Advanced Topics in the Theory of Fundamental Interactions

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1. The non-relativistic Lagrangian

$$\mathcal{L}_{NR} = i\phi^*\partial_t\phi + \phi^*\frac{\partial_k\partial_k}{2m}\phi - c_2\phi^*\phi - c_4(\phi^*\phi)^2$$

is defined in d space-time dimensions. Take as dimensions defining the scaling properties $[\hbar] = 0$ and generic $[t]$, $[l]$, $[t] \neq [l]$. (In terms of scaling this means $t = e^{-[t]\alpha}t'$ and $x = e^{-[l]\alpha}x'$, t' and x' kept fixed). Determine the dimensions of the field φ as well as of momenta p, velocities v, masses m and energies E in terms of $[t]$ and $[x]$.

2. Find the dimensions of $c_{2,4}$. Assuming $c_{2,4}$ are powers of a fundamental mass m and the light velocity c, $c_i \propto m^{\alpha_i}c^{\beta_i}$, determine the correct powers α_i and β_i.

3. Classify the operators $(\phi^*\phi)$ and $(\phi^*\phi)^2$ as relevant/marginal/irrelevant, with respect to the scaling $t = e^{-[t]\alpha}t'$ and $x = e^{-[l]\alpha}x'$, assuming $[l] = -1$ and $[t] = -2$, that is smaller velocities for larger α.

4. From now on set $d = 4$. Let φ describe neutral atoms. Determine the dimensions of the electric and magnetic fields E_k and B_k, using the Lorentz force $F = e(E + v/cB)$.

4. List the most important (that is, less irrelevant) operators describing the interaction between neutral atoms and the electric/magnetic fields, requiring invariance under parity and time reversal.

5. Assign the operators of the previous point the correct power counting (in terms a relevant mass scale M and the light velocity c) and estimate the cross section of elastic scattering of light on neutral atoms, as a function of the photon energy E

6. Which meaning has M? At which anergy the above approximation breaks down?