\right)$$

we have omitted the tiny correction $\frac{z}{R}$

$R \simeq 6371$ Km $\gg z \simeq 10-100$ m

and we used an "effective" theory $F = mg$

If our instruments are extremely precise $\delta g / g \simeq 10^{-3}$ and z_0 is sufficiently large, we can include the 1st correction term to match the exp. precision

$$F = mg\left(1 - 0.31 \cdot 10^{-6} \, z(m) + \ldots\right)$$

Properties of the effective theory:

1. degrees of freedom : 1 body of mass m
2. symmetry: $T(2) \times O(2)$
   Translations in x, y directions $T(2)$
   Rotation around the z axes $O(2)$
   differs from the symmetry of the "full" theory: $O(3)$ rotation in 3 dimensional euclidean space
3. expansion parameter: $\left(\frac{z}{R}\right)$
   the effective theory is no more valid when $z \simeq O(R)$
   As long $z \ll R$ we can compute to any desire precision physical effects, without referring to the full theory.

Notice also that, apart from 3., all the details of the full theory go into $g = GM/R^2$ a parameter that can be defined and measured to arbitrary precision in the effective theory.
3. remembers the full theory through the constant R telling us where the EFT ceases its validity.

1 + 2 + 3 define the ET

$$F = F_0 \left(1 + c_1 \frac{z}{R} + c_2 \left(\frac{z}{R}\right)^2 + \dots \right)$$

most general theory with 1 dot, invariant under $T(2) \times O(2)$ with expansion parameter (z/R).
All the constants $F_0, c_1, \dots$ can be determined

by measurement without reference to the full theory.
As long as $z/R \ll 1$ a computation to a given precision requires only a finite number of constants (F_0, c_i).

Tentative definition of ET

Simplest theory characterized by a # of dof, a symmetry and (a set of) expansion parameter(s), allowing computation of physical effects to a given precision in terms of a finite set of parameters directly deducible from observation.

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The above picture is also denoted as top-down approach in the sense that we know the full theory and we derive from it the corresponding EFT.
But we can also think of a bottom-up perspective.
Suppose we do not know Newton's law we start making measurements and we discover the falling bodies are described by a force

$$F = m g$$

m → inertial mass

g → "universal constant"

oriented along the vertical axis, everywhere
Assuming 1. and 2. (but ignoring N's law) we can still write:

$$F = mg\left(1 + c_1 \frac{z}{z_M} + c_2 \left(\frac{z}{z_M}\right)^2 + \dots\right)$$

and we can look for the fundamental or full theory through a series of precision measurements in the hope of reconstructing the complete functional dependence $F(z)$.
A major difference with the top-down approach is that now we do not know what z_M is.
We can always change z_M by redefining the c_i without changing the physics.
We do not know what is the expansion parameter and where the ET ceases to work.

Suppose that with a precise measurement we found:

$$F = mg\left(1 - 0.31\ 10^{-6}\ z\,(m) + O(z^2)\right)$$

The definition of expansion parameter requires a guess on the full theory.
① For instance we can assume that the full theory produces coefficients $c_i \simeq O(1)$ in the expansion (like in the case of Newton's law)
in this case we define $z_M \equiv (0.31 \cdot 10^{-6})^{-1}\ m$ and we expect the ET holds in the regime $z \ll z_M$ by including a sufficient number of terms c_i.

(2) The full theory can produce coefficients c_i very small e.g $O(10^{-3})$. In this case

$$-0.31 \cdot 10^{-6} z \approx -10^{-3} \frac{z}{z_M} \qquad z_M \equiv (0.31\, 10^{-3})^{-1}_{m}$$

and the ET is expected to break down at $z_M \approx 1 km$.

This happens in the crazy full theory

$$F(r) = \frac{GMm}{R^2} e^{-\xi \frac{r}{z_M}} \qquad \begin{cases} \xi \simeq 10^{-3} \ll 1 \\ z_M \simeq 1km \end{cases}$$

$$\approx g m \left(1 - \xi \frac{r}{z_M} + \dots\right)$$

In bottom-up approach:
ET (defined by dof + symmetry + exp param) can be very useful both for its
- predictability: predictions to a given precision in terms of a finite set of param.
- parametrization of our ignorance about the fund. theory

However, the definition of the expansion parameter requires a guess on the full theory.