Advanced Topics in the Theory of Fundamental Interactions

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1. The Lagrangian

$$\int dt d^3x \left[\frac{1}{2}(\partial_t\varphi)^2 - \frac{1}{2}\varphi(\partial_k\partial_k)^2\varphi + c_1\varphi\partial_k\partial_k\varphi + c_2\varphi^2 \right]$$

describes an anisotropic four-dimensional theory where space and time scale differently:

$$\vec{x} = e^{\alpha}\vec{x}' \quad , \qquad t = e^{z\alpha}t' \quad .$$

Keeping $\hbar$ dimensionless, we can use momenta p as basic unit. In these units, from the previous relations we have

$$[p] = +1 \quad , \qquad [\vec{x}] = -1 \quad , \qquad [t] = -z \quad .$$

Determine the p-dimension of φ.

2. Find z such that the operator $\varphi(\partial_k\partial_k)^2\varphi$ is marginal.

3. With the previously determined value of z, determine the p-dimension of the coefficients c_1 and c_2.

Classify the corresponding operators as relevant/marginal/irrelevant.

4. With the value of z found in **2.**, determine the highest power n such that the operator φ^n is marginal.

5. The generators of space and time translations are

$$P_i = \partial_i \quad , \qquad P_0 = \partial_t \quad ,$$

while the scaling transformations are generated by

$$D = -zt\partial_t - x^i\partial_i$$

Derive the algebra (commutation relations) among P_i, P_0 and D.