Advanced Topics in the Theory of Fundamental Interactions

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January 3, 2020

1. The Lagrangian

$$
\begin{aligned}
\mathcal{L}(\lambda, c_i) &= (D_\mu\varphi)^\dagger D^\mu\varphi - \lambda(\varphi^\dagger\varphi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&+ \frac{c_1}{\Lambda^2}(\varphi^\dagger\varphi)^3 + \frac{c_2}{\Lambda^2}(\varphi^\dagger\varphi)F_{\mu\nu}F^{\mu\nu} \\
&+ \frac{c_3}{\Lambda^2}(\varphi^\dagger\varphi)(D_\mu\varphi)^\dagger D^\mu\varphi + \frac{c_4}{\Lambda^2}[(\varphi^\dagger D^\mu\varphi)^2 + h.c.]
\end{aligned}
$$

with $\lambda > 0$, $D^\mu\varphi = (\partial_\mu + ieA_\mu)\varphi$, describes a gauge theory invariant under a local U(1) group. Make the field redefinition

$$
\varphi \to \varphi + \frac{\alpha}{\Lambda^2}(\varphi^\dagger\varphi)\varphi
$$

and find the value of α that eliminates the operator $(\varphi^\dagger\varphi)(D_\mu\varphi)^\dagger D^\mu\varphi$, working at the order $1/\Lambda^2$.

2. Find the relation between (λ, c_i) and (λ', c'_i) when $\mathcal{L}(\lambda, c_i) \to \mathcal{L}(\lambda', c'_i)$ with $c'_3 = 0$.

3. The UV Lagrangian

$$
\begin{aligned}
\mathcal{L}_{UV} &= (D_\mu\varphi)^\dagger D^\mu\varphi - \lambda_0(\varphi^\dagger\varphi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&+ \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{1}{2}M^2S^2 \\
&+ S(MJ_1 + \frac{1}{M}J_2)
\end{aligned}
$$

where $\lambda_0 > 0$ and

$$
J_1 \equiv a_1(\varphi^\dagger\varphi) \quad , \qquad J_2 \equiv a_2F_{\mu\nu}F^{\mu\nu} + a_3(D_\mu\varphi)^\dagger D^\mu\varphi + a_4(\varphi^\dagger\varphi)^2
$$

contains an additional massive real scalar field S, neutral under the gauge group. Derive the TL IR Lagrangian $\mathcal{L}_{IR}$ valid at energies smaller than M, showing that it can be cast in the form shown in **1.** Identify the IR parameters (λ, c_i) in terms of the UV ones (λ_0, a_i), setting $\Lambda = M$.