$\overbrace{\lambda v^2}^{m_h^2} \gg m^2$

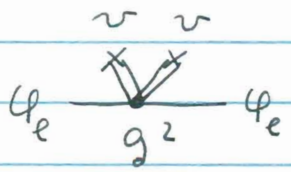
AT

Back to: φ_e, φ_h $\qquad G = Z_2 \times Z_2$ $\qquad \lambda \gg g^2$

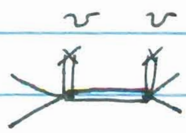
$$\mathcal{L}_{UV}(\varphi_e, \varphi_h) = \frac{1}{2}(\partial\varphi_e)^2 - \frac{1}{2} m^2 \varphi_e^2 + \frac{1}{2}(\partial\varphi_h)^2 - \frac{\lambda}{4}(\varphi_h^2 - v^2)^2 - g^2 \varphi_e^2 \varphi_h^2 + \dots$$

$$\mathcal{L}_{IR}(\varphi_e) = \frac{1}{2}(\partial\varphi_e)^2 - \frac{1}{2}(m^2 + 2g^2v^2)\varphi_e^2 - \frac{g^4}{\lambda}\varphi_e^4 + \dots$$

Estimate $\varphi_e^6, \dots \varphi_e^{2n}$



$$\approx g^2 v^2 \varphi_e^2$$



$$\approx (g^2)^2 \frac{v^2}{\lambda v^2} \qquad \varphi_e^4$$

$$\frac{g^4}{\lambda}$$

$$\approx (g^2)^3 \frac{v^2}{\underbrace{(\lambda v^2)^2}_{m_h^2}} \qquad \varphi_e^6$$

$$\approx g^{2n} \frac{v^2}{(\lambda v^2)^{n-1}} \qquad \varphi_e^{2n}$$

$$\varphi^{2n} \qquad = \frac{g^{2n}}{\lambda} \cdot \frac{1}{(\lambda v^2)^{n-2}} \equiv \frac{g^{2n}}{\lambda} \frac{1}{(m_h^2)^{n-2}}$$

Remarks

* $\mathcal{L}_{IR}(\varphi_\ell)$ can be diagramatically obtained by summing all tree-level diagrams with heavy internal lines.
These are 1PI with respect to the light lines

* The power counting is not completely fixed by dimensional analysis

$$m_h^2 \approx \lambda v^2$$

We could have expected, on dimensional grounds

$$\frac{\varphi^{2n}}{(m_h^2)^{n-2}}$$

Indeed we have : $\dfrac{g^{2n}}{\lambda (m_h^2)^{n-2}} \quad \varphi^{2n}$

The precise powers of g and λ can only come from the UV theory.

The EFT description breaks down at

$$E \approx m_h$$

since reaching this energy we start producing the heavy particles not included in $\mathcal{L}_{IR}$.

On the other hand, if $g^2 \ll 1$ and $g^2 \ll \lambda$ the effects of the irrelevant operators are suppressed by a larger effective scale

$$\Lambda_{eff}^2 \approx \left(\frac{\lambda}{g^{2n}}\right)^{\frac{1}{n-2}} m_h^2 \gg m_h^2$$

This is like in the Fermi theory where the correct power counting gives

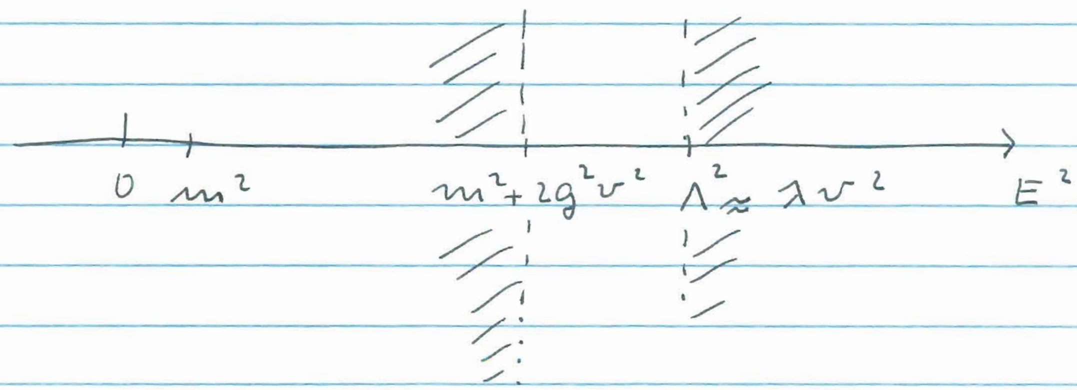
$$\frac{g^2}{M_W^2} \bar{\Psi}\Psi\bar{\Psi}\Psi \approx G_F \bar{\Psi}\Psi\bar{\Psi}\Psi$$

The IR description breaks at $E \approx M_W$ but $\sqrt{1/G_F} \approx v_{EW} \approx 250$ GeV $> M_{W,Z}$.

* A relevant operator has been generated

$$m^2 \to m^2 + 2g^2v^2$$

Even if $g^2 \ll 1$ $g^2v^2 \gg m^2$ thus restricting the window where our EFT applies



Use of classical equation of motion is equivalent to evaluation of tree-level diagrams with heavy internal lines. Which is better / more practical?

→ I prefer e.o.m. since it automatically accounts for combinatorial factors.
Example: back to

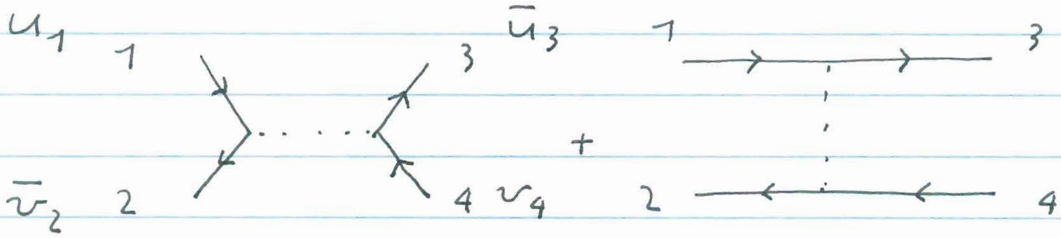
$$\mathcal{L}_{UV}(\varphi, \Psi) = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}M^2\varphi^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - g\varphi\bar{\Psi}\Psi$$

$M^2 \gg m^2$

Call the particles described by Ψ electrons e and consider

$e^+e^- \to e^+e^-$

to derive $\mathcal{L}_{IR}(\Psi)$.
At the tree level we have 2 diagrams



as usual $s = (k_1 + k_2)^2$ $t = (k_1 - k_3)^2$

and

$A(e^+e^- \to e^+e^-) =$

$$(-ig^2)\left[\frac{\bar v_2 u_1 \bar u_3 v_4}{s - M^2} + \frac{\bar u_3 u_1 \bar v_2 v_4}{t - M^2}\right]$$

$$\xrightarrow{s,t \ll M^2} \quad + \frac{ig^2}{M^2}\left(\bar v_2 u_1 \bar u_3 v_4 + \bar u_3 u_1 \bar v_2 v_4\right)$$

which lagrangian reproduces this?
If we are sloppy, we write

$$\mathcal{L}_{IR} = \dots \quad + \frac{g^2}{M^2} \bar\Psi \Psi \bar\Psi \Psi + \dots$$

missing a factor $\frac{1}{2}$ here
Indeed:

$\langle 3 \bar 4 | \bar\Psi \Psi \bar\Psi \Psi | 1 \bar 2 \rangle$ produces 4 contributions:

2 s-channel: $\langle 3 \bar 4 | \bar\Psi \Psi \bar\Psi \Psi | 1 \bar 2 \rangle$

$\langle 3 \bar 4 | \bar\Psi \Psi \bar\Psi \Psi | 1 \bar 2 \rangle$

2 t-channel: $\langle 3 \bar 4 | \bar\Psi \Psi \bar\Psi \Psi | 1 \bar 2 \rangle$

$\langle 3 \bar 4 | \bar\Psi \Psi \bar\Psi \Psi | 1 \bar 2 \rangle$

and we should put a factor $\frac{1}{2}$.
This comes automatically by using the e.o.m.

SUMMARY

- For a rapid estimate of power counting diagrams are O.K.
- To fix coefficients e.o.m. are better unless you are very good at computing diagrams

Exercise

Yukawa theory with $\lambda\varphi^4$ interaction:

$$\mathcal{L}_{UV} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}M^2\varphi^2 - \frac{\lambda}{4}\varphi^4 + \bar\Psi(i\partial\!\!\!/ - m)\Psi - g\varphi\bar\Psi\Psi$$

Derive $\mathcal{L}_{IR}$ at tree level keeping $O(\lambda)$ terms

$m \ll M$, neglecting the kinetic term

EOM in the static limit:

$$M^2\varphi + \lambda\varphi^3 + g\bar\Psi\Psi = 0$$

0^{th} order in λ:

$$\varphi_0 = -\frac{g\bar\Psi\Psi}{M^2} = \frac{J}{M^2} \qquad J \equiv -g\bar\Psi\Psi$$

1^{st} order:

$$\varphi_1 = \varphi_0 + \lambda\delta\varphi$$

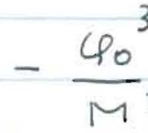
$$M^2\cancel{\varphi_0} + \lambda M^2\delta\varphi + \lambda\varphi_0^3 - \cancel{J} = 0 \qquad \delta\varphi = -\frac{\varphi_0^3}{M^2}$$

$$\boxed{\varphi_1 = \frac{J}{M^2}\left(1 - \lambda\frac{J^2}{M^6} + \dots\right)} \qquad \delta\varphi = -\frac{J^3}{M^8}$$

Back into $\mathcal{L}_{UV}$, neglecting the k. term

$$\mathcal{L}_{IR} = -\frac{1}{2}M^2\frac{J^2}{M^4}\left(1 - 2\lambda\frac{J^2}{M^6} + \dots\right) - \frac{\lambda}{4}\frac{J^4}{M^8} + \bar\Psi(i\partial\!\!\!/ - m)\Psi$$

$$\underbrace{-g\bar\Psi\Psi}_{+J}\cdot\frac{J}{M^2}\left(1 - \lambda\frac{J^2}{M^6} + \dots\right)$$



$$= \frac{J^2}{M^2}\left(-\frac{1}{2} + 1\right) + \lambda\frac{J^4}{M^8}\left(+\cancel{1} - \frac{1}{4} - \cancel{1}\right) + \dots$$

$$= \frac{1}{2}\frac{J^2}{M^2} - \frac{\lambda}{4}\frac{J^4}{M^8} + \dots = \frac{1}{2}\frac{g^2(\bar\Psi\Psi)^2}{M^2} - \frac{\lambda g^4(\bar\Psi\Psi)^4}{4M^8} + \dots$$

$$g^6(\bar\Psi\Psi)^6\lambda^2 \qquad \boxed{\lambda^{n-1}g^{2n}\frac{(\bar\Psi\Psi)^{2n}}{M^{2(3n-2)}}}$$

but how we proceed if $\lambda/g \gg 1$? (and/or)
We cannot rely on perturbation theory

1. Assume that at $E \ll M$ the relevant particles are still described by Ψ
Not guaranteed. In a strong coupling regime particles other than φ and Ψ can be formed. Some of them can be light. For instance in QCD the fundamental fields are quarks q and gluons G but in the low-energy regime we have light particles such as π's, etc...

2 Assume Lorentz invariance

$$\mathcal{L}_{IR} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m')\Psi + \frac{C_4^{(1)}}{\Lambda^2}(\bar{\Psi}\Psi)^2 + \dots$$
$$+ \frac{C_4^{(2)}}{\Lambda^2}(\bar{\Psi}\gamma^\mu\Psi)(\bar{\Psi}\gamma_\mu\Psi) + \dots$$
$$+ \frac{C_4^{(3)}}{\Lambda^2}(\bar{\Psi}\sigma^{\mu\nu}\Psi)(\bar{\Psi}\sigma_{\mu\nu}\Psi) + \dots$$

Exercise

Spontaneously broken $U(1)$ theory

φ complex scalar $\quad Q(\varphi) = +1$

Ψ Dirac fermion $\quad Q(\Psi) = +1$

$$\mathcal{L}_{UV} = \bar{\Psi}(i \not{D} - m)\Psi + (D_\mu \varphi)^\dagger D^\mu \varphi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4}\left(|\varphi|^2 - \frac{v^2}{2}\right)^2$$

$$D_\mu \varphi = (\partial_\mu + i g A_\mu)\varphi \qquad D_\mu \Psi = (\partial_\mu + i g A_\mu)\Psi$$

Determine $\mathcal{L}_{IR}$ at tree-level in the static approximation in the regimes

(1) $g^2 v^2 \gg m^2, \lambda v^2$

(2) $\lambda v^2 \gg m^2, g^2 v^2$

Mass spectrum

fermion ψ has mass m

$\lambda > 0$ $\quad \langle \varphi \rangle = \frac{v}{\sqrt{2}}$: gauge symmetry is spontaneously broken

Use new fields:

$$\varphi = \frac{\rho}{\sqrt{2}} e^{i\frac{\xi}{v}}$$

$$V = \frac{\lambda}{16}(\rho^2 - v^2)^2 \qquad \xi\text{-independent}$$

$$V_\xi = 0 \qquad V_\rho = \frac{\lambda}{4}(\rho^2 - v^2)\rho = 0$$

$\rho = 0$ local max $\qquad \rho = v$ local minimum

$$V_{\rho\rho} = \frac{\lambda}{4}(\rho^2 - v^2) + \frac{\lambda}{2}\rho^2$$

$$V_{\rho\rho}\Big|_0 = -\frac{\lambda}{4}v^2 < 0 \qquad V_{\rho\rho}\Big|_v = \frac{\lambda}{2}v^2 > 0$$

$m_\xi = 0$ $\qquad$ redefine $\rho = h + v$

$$m_h^2 = \frac{\lambda}{2}v^2$$

Finally

$$(D_\mu\varphi)^+ D^\mu\varphi = \left|(\partial_\mu + igA_\mu)\frac{h+v}{\sqrt{2}} e^{i\xi/v}\right|^2$$

$\rightarrow$ unitary gauge $\xi = 0$

$$\left|\partial_\mu \frac{h}{\sqrt{2}} + igA_\mu\left(\frac{h+v}{\sqrt{2}}\right)\right|^2 =$$

$$= \frac{1}{2}\partial_\mu h\,\partial^\mu h + \frac{g^2}{2}(h+v)^2 A_\mu A^\mu \rightarrow m_A^2 = g^2 v^2$$

SUMMARY

field	ψ	ξ	h	A
mass^2	m^2	eaten up by A_μ	$\frac{1}{2}\lambda v^2$	$g^2 v^2$

$$\boxed{\begin{aligned}\mathcal{L}_{UV} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu h\,\partial^\mu h + \bar\psi i\gamma^\mu\partial_\mu\psi - \bar\psi m\psi \\ & - g A_\mu \bar\psi\gamma^\mu\psi + \frac{g^2}{2}(h+v)^2 A_\mu A^\mu \\ & - \frac{\lambda}{16}\left((h+v)^2 - v^2\right)^2\end{aligned}}$$

① $g^2v^2 \gg m^2, \lambda v^2$

$\rightarrow$ m_A is the heaviest mass

e.o.m. for A_μ in the static limit (i.e. neglecting $F_{\mu\nu}F^{\mu\nu}$)

$$-g\,\bar\psi\gamma^\mu\psi + g^2(h+v)^2 A^\mu = 0$$

$$\boxed{A_\mu = \frac{g\,\bar\psi\gamma^\mu\psi}{g^2(h+v)^2} = \frac{g}{m_A^2}\cdot\frac{\bar\psi\gamma^\mu\psi}{\left(1+\frac{h}{v}\right)^2}}$$

back into the Lagrangian $\mathcal{L}_{UV}$:

$$\mathcal{L}_{IR} = \frac{1}{2}\partial_\mu h\,\partial^\mu h + \bar\psi i\gamma^\mu\partial_\mu\psi - \bar\psi m\psi - V(h)$$

$$-\frac{g^2}{m_A^2}\cdot\frac{(\bar\psi\gamma^\mu\psi)^2}{\left(1+\frac{h}{v}\right)^2} + \frac{g^{\cancel{2}}}{2}\cancel{(h+v)^2}\,\frac{g^2}{m_A^{\cancel{4}\,2}}\,\frac{(\bar\psi\gamma^\mu\psi)^2}{\left(1+\frac{h}{v}\right)^{\cancel{4}\,2}}$$

$$\boxed{\mathcal{L}_{IR} = \frac{1}{2}\partial_\mu h \partial^\mu h + \bar{\psi} i \gamma^\mu \partial_\mu \psi - \bar{\psi} m \psi - V(h) - \frac{1}{2}\frac{g^2}{m_A^2}\cdot\frac{(\bar{\psi}\gamma^\mu\psi)^2}{\left(1+\frac{h}{v}\right)^2}}$$

how should we interpret the last term

$$\frac{h}{v} = \frac{g\,h}{g\,v} = \frac{g\,h}{m_A}$$

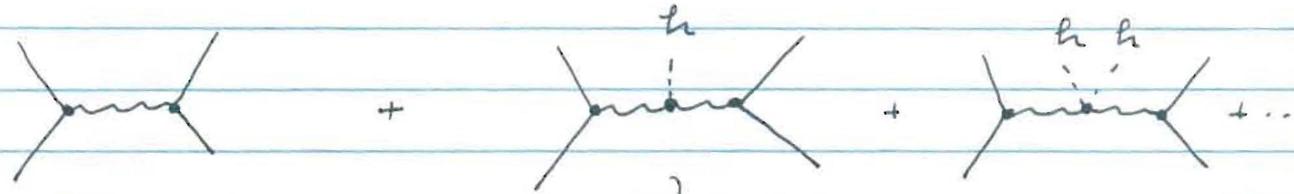
and we can expand in $\frac{1}{m_A}$

$$\left(1+\frac{h}{v}\right)^{-1} = 1 - \frac{g\,h}{m_A} + O(h^2)$$

The last term reads

$$-\frac{1}{2}\frac{g^2}{m_A^2}(\bar{\psi}\gamma^\mu\psi)^2\left[1 - 2\frac{g\,h}{m_A} + \dots\right]$$

In terms of diagrams



$$\frac{g^2(g^2 v)}{m_A^4} \simeq \frac{g^3}{m_A^3}$$

(2) $\lambda v^2 \gg g^2 v^2, m^2$

h is the heaviest state

$$V = \frac{\lambda}{16}(h^2 + 2vh)^2$$

$$= \frac{\lambda}{16}(h^4 + 4vh^3 + 4v^2h^2)$$

$$g^2 A_\mu A^\mu - \frac{\lambda}{4}(\rho^2 - v^2) = 0$$

$$\rho = v\left(1 + \frac{g^2 A^2}{m_h^2}\right)$$

$$g^2 A^2 - \frac{\lambda}{4}\left(v^2\left(\cancel{1} + \frac{2g^2A^2}{m_h^2}\right) - \cancel{v^2}\right) = 0$$

better to use the variables: $\rho \equiv h + v$

e.o.m. in the static limit

$$g^2 \rho A_\mu A^\mu - \frac{\lambda \rho}{4}(\rho^2 - v^2) = 0$$

we are working in the regime $\lambda v^2 \gg g^2 v^2$
$\lambda \gg g^2$

we can solve this by expanding in series of g^2

0^{th} order

$$\rho^2 - v^2 = 0 \qquad \rho = v \quad \leftrightarrow \quad h = 0$$

back into $\mathcal{L}_{UV}$:

$$\mathcal{L}_{IR} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} i \gamma^\mu \partial_\mu \Psi - \bar{\Psi} m \Psi$$

$$- g A_\mu \bar{\Psi} \gamma^\mu \Psi + \frac{1}{2} m_A^2 A_\mu A^\mu$$

1^{st} order

$$\rho = v(1 + x g^2)$$

$$g^2 v A_\mu A^\mu - \frac{\lambda v}{4}\left(v^2(\cancel{1} + 2 x g^2) - \cancel{v^2}\right) = 0$$

$$\cancel{v} A_\mu A^\mu - \frac{\lambda v^{\cancel{3}\,2}}{2} x = 0$$

$$x = \frac{A_\mu A^\mu}{m_h^2} \qquad \rho = v\left(1 + \frac{g^2}{m_h^2} A_\mu A^\mu + \dots\right)$$

$$\mathcal{L}_{IR} = 1^{st} \text{ line } - g A_\mu \bar{\Psi} \gamma^\mu \Psi$$

$$+ \frac{g^2}{2} v^2 \left(1 + \frac{2g^2}{m_h^2} A_\mu A^\mu + \dots\right) A_\nu A^\nu$$

$$- \frac{\lambda}{16}\left[v^2\left(\cancel{1} + \frac{2g^2}{m_h^2} A_\mu A^\mu + \dots\right) - \cancel{v^2}\right]^2$$

$$\mathcal{L}_{IR} = 1^{st}\text{ line} - g A_\mu \bar{\Psi}\gamma^\mu\Psi + \frac{1}{2} m_A{}^2 A_\nu A^\nu$$

$$+ (A_\mu A^\mu)^2 \left[\frac{g^4 v^2}{m_h{}^2} - \frac{\lambda}{16}\frac{v^4 4 g^4}{m_h{}^4} \right]$$

$$\downarrow$$

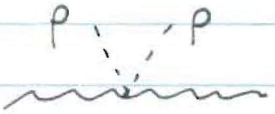
$$4\;\frac{1}{\cancel{16}}\frac{\lambda v^2}{m_h{}^4}\cdot\cancel{4}\, g^4 v^2$$

$$\equiv \frac{1}{2 m_h{}^2}\, g^4 v^2$$

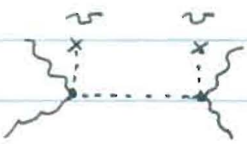
$$\boxed{\begin{aligned}\mathcal{L}_{IR} &= 1^{st}\text{ line} - g A_\mu \bar{\Psi}\gamma^\mu\Psi + \frac{1}{2} m_A{}^2 A_\mu A^\mu \\ &\quad + \frac{1}{2}\frac{g^4 v^2}{m_h{}^2}(A_\mu A^\mu)^2 + \dots\end{aligned}}$$

$\uparrow$ marginal operator

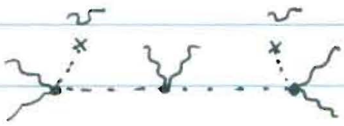
in terms of diagrams



g^2 $\qquad \approx g^2 v^2 \qquad A_\mu A^\mu$



$$\approx \frac{g^4 v^2}{m_h{}^2} \qquad (A_\mu A^\mu)^2$$



$$\approx \frac{g^6 v^2}{m_h{}^4} \qquad (A_\mu A^\mu)^3$$

Notice the peculiar power counting

$$\frac{g^{2n} v^2}{(m_h^2)^{n-1}} \approx (A_\mu A^\mu)^n$$

$$\boxed{\text{CASE} \quad g^2 v^2 \approx \lambda v^2 \gg m^2}$$

$$\begin{cases} -\bar\Psi \gamma^\mu \Psi + g\rho^2 A^\mu = 0 \\ g^2 A_\mu A^\mu - \frac{\lambda}{4}(\rho^2 - v^2) = 0 \end{cases}$$

1^{st} gives $\quad A_\mu = \dfrac{\bar\Psi \gamma_\mu \Psi}{g\rho^2}$

in the 2^{nd} gives:

$$\frac{\cancel{g^2}}{\cancel{g^2}\rho^4}(\bar\Psi\gamma_\mu\Psi)^2 - \frac{\lambda}{4}(\rho^2 - v^2) = 0$$

$$(\bar\Psi\gamma_\mu\Psi)^2 = \frac{\lambda}{4}\rho^4(\rho^2 - v^2)$$

To formally solve this, introduce a fake expansion parameter u

$$u(\bar\Psi\gamma_\mu\Psi)^2 \equiv u J^2 = \frac{\lambda}{4}\rho^4(\rho^2 - v^2)$$

Expanding the solution in powers of u:

$$\rho = v(1 + x u)$$

$$\cancel{u} J^2 = \frac{\lambda}{4} v^6 (1 + 4xu) 2x\cancel{u} = \frac{\lambda}{2} x \cancel{u} v^6$$

$$\rightarrow \quad x = \frac{2}{\lambda} \cdot \frac{J^2}{v^6}$$

$$\boxed{\begin{aligned} \rho &\equiv v\left(1 + \frac{2}{\lambda}\frac{J^2}{v^6} + \dots\right) \\ A_\mu &\equiv \frac{J_\mu}{g v^2}\left(1 - \frac{4}{\lambda}\frac{J^2}{v^6} + \dots\right) \end{aligned}} \qquad A_\mu = \frac{J_\mu}{g v^2\left(1 + \frac{4}{\lambda}\frac{J^2}{v^6}\right)}$$

$$\mathcal{L}_{IR} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\, i\gamma^{\mu}\partial_{\mu}\psi - \bar{\Psi}\, m\,\psi$$

$$-\cancel{g}\,\frac{J^2}{\cancel{g}\,v^2}\left(1-\frac{4}{\lambda}\,\frac{J^2}{v^6}+\dots\right)$$

$$+\frac{\cancel{g^2}}{2}\,\cancel{v^2}\left(1+\frac{4}{\lambda}\,\frac{J^2}{v^6}+\dots\right)\frac{J^2}{\cancel{g^2}\,v^{\cancel{4}}\,2}$$

$$-\frac{\lambda}{16}\left[v^2\left(\cancel{1}+\frac{4}{\lambda}\,\frac{J^2}{v^6}+\dots\right)-\cancel{v^2}\right]^2$$

$= 1^{st}$ line

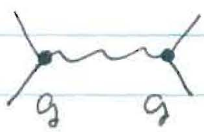
$$+\frac{J^2}{v^2}\left(-1+\frac{1}{2}\right)$$

$$+\frac{J^4}{\lambda v^8}\left(+4+2-\frac{16}{16}\right)$$

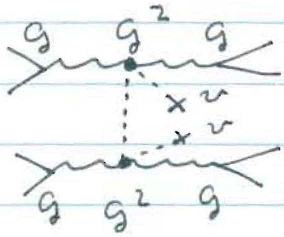
$= 1^{st}$ line

$$-\frac{1}{2v^2}\,J^2+\frac{5}{\lambda v^8}\,J^4+\dots$$

$\updownarrow$



$$\frac{g^2}{g^2v^2}\approx\frac{1}{v^2}$$



$$\approx\frac{g^8v^2}{m_A^8\,m_h^2}$$

$$\approx\frac{\cancel{g^8}\,\cancel{v^2}}{\cancel{g^8}\,v^8\,\lambda\,\cancel{v^2}}$$

$$\approx\frac{1}{\lambda v^8}$$

Derivation of Fermi Lagrangian

Recap of the SM

① Gauge symmetry $SU(3)\times SU(2)\times U(1)$

② Field content

- gauge vector bosons W^a_μ, B_μ, $G_{a\mu}$
- scalars: $\varphi \sim (1,2,+1/2)$
- fermions: 3 generations of quarks and leptons

$$q_L \sim (3,2,+1/6) = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$
$$u_R \sim (3,1,+2/3)$$
$$d_R \sim (3,1,-1/3)$$
$$l_L \sim (1,2,-1/2) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$
$$e_R \sim (1,1,-1)$$

③ Renormalizability (only relevant and marginal op.)

① + ② + ③ $\rightarrow \mathcal{L}_{SM} =$

$$= -\frac{1}{4}F^2 + i\bar{\Psi}\gamma^\mu D_\mu \Psi$$
$$+ (D_\mu\varphi)^\dagger D^\mu\varphi - V(\varphi^\dagger\varphi)$$
$$-(\bar{\Psi}\, y\, \varphi\, \Psi + \text{h.c.})$$

More in detail:

$$i\bar{\Psi}\gamma^\mu D_\mu\Psi = \text{kinetic terms} - e A_\mu J^\mu_{em}$$
$$-\frac{g}{\sqrt{2}}(W^+_\mu J^{-\mu} + W^-_\mu J^{+\mu})$$
$$-\sqrt{g^2+g'^2}\, Z_\mu J_Z^\mu$$
$$+ \text{gluon interactions}$$
$$- g_s G_{a\mu} \sum_q \bar{q}\gamma^\mu t^a q$$

where $$J^\mu_{em} = \sum_f \bar{f} \gamma^\mu Q_f f$$

$$= -\bar{e}\gamma^\mu e + \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d$$

$$J^{+\mu} = \bar{d}_L \gamma^\mu u_L + \bar{e}_L \gamma^\mu \nu_L$$
$$J^{-\mu} = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L$$

$$J^\mu_{3L} = \sum_f \bar{f}\, T_{3L}^f f$$

$$= \frac{1}{2}(\bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L)$$

$$+ \frac{1}{2}(\bar{\nu}_L \gamma^\mu \nu_L - \bar{e}_L \gamma^\mu e_L)$$

$$J_Z^\mu = J^\mu_{3L} - s^2_\theta J^\mu_{em}$$

$tg\,\vartheta \equiv g'/g$

$m_A = m_g = 0$ $\qquad m_W^2 = \frac{g^2 v^2}{4}$ $\qquad m_Z^2 = \frac{(g^2+g'^2)v^2}{4}$

$$\frac{m_W^2}{m_Z^2 c_\theta^2} = 1$$

Moreover, from $(D_\mu\varphi)^+ D^\mu\varphi$ we get

$$(D_\mu\varphi)^+ D^\mu\varphi = m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

The Fermi Lagrangian refers to an IR EFT derived in the limit $E \ll m_{W,Z}$

1 at the TL

2 in the static approximation

3 at LO in g expansion

Since $F_{\mu\nu} F^{\mu\nu} \sim \underbrace{(\partial A)^2 + g A^2 \partial A}_{\text{neglected (2)}} + \underbrace{g^2 A^4}_{\text{neglected (3)}}$

Relevant terms:

$$\mathcal{L}_{SM} = m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$
$$- \frac{g}{\sqrt{2}} \left(W^+_\mu J^{-\mu} + W^-_\mu J^{+\mu} \right)$$
$$- \sqrt{g^2 + g'^2}\, Z_\mu J_Z^\mu + \ldots$$

e.o.m.

$W^\pm_\mu$ $\qquad m_W^2 W^{\mp\mu} - \frac{g}{\sqrt{2}} J^{\mp\mu} = 0$

Z_μ $\qquad m_Z^2 Z^\mu - \sqrt{g^2 + g'^2}\, J_Z^\mu = 0$

$$W^{\mp\mu} = \frac{g}{\sqrt{2}\, m_W^2} J^{\mp\mu}$$
$$Z^\mu = \frac{\sqrt{g^2 + g'^2}}{m_Z^2} J_Z^\mu$$

back into $\mathcal{L}_{SM}$:

$$\mathcal{L}_{Fermi} = \frac{\cancel{m_W^2}\, g^2}{2 m_W^{\cancel{4}\,2}} J^+ J^- + \frac{1}{2} \frac{m_Z^2 (g^2 + g'^2)}{m_Z^4} J_Z^2$$
$$- \frac{g}{\sqrt{2}} \frac{g}{\sqrt{2}\, m_W^2} 2 J^+ J^- - \frac{(g^2 + g'^2)}{m_Z^2} J_Z^2$$

original Fermi Lagrangian was of $V \times V$ type

1957 Sudarshan Marshak $(V-A)\times(V-A)$

Feynman Gell-Mann

$$= -\frac{g^2}{2 m_W^2} J^+ J^- - \frac{(g^2+g'^2)}{2 m_Z^2} J_Z^2 + \dots = \mathcal{L}_{Fermi}$$

$$\frac{g^2}{2 m_W^2} = \frac{4 g^2}{2 g^2 v^2} = \frac{2}{v^2}$$

$$\frac{(g^2+g'^2)}{2 m_Z^2} = \frac{4\,(\cancel{g^2+g'^2})}{2\,(\cancel{g^2+g'^2})\,v^2} = \frac{2}{v^2}$$

Define $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$ $\qquad \frac{2}{v^2} = 2\sqrt{2}\, G_F$

$$\mathcal{L}_{Fermi} = -2\sqrt{2}\, G_F \left(J^+ J^- + J_Z^2 \right)$$

For instance, in the lepton sector

$$J^+_\mu = \bar{e}_L \gamma_\mu \nu_{eL} + \dots = \frac{1}{2} \bar{e} \gamma_\mu (1-\gamma_5) \nu + \dots$$

$$J^-_\mu = \bar{\nu}_{\mu L} \gamma_\mu \mu_L + \dots = \frac{1}{2} \bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu + \dots$$

$$\mathcal{L}_{Fermi} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu * \bar{e} \gamma^\mu (1-\gamma_5) \nu + \dots$$

describes μ-decay

<u>Remarks</u>

→ Same weight between $J^+ J^-$ and J_Z^2 as a consequence of $\frac{m_W^2}{m_Z^2 \cos^2\theta} = 1$

$\leftrightarrow$ EW symmetry broken by $\varphi \sim (2, +1/2)$