Advanced Topics in the Theory of Fundamental Interactions

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1. List all operators (redundant or not) up to dimension 8 we can build out of a real scalar field ξ, and invariant under Z_2, $\xi \to -\xi$, and under a global continuous shift symmetry:

$$\xi \to \xi + \alpha \qquad\qquad \alpha \in \mathbb{R}$$

item[**2.**] Write the most general Lagrangian including operators up to dimension 8, eliminating the redundant ones.

3. Derive the classical equations of motion for ξ

4. Consider the UV Lagrangian of a complex scalar field φ

$$\mathcal{L}_{UV} = \partial_\mu \varphi^\dagger \partial^\mu \varphi - \frac{\lambda}{4}\left(\varphi^\dagger \varphi - \frac{v^2}{2}\right)^2$$

with $\lambda > 0$, $v^2 > 0$, invariant under the global U(1) symmetry

$$\varphi \to e^{i\alpha}\varphi$$

Using the parametrization

$$\varphi(x) = \frac{\sigma(x) + v}{\sqrt{2}} e^{\,i\frac{\xi(x)}{v}} \quad ,$$

determine the masses of the ξ and σ particles.

5. Find the tree-level IR Lagrangian $\mathcal{L}_{IR}$, describing the physics at energies much smaller than $\sqrt{\lambda}v$, containing up to dimension 8 operators. [Hint: solve the classical equations of motion for the heavy particle iteratively, by expanding them in $1/(\lambda v^2)$.]

6. Estimate the cross section of the elastic scattering $\xi\xi \to \xi\xi$, at energies much smaller than $\sqrt{\lambda}v$.