FORMULE UTILI IN MECCANICA QUANTISTICA

Buca di potenziale simmetrica e infinita [-a/2,+a/2] 1D

Autovalori e autofunzioni dell'Hamiltoniano ($2n$ e $2n+1$ sono interi positivi):

$$E_{2n+1} = \frac{\hbar^2}{2m}(2n+1)^2\left(\frac{\pi}{a}\right)^2 \qquad \varphi_{2n+1} = \sqrt{\frac{2}{a}}\cos\left((2n+1)\frac{\pi}{a}x\right)$$

$$E_{2n} = \frac{\hbar^2}{2m}(2n)^2\left(\frac{\pi}{a}\right)^2 \qquad \varphi_{2n} = \sqrt{\frac{2}{a}}\sin\left(2n\frac{\pi}{a}x\right)$$

Oscillatore Armonico 1D

$$a = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}X + i\frac{1}{\sqrt{m\omega\hbar}}P\right) \qquad a^+ = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}X - i\frac{1}{\sqrt{m\omega\hbar}}P\right) \qquad [a, a^+] = 1$$

$N = a^+a \qquad N|n >= n|n > \quad (n = 0, 1, 2, ...) \qquad a|n >= \sqrt{n}|n-1\rangle \qquad a^+|n >= \sqrt{n+1}|n+1\rangle$

$$\varphi_n(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n(\beta x)\; e^{-\beta^2x^2/2} \qquad \beta = \sqrt{\frac{m\omega}{\hbar}}$$

$$H_0(z) = 1 \qquad H_1(z) = 2z \qquad H_2(z) = 4z^2 - 2 \qquad H_3(z) = 8z^3 - 12z$$

Momenti Angolari

$$[J_i, J_j] = i\hbar\epsilon_{ijk}J_k \quad (i, j, k, = 1, 2, 3) \qquad [J^2, J_i] = 0 \qquad J_\pm = J_1 \pm iJ_2$$

$J^2|j, m\rangle = \hbar^2 j(j+1)|j, m\rangle \qquad J_3|j, m\rangle = \hbar m|j, m\rangle \qquad J_\pm|j, m\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}|j, m\pm 1\rangle$

spin 1/2

$$J_1 = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad J_2 = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad J_3 = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

spin 1

$$J_1 = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad J_2 = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad J_3 = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

momento angolare orbitale

$$L_3 = -i\hbar\frac{\partial}{\partial\varphi} \qquad L_\pm = \hbar e^{\pm i\varphi}\left(\pm\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\varphi}\right)$$

$$L^2 Y_l^m(\theta, \varphi) = \hbar^2 l(l+1)Y_l^m(\theta, \varphi) \qquad L_3 Y_l^m(\theta, \varphi) = \hbar m Y_l^m(\theta, \varphi)$$

$$Y_0^0 = \frac{1}{\sqrt{4\pi}} \qquad Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi}$$

Atomo di Idrogeno

autovalori e autofunzioni dell'hamiltoniano (elettrone senza spin e non-relativistico)

$$E_n = -\frac{E_I}{n^2} \qquad E_I = m_e c^2\alpha^2 \qquad \alpha = \frac{e^2}{\hbar c} \qquad a_0 = \frac{\hbar^2}{m_e e^2} \qquad \varphi_{nlm}(r, \theta, \varphi) = R_{nl}(r)Y_l^m(\theta, \varphi)$$

$$R_{1,0}(r) = \frac{2}{(a_0)^{3/2}} e^{-r/a_0} \qquad R_{2,0}(r) = \frac{2}{(2a_0)^{3/2}} \left(1 - \frac{r}{2a_0}\right) e^{-r/2a_0} \qquad R_{2,1}(r) = \frac{1}{(2a_0)^{3/2}} \frac{1}{\sqrt{3}} \frac{r}{a_0} e^{-r/2a_0}$$

Teoria delle perturbazioni (spettro discreto e non degenere)

$$H = H_0 + W \qquad H_0|E_n^0\rangle = E_n^0|E_n^0\rangle$$

$$E_n = E_n^0 + W_{nn} + ... \qquad |E_n\rangle = |E_n^0\rangle - \sum_{m\neq n} \frac{W_{mn}}{E_m^0 - E_n^0}|E_m^0\rangle + ... \qquad W_{mn} = \langle E_m^0|W|E_n^0\rangle$$