Advanced Topics in the Theory of Fundamental Interactions

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1. Consider the following theory in $d = 4$ with a heavy scalar ϕ_H and a light scalar ϕ_L:

$$\begin{aligned}\mathcal{L}_{UV} &= \frac{1}{2}\partial_\mu\phi_L\partial^\mu\phi_L - \frac{1}{2}m^2\phi_L^2 + \frac{1}{2}\partial_\mu\phi_H\partial^\mu\phi_H - \frac{1}{2}M^2\phi_H^2 - \frac{\lambda_L}{4!}\phi_L^4 \\ &- \frac{\lambda_{LH}}{4}\phi_L^2\phi_H^2 - \frac{gM}{2}\phi_H\phi_L^2\end{aligned}$$

At low energies $E \approx m \ll M$, the physics is described by an effective Lagrangian $\mathcal{L}_{IR}$ which depends only on the light field and has the form

$$\begin{aligned}\mathcal{L}_{IR} &= \frac{1}{2}\partial_\mu\phi_L\partial^\mu\phi_L - \frac{1}{2}\tilde{m}^2\phi_L^2 - \frac{\tilde{\lambda}_L}{4!}\phi_L^4 \\ &- \frac{C_{(4,2)}}{M^2}\phi_L^2\Box\phi_L^2 - \frac{C_{(6,0)}}{M^2}\phi_L^6\end{aligned}$$

up to terms suppressed M^4.

2. Derive the coefficients $\tilde{m}^2$, $\tilde{\lambda}_L$, $C_{(4,2)}$ and $C_{(6,0)}$ at the tree-level.

3. Perform a field redefinition in the effective theory

$$\phi_L(x) \to \phi_L(x) + \frac{\alpha}{M^2}\phi_L(x)^3$$

dropping any $1/M^4$ terms. Show that a suitable choice of α and eliminates the operator $\phi_L^2\Box\phi_L^2$ from $\mathcal{L}_{IR}$.(Remember that, up to total derivatives, $\phi_L^2\Box\phi_L^2 = 4/3\phi_L^3\Box\phi_L$.)

4. Now consider the more general IR Lagrangian:

$$\begin{aligned}\mathcal{L}_{IR} &= \frac{1}{2}\partial_\mu\phi_L\partial^\mu\phi_L - \frac{1}{2}\tilde{m}^2\phi_L^2 - \frac{\tilde{\lambda}_L}{4!}\phi_L^4 \\ &- \frac{C_{(2,4)}}{M^2}\phi_L\Box^2\phi_L - \frac{C_{(4,2)}}{M^2}\phi_L^2\Box\phi_L^2 - \frac{C_{(6,0)}}{M^2}\phi_L^6\end{aligned}$$

Perform a field redefinition in the effective theory

$$\phi_L(x) \to \phi_L(x) + \frac{\beta}{M^2}\Box\phi_L(x) + \frac{\alpha}{M^2}\phi_L(x)^3$$

dropping any $1/M^4$ terms. Show that a suitable choice of α and β eliminates the operators $\phi_L^2\Box\phi_L^2$ and $\phi_L\Box^2\phi_L$ from $\mathcal{L}_{IR}$.

5. Draw the diagrams contributing to the two-point function up to one loop in the full and the effective theory. Use a double line to denote the heavy field and a single line for the light field.

6. Draw the diagrams contributing to the four-point function at one loop in the full and the effective theory. One representative of each topology is enough (additional diagrams, related by the exchange of external legs, need not be drawn).