Advanced Topics in the Theory of Fundamental Interactions

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1. The field content of a QFT in d spacetime dimensions consists of a scalar field φ and a vector boson A_μ, subject to the following local transformation:

$$
\begin{cases}
\varphi(x) \rightarrow e^{-i\alpha(x)}\varphi(x) \\
A_\mu(x) \rightarrow A_\mu(x) + \dfrac{\partial_\mu \alpha(x)}{e}
\end{cases}
$$

2. List all possible relevant and marginal operators invariant under CP and the above local transformation, in $d = 3, 4, 6$.

3. Working in $d = 4$, focus on the Lagrangian

$$
\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\varphi^\dagger D^\mu\varphi - m^2(\varphi^\dagger\varphi) - \frac{\lambda}{4}(\varphi^\dagger\varphi)^2
$$

where $D_\mu\varphi = (\partial_\mu + ieA_\mu)\varphi$, $\lambda > 0$ and m^2 is a real parameter. List the operators of the low-energy EFT in the two cases:

   i) $m^2 > 0$, for energies $E << \sqrt{m^2}$. Stop at dimension 8, included.

   ii) $m^2 < 0$ and $e << \lambda$, for energies $e/\lambda\sqrt{-m^2} << E << \sqrt{-m^2}$. Stop at dimension 6, included.

4. Estimate the power counting of the operators found in **3.**

5. In case ii), evaluate explicitly $\mathcal{L}_{IR}$ at the tree-level, including all marginal and irrelevant operators.