Again on loop effects

$$\boxed{\text{Symmetries of the IR description that are not symmetries of the UV theory.}}$$

Consider the following renormalizable theory in $d=4$
1 scalar φ

$$\mathcal{L} = \frac{1}{2}\partial_0\varphi\,\partial^0\varphi - \frac{b_\varphi^2}{2}\partial_k\varphi\,\partial_k\varphi - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

$b_\varphi > 0$ $\quad b_\varphi \neq 1$ $\quad$ <u>constant</u>

Is this theory Lorentz invariant?
symmetries
1. $\varphi \to -\varphi$
2. Time translation, space-translations
   $x^0 \to x^0 + a^0$ $\qquad x^k \to x^k + a^k$
   $\varphi(x^0,\vec{x}) \to \varphi(x^0+a^0,\vec{x})$ $\qquad \varphi(x^0,\vec{x}) \to \varphi(x^0,\vec{x}+\vec{a})$
3. 3D rotations $\qquad x^k \to \mathbb{O}^k{}_\ell\, x^\ell \qquad \mathbb{O}^T\mathbb{O} = \mathbb{1}$
   $\varphi(x^0,\vec{x}) \to \varphi(x^0,\mathbb{O}\vec{x})$

The above lagrangian corresponds to a system of units where $c \neq 1$. Of course this does not violate Lorentz symmetry.
The factor b_φ can be eliminated by means of a rescaling of the x^k coordinates

$$x^k \to b_\varphi\, x^k \qquad \partial_k \to b_\varphi^{-1}\partial_k$$

$$\mathcal{L} = \frac{1}{2}\partial_0\varphi\,\partial_0\varphi - \frac{1}{2}\partial_k\varphi\,\partial_k\varphi - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

where $\varphi \equiv \varphi(x^0, b_\varphi x^k) \equiv \varphi'(x^0, x^k)$

and $\mathcal{L}(\varphi', \partial_k\varphi')$ is Lorentz invariant.

Message: b_φ is not a true parameter
It is related to a choice of units
We can choose b_φ as we want

Consider now:

$$\mathcal{L}_{UV} = \frac{1}{2}\partial_0\varphi\,\partial_0\varphi - \frac{b_\varphi^2}{2}\partial_k\varphi\,\partial_k\varphi - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

$$+ \bar{\Psi}(i\gamma^0\partial_0 + i\,b_\psi\gamma^k\partial_k - M)\Psi$$

$$+ i g\,\bar{\Psi}\gamma^5\Psi\varphi$$

$b_\varphi,\ b_\psi > 0 \qquad b_\varphi \neq b_\psi \neq 1$

In this case we cannot rescale x^k to eliminate both b_φ and b_ψ. The UV theory breaks Lorentz-invariance.

What happens at low energies?
We expect that b_φ and b_ψ are true parameters.
Therefore, in general $b_\varphi = b_\varphi(\mu)$ and $b_\psi = b_\psi(\mu)$. They will satisfy RGE:

$$\mu\frac{\partial}{\partial\mu} b_\varphi^2 = \beta_{b_\varphi^2}(g) \qquad \mu\frac{\partial}{\partial\mu} b_\psi = \beta_{b_\psi}(g)$$

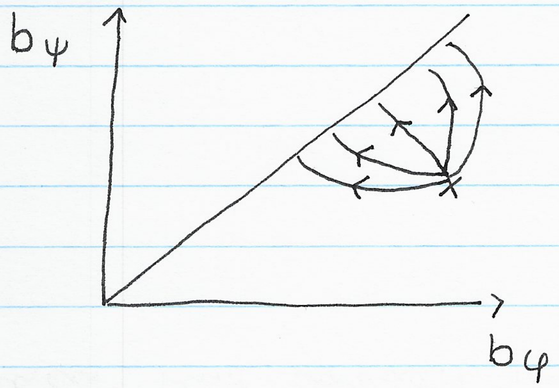
We know that, if at some scale Λ the two constants are equal:

$$b_\varphi(\Lambda) = b_\psi(\Lambda)$$

then they are equal at any scale μ.
Indeed if $b_\varphi(\Lambda) = b_\psi(\Lambda)$, the theory is Lorentz invariant at the scale Λ and loop corrections do not break this property

$$\rightarrow \begin{cases} \beta_{b_\varphi^2} \propto (b_\varphi - b_\psi) \\ \beta_{b_\psi} \propto (b_\varphi - b_\psi) \end{cases}$$

In other words $b_\varphi = b_\psi$ is a fixed point, or better line, of the RGE evolution.
The crucial aspect is whether it is an UV stable or an IR stable fixed point



UV $\longrightarrow$ IR

flow convention

← IR stable: the theory flows to Lorentz invariance in the IR

← UV stable

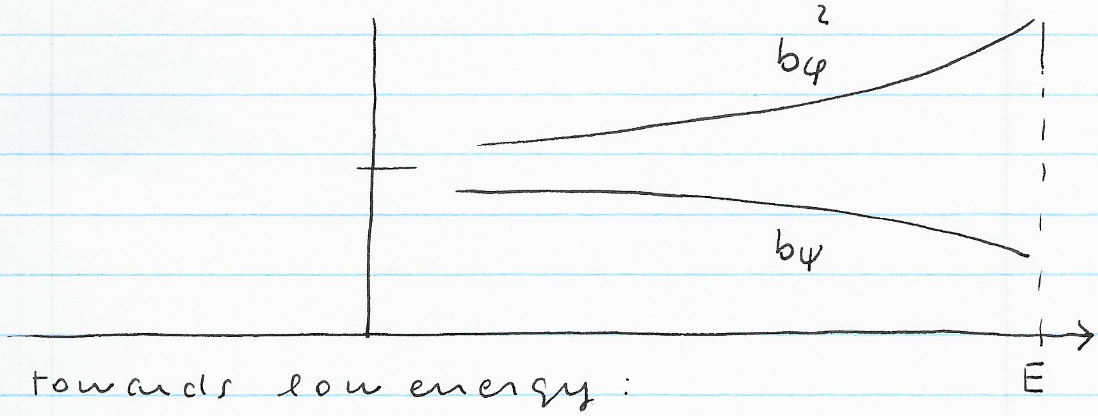
We find:

$$\begin{cases} \beta_{b_\varphi^2} = \dfrac{1}{4\pi^2} \cdot \dfrac{b_\varphi + b_\psi}{b_\psi^3} \cdot (b_\varphi - b_\psi)\, g^2 \\[2ex] \beta_{b_\psi} = -\dfrac{1}{6\pi^2} \dfrac{1}{b_\varphi (b_\varphi + b_\psi)^2} (b_\varphi - b_\psi)\, g^2 \end{cases}$$

1. As expected if $b_\varphi = b_\psi \rightarrow \beta_{b_\varphi^2} = \beta_{b_\psi} = 0$

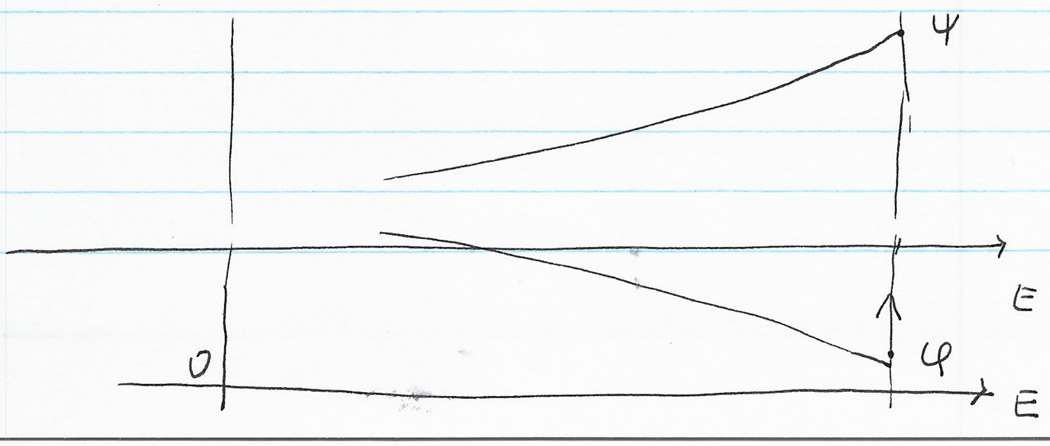
→ Starting on the fixed line, we remain there and the theory is Lorentz-invariant.

2. If we start with $b_\varphi > b_\psi$ (both positive) b_φ^2 grows, while b_ψ decreases with μ (E)



towards low energy:
until they reach a common value. Λ
From there on the theory is Lorentz invariant

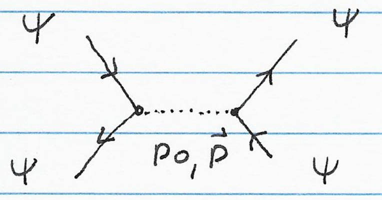
3. start with $b_\psi > b_\varphi > 0$



This shows that the line $b_\varphi^2 = b_\psi$ is IR stable and the theory becomes Lorentz-invariant in the IR.

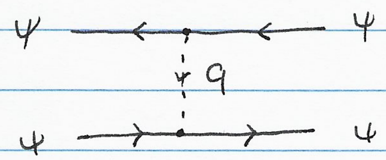
$\propto$

Process sensitive to b_φ^2:



in CM frame

$$\approx g^2 \frac{1}{p_0^2 - M^2}$$



$$\approx g^2 \frac{1}{+b_\varphi^2\, 2p^2(1-c_\theta) - M^2}$$

$(p_0, 0, 0, p)$ $\qquad$ $(p_0, p \cdot s_\theta, 0, p\, c_\theta)$

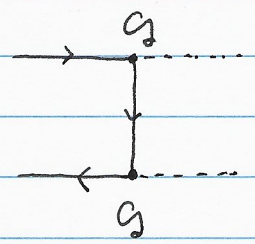
$(p_0, 0, 0, -p)$ $\qquad$ $(p_0, -p \cdot s_\theta, 0, -p\, c_\theta)$

$$q = (0, -p \cdot s_\theta, 0, p(1-c_\theta))$$

$$q^2 = 0 - p^2 s_\theta^2 - p^2(1 - 2c_\theta + c_\theta^2) = -2p^2(1-c_\theta)$$

If we redefine spatial momenta $b_\varphi^2 p^2 \to p^2$ the b_φ^2 dependence goes in the phase space.

cf.



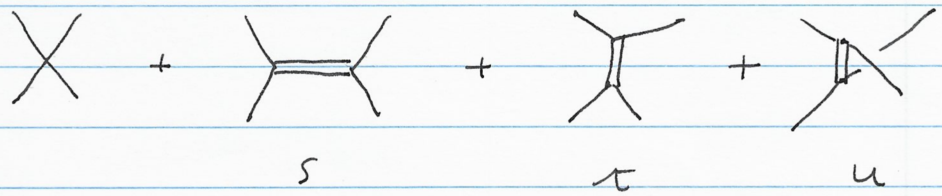
whose matrix element depends on b_ψ

Check that at the TL

$$\mathcal{A}(\varphi\varphi \to \varphi\varphi)\Big|_{IR} = \mathcal{A}(\varphi\varphi \to \varphi\varphi)\Big|_{UV} \qquad \text{for } E \ll M$$

$$\mathcal{A}_{IR} = -i\left(\lambda_0 - 3\lambda_1^2 - 4\lambda_1^2 \frac{m_L^2}{M^2}\right)$$

$\mathcal{A}_{UV} =$



$$= -i\lambda_0 + (-i\lambda_1 M)^2 \frac{i}{s - M^2} + \text{"}t\text{"} + \text{"}u\text{"}$$

$$= -i\left(\lambda_0 + \lambda_1^2\left(\frac{M^2}{s - M^2} + \text{"}t\text{"} + \text{"}u\text{"}\right)\right)$$

$$= -i\left(\lambda_0 - \lambda_1^2 \frac{M^2}{M^2}\left(\frac{1}{1 - \frac{s}{M^2}} + \text{"}t\text{"} + \text{"}u\text{"}\right)\right)$$

$$= -i\left(\lambda_0 - \lambda_1^2\left(3 + \frac{s + t + u}{M^2}\right)\right)$$

$$= -i\left(\lambda_0 - 3\lambda_1^2 - \frac{4 m_L^2}{M^2}\right) \qquad \text{o.k.}$$