Probability for Neutrino Oscillation in Vacuum

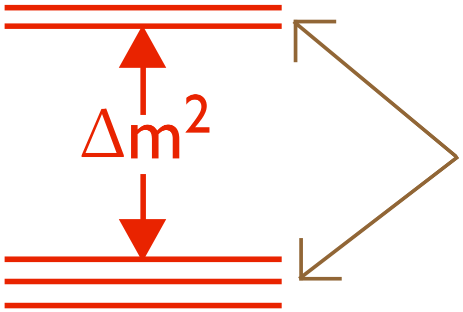
$$P(\nu_\alpha \rightarrow \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$

When One Big Δm^2 Dominates



These splittings are invisible if $\Delta m^2 \dfrac{L}{E} = \mathcal{O}(1)$.

For $\beta \neq \alpha$,

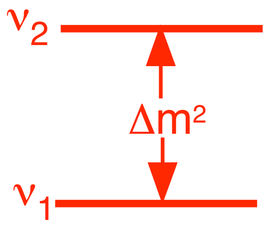
$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \cong S_{\alpha\beta} \sin^2(\Delta m^2 \frac{L}{4E}) \; ; \; S_{\alpha\beta} \equiv 4 \left| \sum_{i \text{ Clump}} U_{\alpha i}^* U_{\beta i} \right|^2 .$$

For no flavor change,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \cong 1 - 4T_\alpha (1 - T_\alpha) \sin^2(\Delta m^2 \frac{L}{4E}) \; ; \; T_\alpha \equiv \sum_{i \text{ Clump}} |U_{\alpha i}^*|^2 .$$

"i Clump" is a sum over only the mass eigenstates on one end of the big gap Δm^2.

When There are Only Two Flavors and Two Mass Eigenstates

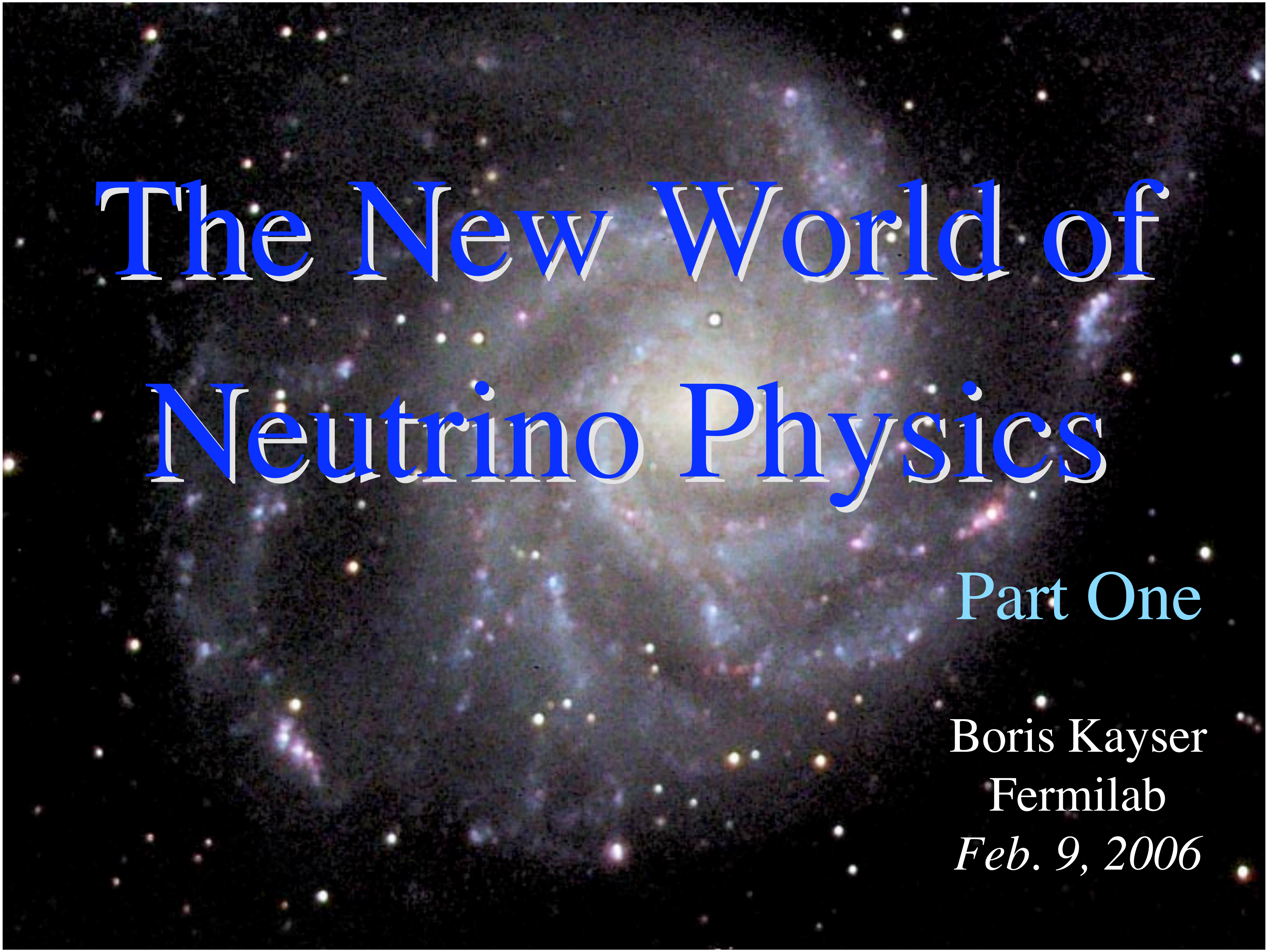


$$U = \begin{array}{cc} & \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{array}{c} \nu_\alpha \\ \nu_\beta \end{array} & \left[\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right] \end{array} \;\; ; \;\; S_{\alpha\beta} = 4T_\alpha(1 - T_\alpha) = \sin^2 2\theta$$

Mixing angle

For $\beta \neq \alpha$, $P(\overset{(-)}{\nu}_\alpha \leftrightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.

For no flavor change, $P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.



The New World of Neutrino Physics

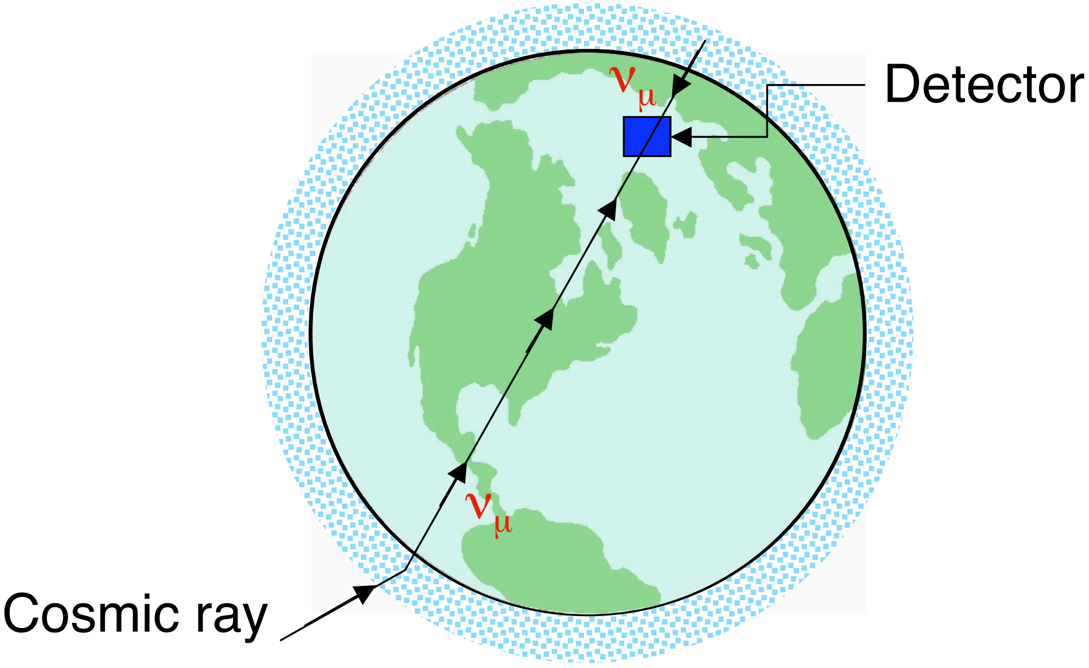
Part One

Boris Kayser
Fermilab
Feb. 9, 2006

Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (L ~ 180 km)	Compelling
Atmospheric	Compelling
Accelerator (L = 250 km)	Very Strong
Stopped μ^+ Decay $\begin{pmatrix}\text{LSND}\\ L \approx 30\text{ m}\end{pmatrix}$	Unconfirmed

Atmospheric Neutrinos



Isotropy of the $\gtrsim$ 2 GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \quad \frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 1\ .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 0.54 \pm 0.04\ .$$

Half of the upward-going, long-distance-traveling ν_μ are disappearing.

Voluminous atmospheric neutrino data are well described by —

$$\nu_\mu \longrightarrow \nu_\tau$$

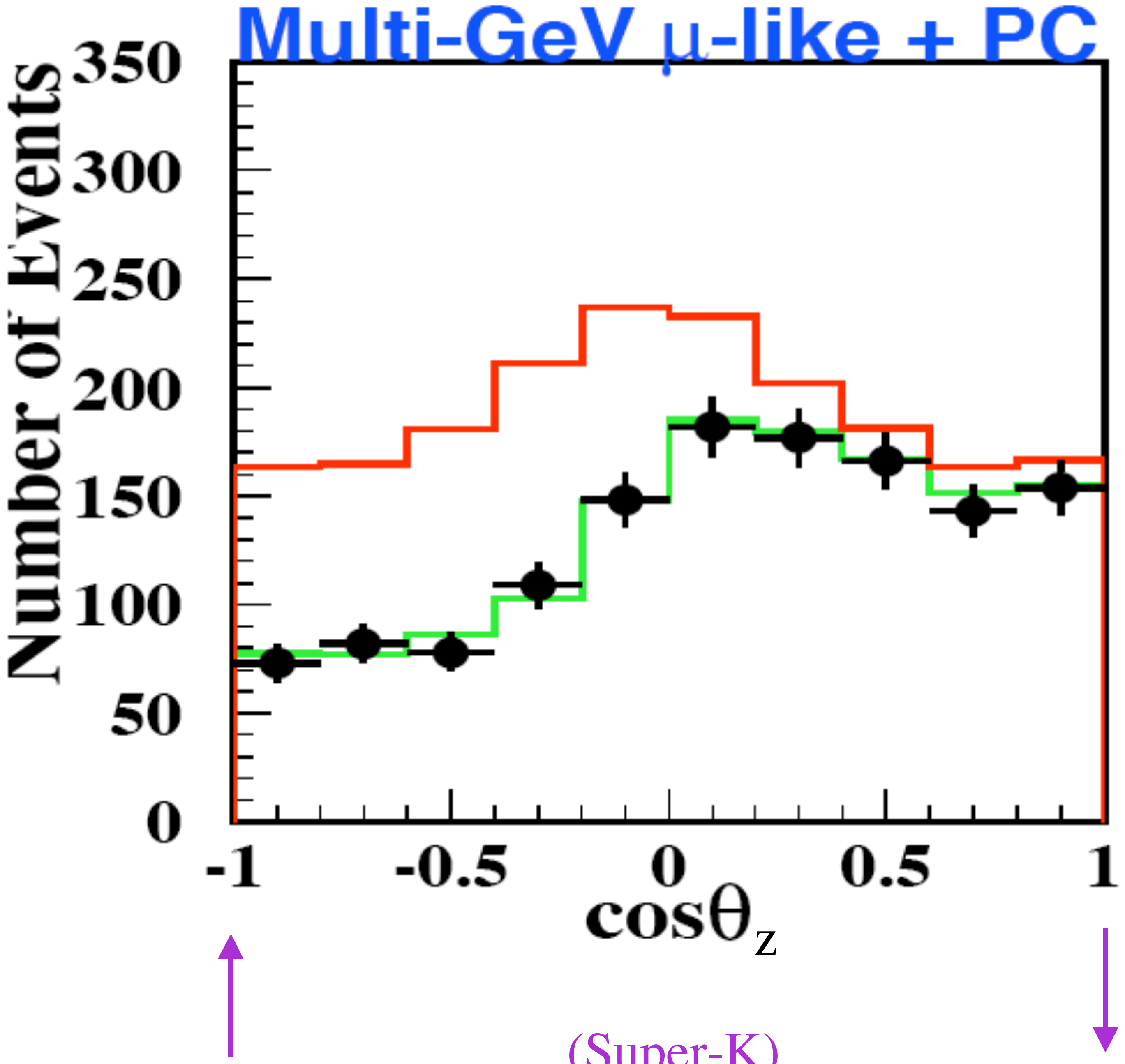
with —

$$1.9 \times 10^{-3} < \Delta m^2_{atm} < 3.0 \times 10^{-3}\ \mathrm{eV}^2$$

and —

$$\sin^2 2\theta_{atm} > 0.92$$

$\left(\begin{array}{c}\text{Super-K}\\ \text{90\%CL}\end{array}\right)$


Multi-GeV μ-like + PC
Number of Events
350
300
250
200
150
100
50
0
-1
-0.5
0
0.5
1
$\cos\theta_z$

(Super-K)

Solar Neutrinos

History –

Nuclear reactions in the core of the sun produce ν_e. Only ν_e.

Theorists, especially John Bahcall, calculated the produced ν_e flux vs. energy E.

Ray Davis' Homestake experiment measured the higher-E part of the ν_e flux ϕ_{ν_e} that arrives at earth.

The Homestake experiment could detect only ν_e. It found:

$$\frac{\phi_{\nu_e}(\text{Homestake})}{\phi_{\nu_e}(\text{Theory})} = 0.34 \pm 0.06$$

The Possibilities:

The theory was wrong.

The experiment was wrong.

Both were wrong.

Neither was wrong. Two thirds of the ν_e flux morphs into a flavor or flavors that the Homestake experiment could not see.

The Resolution —

Sudbury Neutrino Observatory (SNO) measures, for the high-energy part of the solar neutrino flux:

$$\nu_{sol}\, d \rightarrow e\, p\, p \;\Rightarrow \phi_{\nu_e}$$

$$\nu_{sol}\, d \rightarrow \nu\, n\, p \;\Rightarrow \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$$

From the two reactions,

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}} = 0.340 \pm 0.023\,(\text{stat}) \pm 0.030\,(\text{syst})$$

Clearly, $\phi_{\nu_\mu} + \phi_{\nu_\tau} \neq 0$. Neutrinos change flavor.

Change of flavor does not change the total number of neutrinos.

The total flux, $\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$, should agree with Bahcall's prediction.

SNO: $\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} = (4.94 \pm 0.21 \pm 0.36) \times 10^6/\mathrm{cm}^2\mathrm{sec}$

Theory*: $\phi_{\mathrm{total}} = (5.69 \pm 0.91) \times 10^6/\mathrm{cm}^2\mathrm{sec}$

*Bahcall, Basu, Serenelli

John Bahcall and Ray Davis both stuck to their guns for several decades, and both were right all along.

The now-established mechanism for solar $\nu_e \rightarrow \nu_\mu / \nu_\tau$ is not oscillation in vacuum but the —

Large Mixing Angle —

Mikheyev Smirnov Wolfenstein

— Effect.

This effect occurs as the neutrinos stream outward through solar material. It requires both interactions with matter and neutrino mass and mixing.

For the solar neutrinos, the interaction with matter changes the evolution of the neutrino "beam" considerably.

Matter effects on the evolution of ν and $\bar{\nu}$ beams will be covered by Stephen Parke.

Reactor (Anti)Neutrinos

The CHOOZ reactor experiment, with a detector ~ 1km from the source, tells us that, to a good approximation, ν_e is made up of just 2 mass eigenstates.

As a result, solar neutrino behavior is approximately a two-neutrino problem.

The vacuum neutrino properties Δm^2_{sol} and θ_{sol} implied by LMA-MSW are —

$$\Delta m^2_{sol} \sim 8 \times 10^{-5}\ \mathrm{eV}^2\ ;\ \theta_{sol} \sim 35^o\ .$$

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\overbrace{[(G_{\text{Fermi}}/\sqrt{2})N_e]}^{\text{Interaction energy}} \; / \; \overbrace{[\Delta m^2/4E]}^{\text{Vacuum energy}} \equiv x\ .$$

Density of electrons

For $\Delta m^2 = \Delta m^2_{\text{sol}} \sim 8 \text{ x } 10^{-5}\ \text{eV}^2$,

$$x = 2.5 \text{ x } 10^{-3}\ E(\text{MeV})\ .$$

At reactor energies of a few MeV,
this is negligible.

The KamLAND detector is ~ 180 km from reactor $\overline{\nu}_e$ sources.

For KamLAND, at say 3 MeV, the argument of —

$$\sin^2[1.27\Delta m^2_{sol}(eV^2)L(km)/E(GeV)]$$

is —

$$3.9 \text{ x } (\pi/2).$$

The experiment sees an energy-averaged oscillation.

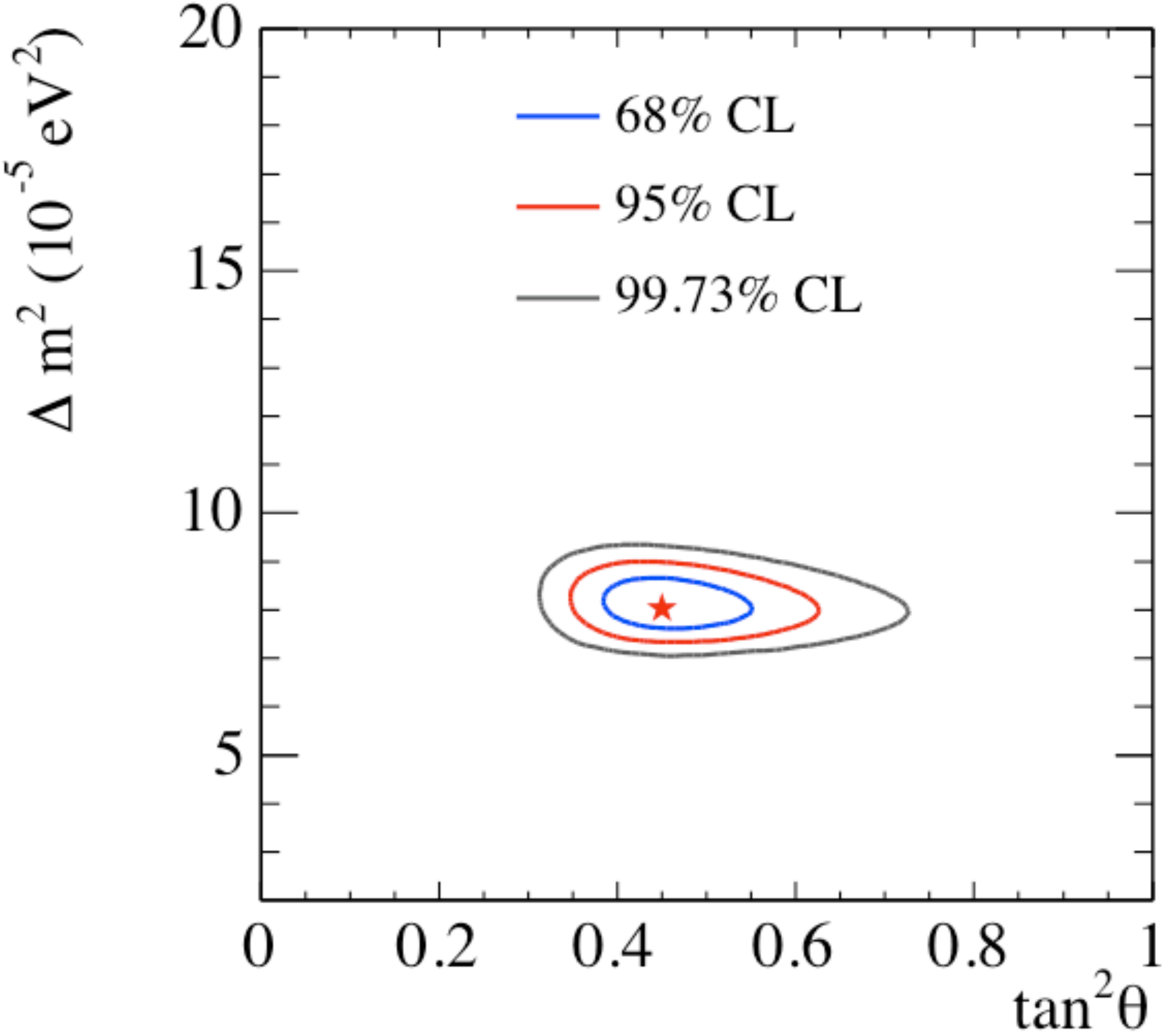
It should see substantial disappearance of $\overline{\nu}_e$ flux.

KamLAND actually does see —

$$\frac{\phi_{\bar{\nu}_e}}{\phi_{\bar{\nu}_e}\Big|_{\substack{\text{No}\\\text{Disappearance}}}} = 0.658 \pm 0.044(\text{stat}) \pm 0.047(\text{syst})\ .$$

Reactor $\bar{\nu}_e$ do disappear.

Flavor change, with Δm^2_{sol} and θ_{sol} in the LMA-MSW range, fits both the solar and reactor data.

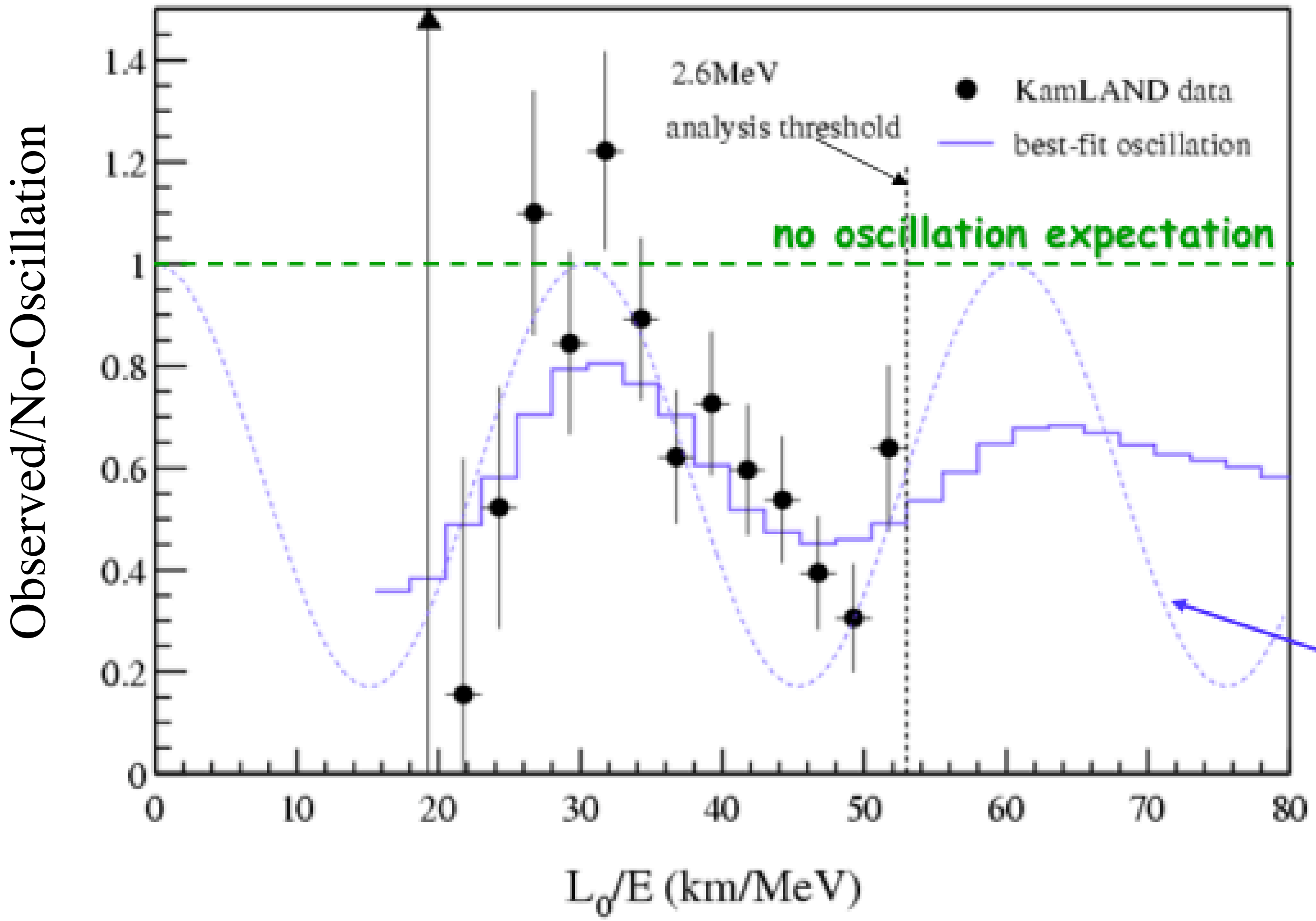


From
nucl-ex/
0502021

Solar Δm^2 and mixing angle from SNO analysis of solar neutrino and KamLAND data

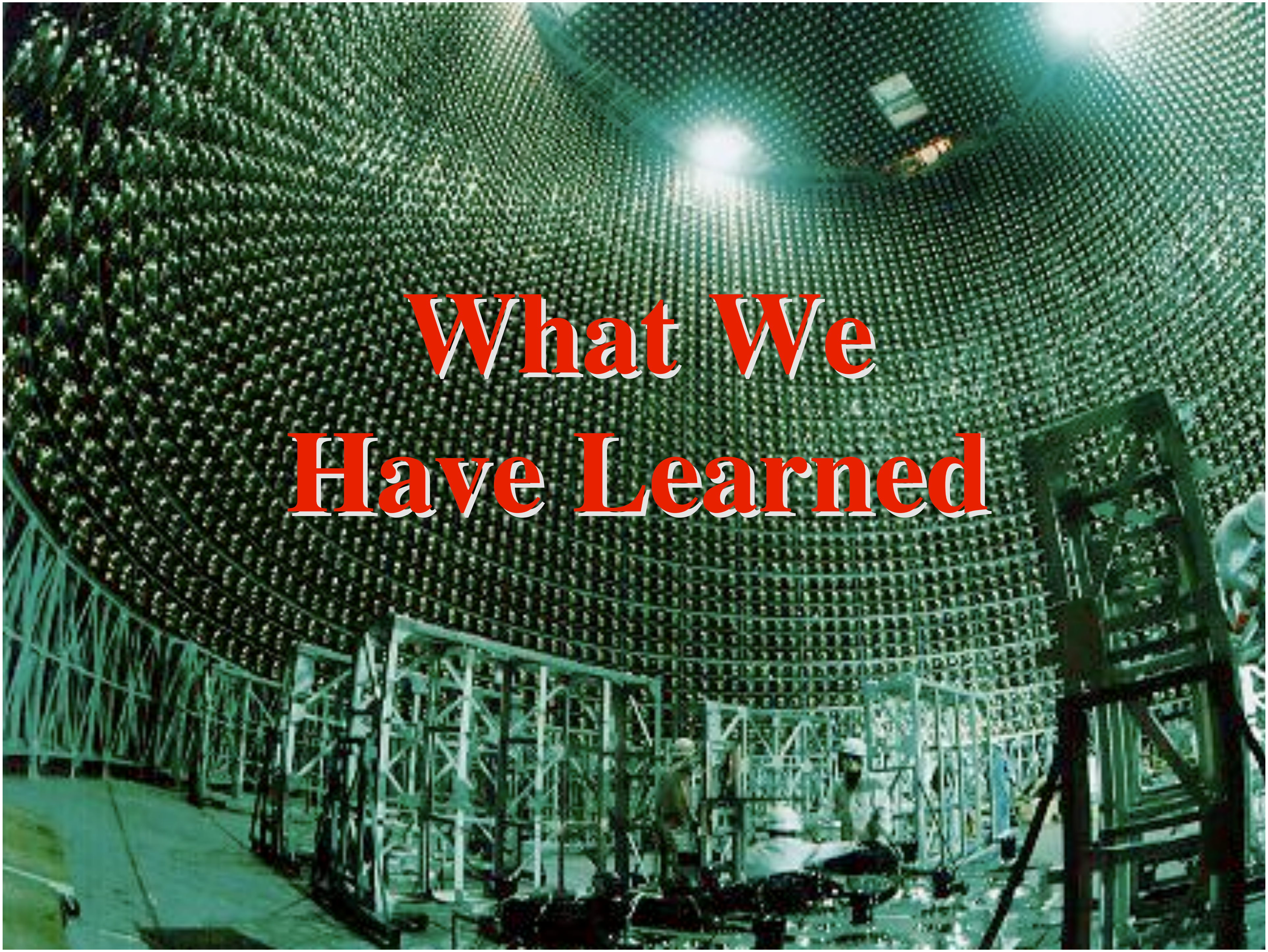
Evidence for the oscillation of flavor change

From G. Gratta at Neutrino 04



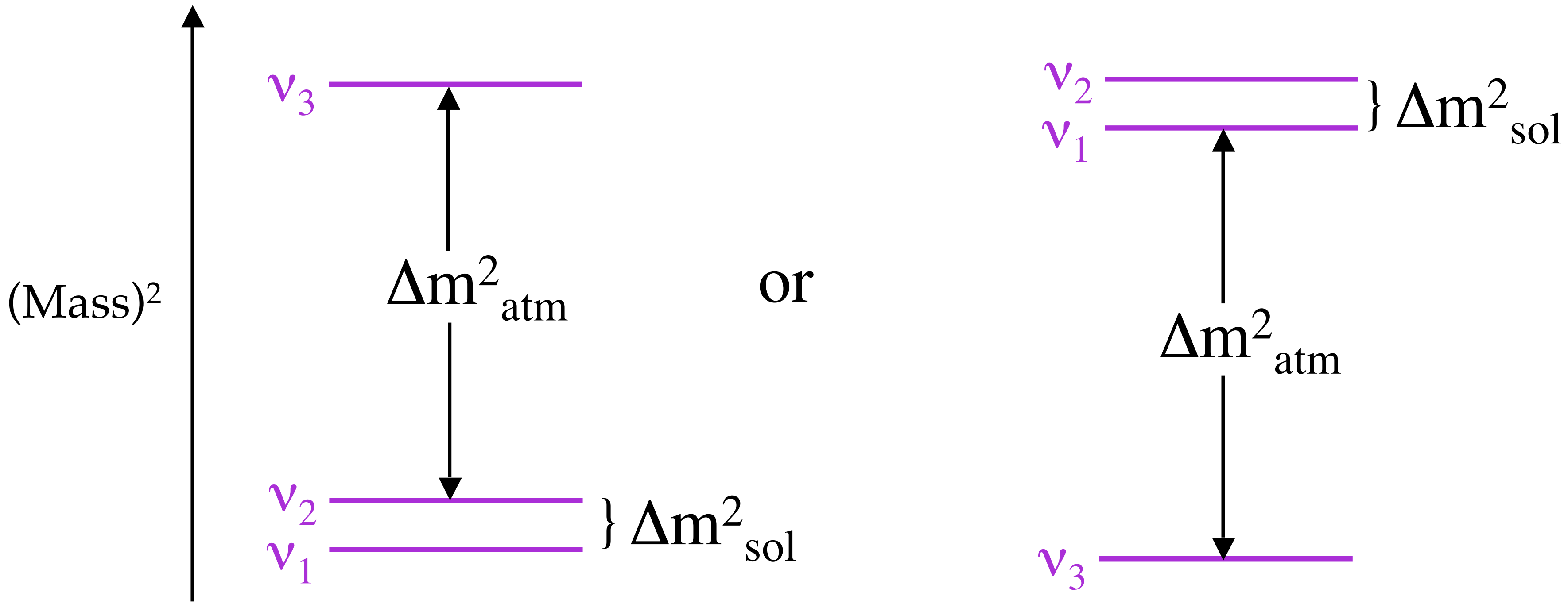
Hypothetical single 180km baseline experiment

KamLAND $\overline{\nu}_e$ event rate vs. L/E, assuming each $\overline{\nu}_e$ traveled $L = L_0 = 180$ km.



What We
Have Learned

The $(\text{Mass})^2$ Spectrum



$(\text{Mass})^2$

ν_3

Δm^2_{atm}

ν_2

ν_1

} Δm^2_{sol}

or

ν_2

ν_1

} Δm^2_{sol}

Δm^2_{atm}

ν_3

Normal

Inverted

$\Delta m^2_{\text{sol}} \cong 8 \text{ x } 10^{-5}\ \text{eV}^2$, $\Delta m^2_{\text{atm}} \cong 2.5 \text{ x } 10^{-3}\ \text{eV}^2$

LSND suggests there is (at least) one more Δm^2, hence one more mass eigenstate.

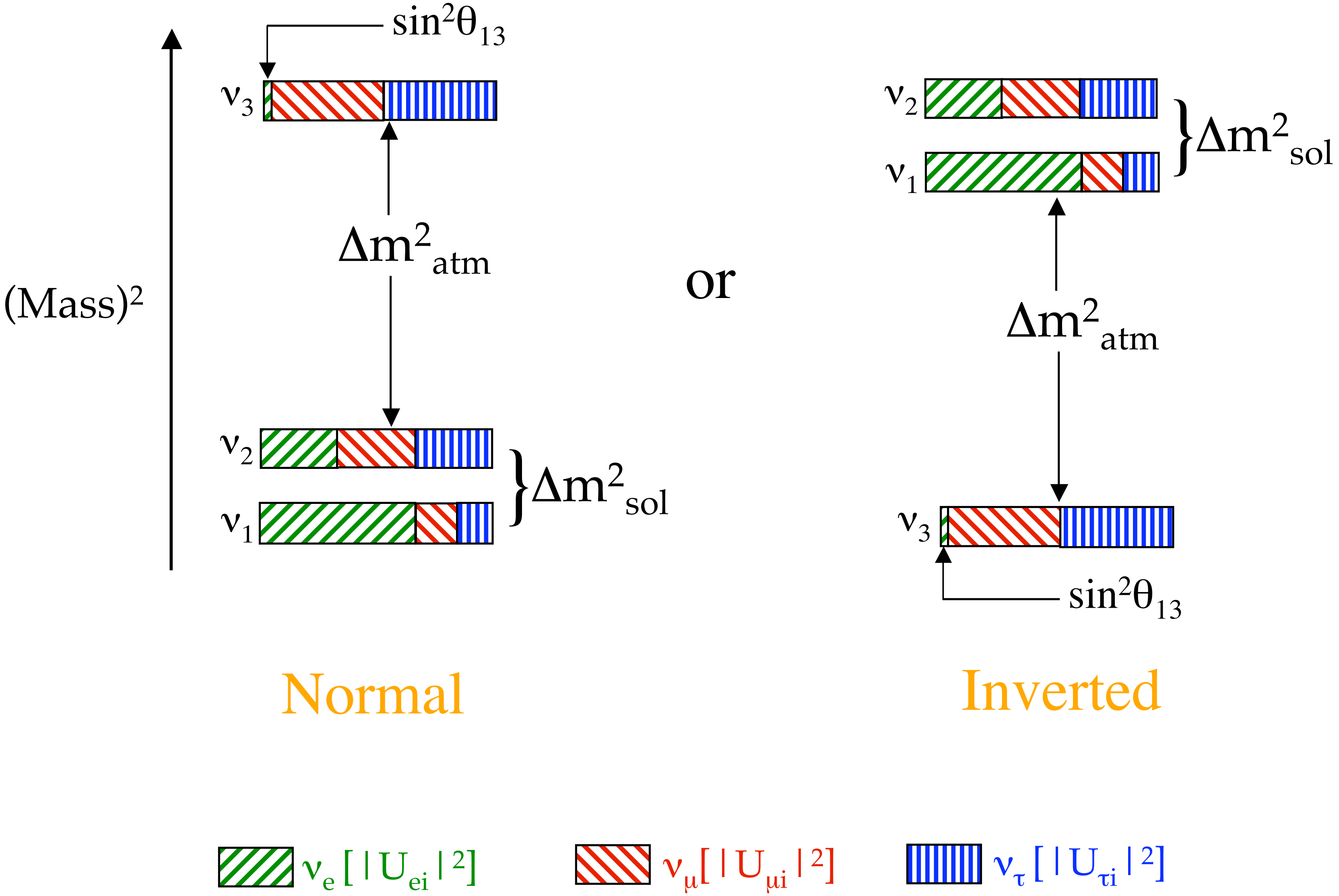
Recall that each mass eigenstate is a superposition of flavors:

$$|\nu_i> = \Sigma_\alpha U_{\alpha i} | \nu_\alpha > .$$

The flavor-α fraction of ν_i is —

$$|<\nu_\alpha|\nu_i>|^2 = |U_{\alpha i}|^2 .$$

Assuming that there are only 3 mass eigenstates, the spectrum, showing its approximate flavor content, is —



$\sin^2\theta_{13}$

ν_3

Δm^2_{atm}

(Mass)^2

ν_2

ν_1

$\}\Delta m^2_{sol}$

or

ν_2

ν_1

$\}\Delta m^2_{sol}$

Δm^2_{atm}

ν_3

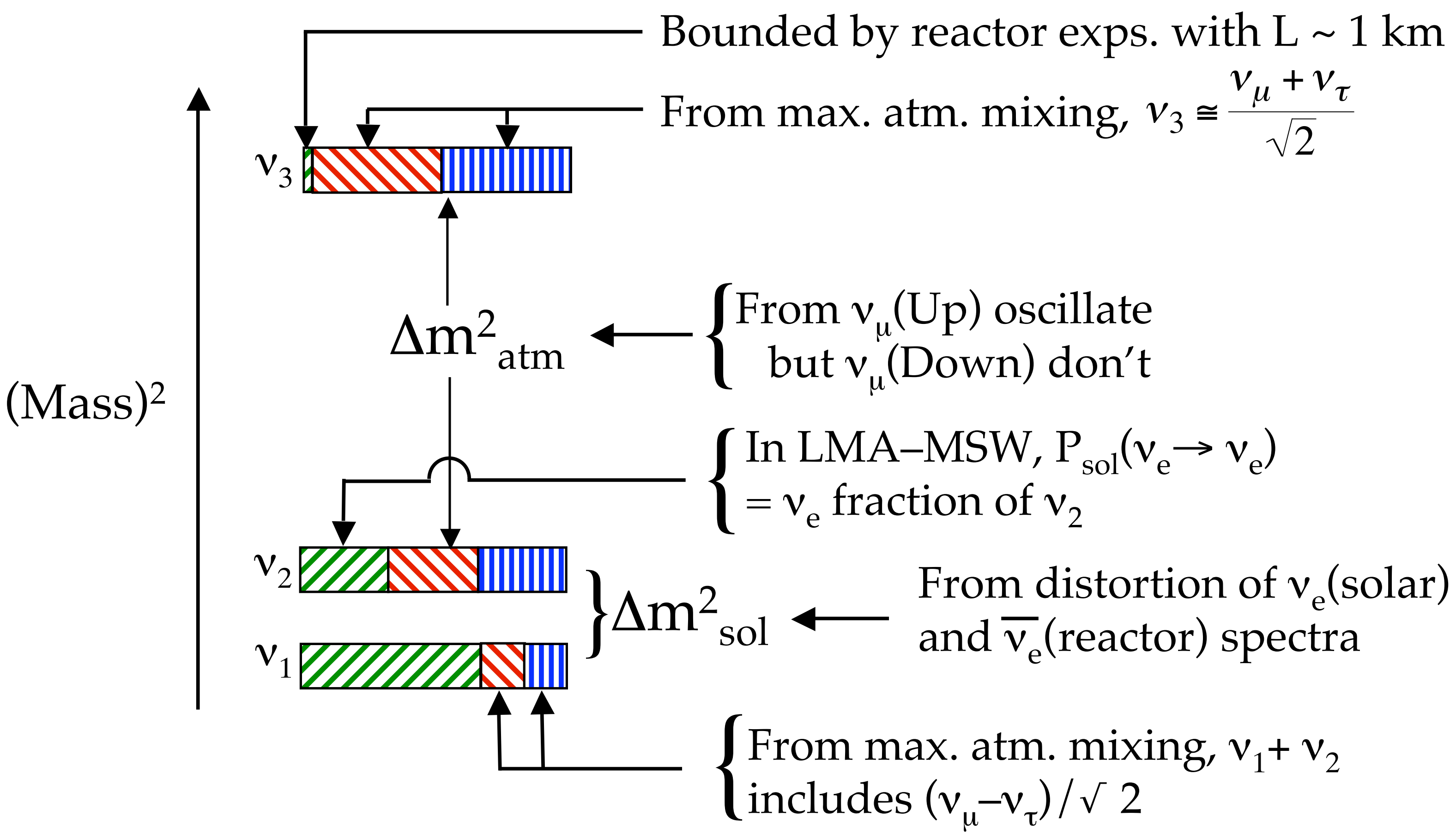
$\sin^2\theta_{13}$

Normal

Inverted

▨ $\nu_e\ [\,|U_{ei}|^2]$ ▨ $\nu_\mu[\,|U_{\mu i}|^2]$ ▥ $\nu_\tau\ [\,|U_{\tau i}|^2]$

How Did We Learn This?



Bounded by reactor exps. with L ~ 1 km

From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$

ν_3

Δm^2_{atm}

From ν_μ(Up) oscillate
but ν_μ(Down) don't

(Mass)^2

In LMA–MSW, $P_{sol}(\nu_e \to \nu_e)$
= ν_e fraction of ν_2

ν_2

ν_1

Δm^2_{sol}

From distortion of ν_e(solar)
and $\overline{\nu}_e$(reactor) spectra

From max. atm. mixing, $\nu_1 + \nu_2$
includes $(\nu_\mu - \nu_\tau)/\sqrt{2}$

🟩 ν_e [$|U_{ei}|^2$]   🟥 ν_μ [$|U_{\mu i}|^2$]   🟦 ν_τ [$|U_{\tau i}|^2$]

The Mixing Matrix

$$U = \overset{\text{Atmospheric}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}} \times \overset{\text{Cross-Mixing}}{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}} \times \overset{\text{Solar}}{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

$$\times \underset{\text{Majorana } \cancel{CP} \text{ phases}}{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$\theta_{12} \approx \theta_{sol} \approx 34°$, $\theta_{23} \approx \theta_{atm} \approx 37\text{-}53°$, $\theta_{13} \lesssim 10°$

δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$. $\cancel{CP}$

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

The Contrast Between Quark and Lepton Mixing

$$V_{\text{quark}} = \begin{pmatrix} 1 & S & s \\ S & 1 & S \\ s & S & 1 \end{pmatrix}$$

s ≡ small

Why?

$$U_{\text{lepton}} = \begin{pmatrix} B & B & \theta_{13} \\ B & B & B \\ B & B & B \end{pmatrix}$$

B ≡ Big

How Can the Standard Model be Modified to Include Neutrino Masses?