Advanced Topics in the Theory of Fundamental Interactions

F. Feruglio

January 6, 2020

1. The Euler-Heisenberg Lagrangian describes electromagnetic interactions below the electron mass

$$\mathcal{L}_{EH} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2(F_{\mu\rho}F^{\rho\nu})(F^{\mu\lambda}F_{\lambda\nu})$$

Assign the correct power counting to the coefficients $c_{1,2}$ in terms of the electric charge e and the electron mass m_e.

2. Estimate the low-energy cross section of light-by-light scattering

3. List all the symmetries of $\mathcal{L}_{EH}$

4. Let

$$\mathcal{L}_{SM} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} + (D_\mu\varphi)^\dagger D_\mu\varphi - V(\varphi^\dagger\varphi)$$

the bosonic part of the SM, φ being the Higgs doublet, $D_\mu\varphi = (\partial_\mu + ig\frac{\sigma^i}{2}W^i_\mu + i\frac{g'}{2}B_\mu)\varphi$, B_μ and W^i_μ the gauge bosons of U(1) and SU(2). List all the symmetries of $\mathcal{L}_{SM}$.

5. Set $g = 0$ (that is ignore the SU(2) part) and consider the unrealistic case $m_h \gg m_Z$. Ignoring fermions, at energies below the Higgs mass the theory is described by an IR Lagrangian $\mathcal{L}_{IR}$ depending on the gauge vector boson fields $Z_\mu = B_\mu$. List all the symmetries of $\mathcal{L}_{IR}$.

6. List all relevant and marginal operators of $\mathcal{L}_{IR}$ and determine their power counting in terms of v^2, m_Z^2 and m_h^2.

7. Determine $\mathcal{L}_{IR}$ at the TL, up to terms containing four gauge vector bosons B_μ.