Measurement of α_s

Key element of QCD is the running coupling $\alpha_s(\mu^2) = \dfrac{1}{b_0 \ln\left(\dfrac{\mu^2}{\Lambda^2}\right)}$

This has been measured in several processes like fragmentation Functions, jets production rate, global fit at LEP and SLAC (Z^0)
At hadron collider can be measured using the inclusive jet cross section.

$$\frac{d\sigma}{dE_T} = \alpha_s^2(\mu_R)\hat{X}^{(0)}(\mu_F, E_T)[1 + \alpha_s(\mu_R)k_1(\mu_R, \mu_F, E_T)]$$

$$\alpha_s^3(\mu_R)\hat{X}^{(0)}(\mu_F, E_T)k_1(\mu_R, \mu_F, E_T)$$

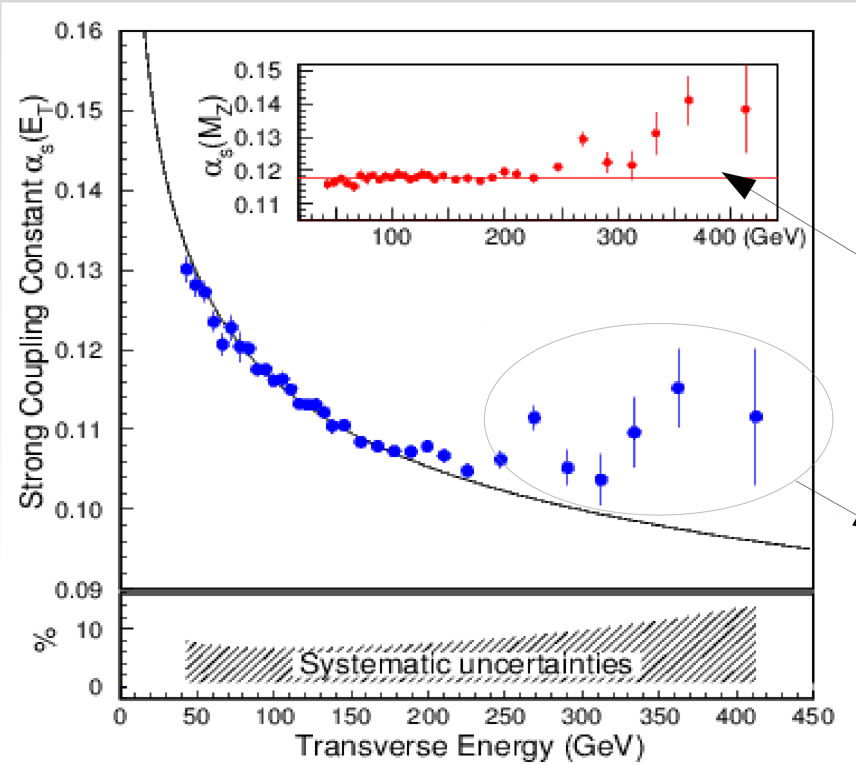
Jets transverse energy distribution

Leading order prediction

Nest to leading order contribution

Jet data are divided into several bins of Et and in each bin α_s is measured.

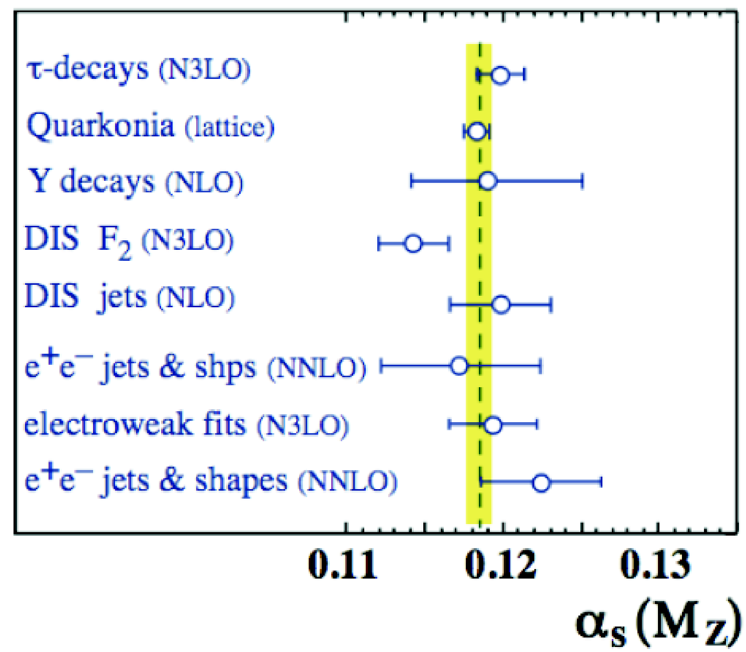
Measurement of α_s



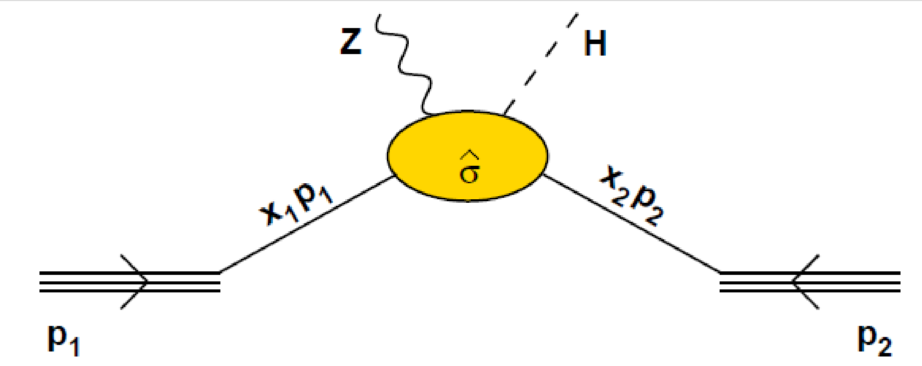
Good agreement between data and and predictions

α_s is evolved to M_Z for all the measurements. Then the values are averaged: $\alpha_s(M_Z) = 0.1178 \pm 0.0001(\mathrm{stat}).$

Problem due to the gluon PDF



Parton Distribution Function

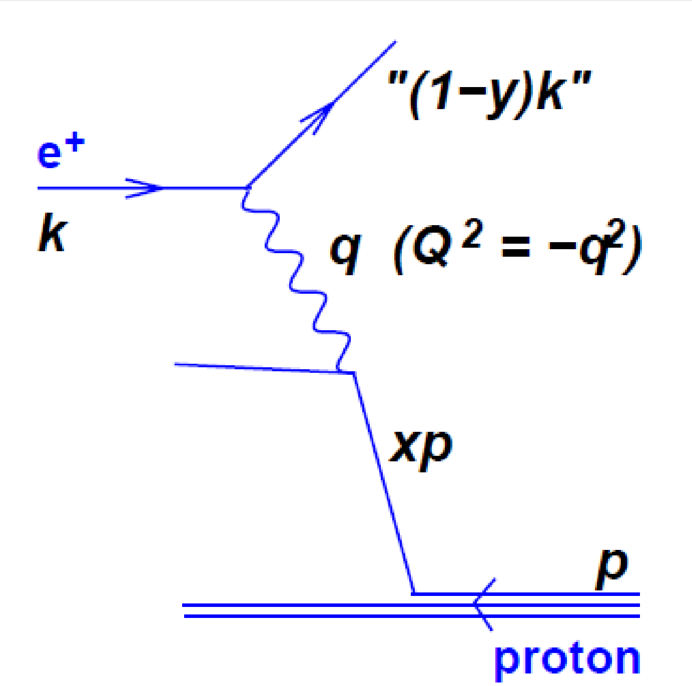


In order to evaluated the cross section of any process involving hadrons in the initial state we must know the parton distribution inside the hadron

PDF are non-perturbative properties, in principle they can be calculated using Lattice QCD but the precision is not enough yet respect to perturbative QCD+experiment measurements

A lot of progress on PDF has come from Deep Inelastic Scattering at HERA, ep collider with 2 experiments: H1 and ZEUS

Deep Inelastic Scattering: description



$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ the center of mass energy

Q^2: photon virtuality ↔ transverse resolution at which it probes proton

x: longitudinal momentum fraction of the struck parton in proton

y: momentum fraction lost by e

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

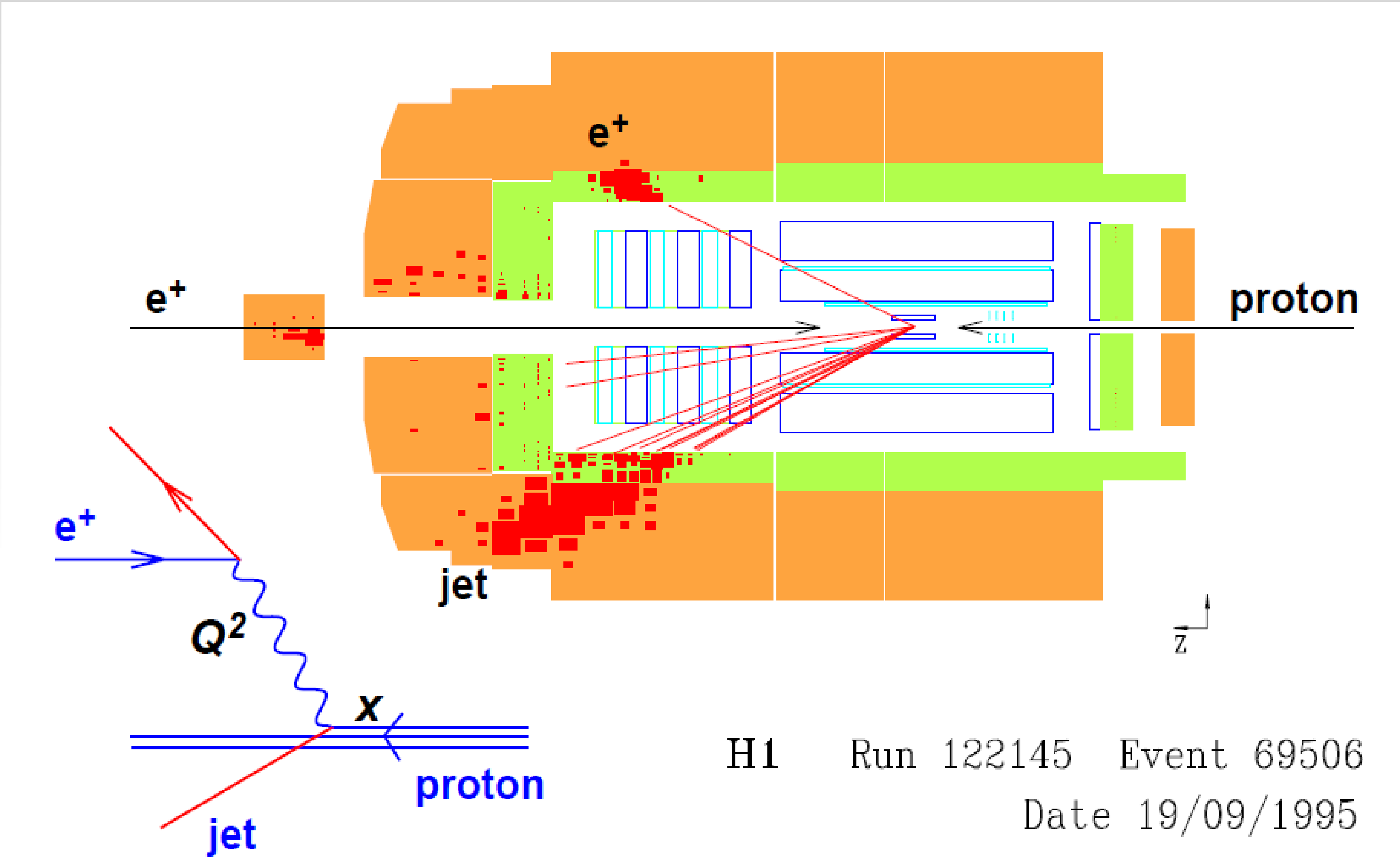
Differential Cross section (what is measured)

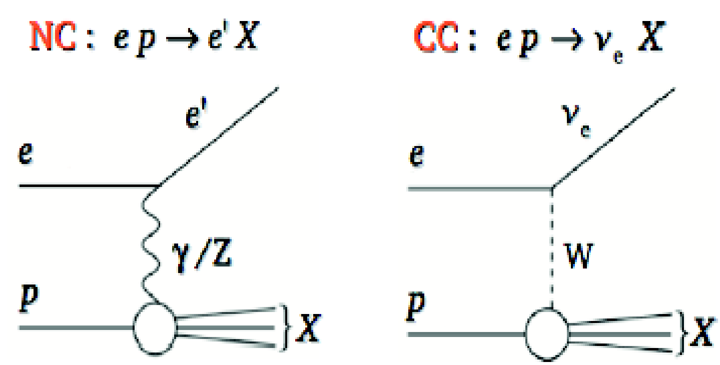
Structure Function

$$F_2^{\text{proton}} = x(e_u^2 u_p(x) + e_d^2 d_p(x)) = x\left(\frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)\right)$$

parton distribution function

Experimental measurements





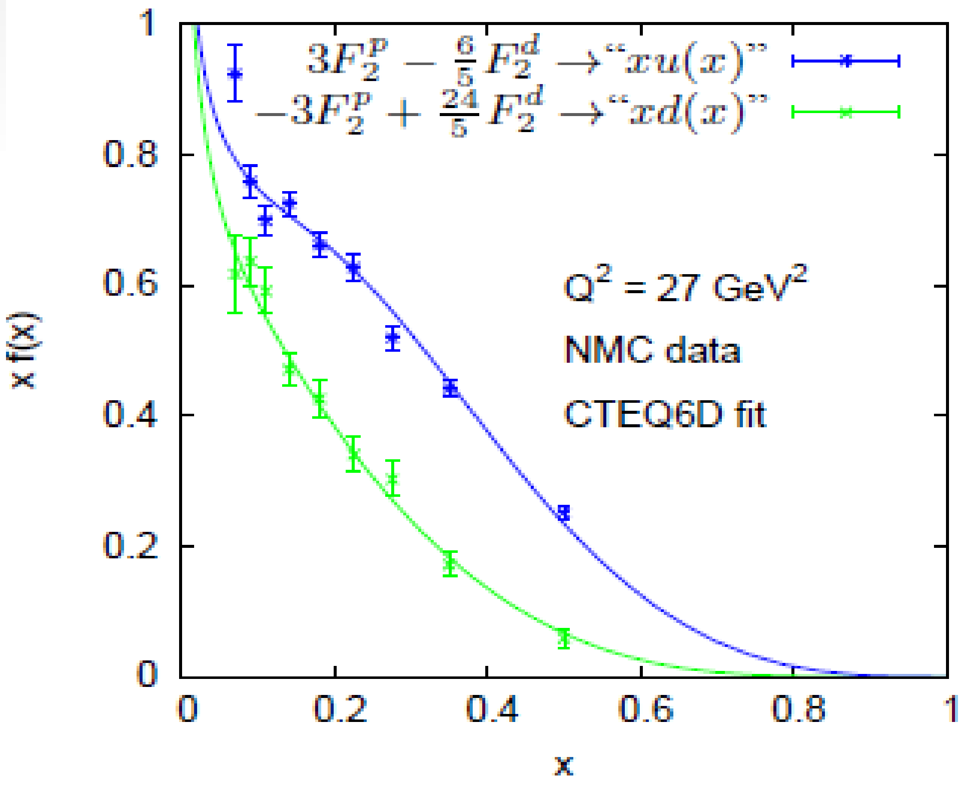
ep measurements are not enough to extract u_p and d_p

Deep Inelastic Scattering: PDF

We need other measurements, the neutron = proton with u ◄—► d

$$\frac{1}{x}F_2^{\text{neutron}} = \frac{4}{9}u_n(x) + \frac{1}{9}u_n(x) \simeq \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

With a linear combination of proton and deuteron data it possible to deduce xu(x) and xd(x)



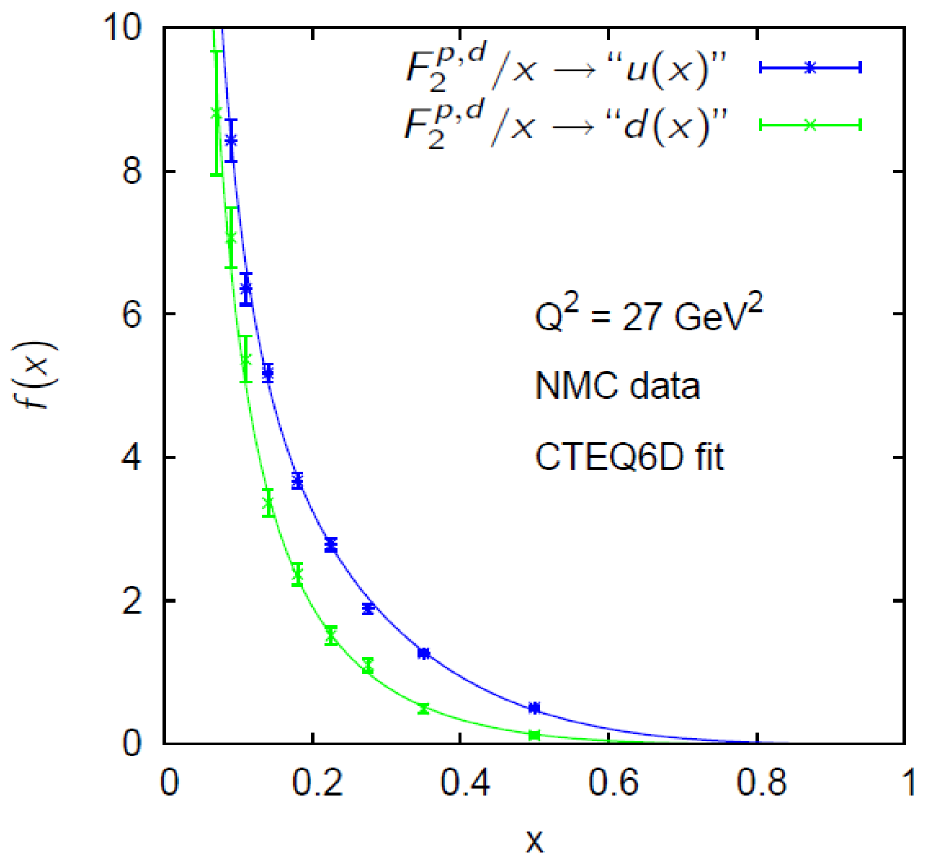
Next question:
How many u and d quarks are present?
Integrate u(x) or d(x) to find the total number of u or d quark.

Deep Inelastic Scattering: PDF - 2

We need other measurements, the neutron = proton with u ←→ d

$$\frac{1}{x}F_2^{\text{neutron}} = \frac{4}{9}u_n(x) + \frac{1}{9}u_n(x) \simeq \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

With a linear combination of proton and deuteron data it possible to deduce xu(x) and xd(x)



Integrate u(x) or d(x) to find the total number of u or d quark.

PDFs seem to diverge for x → 0 .

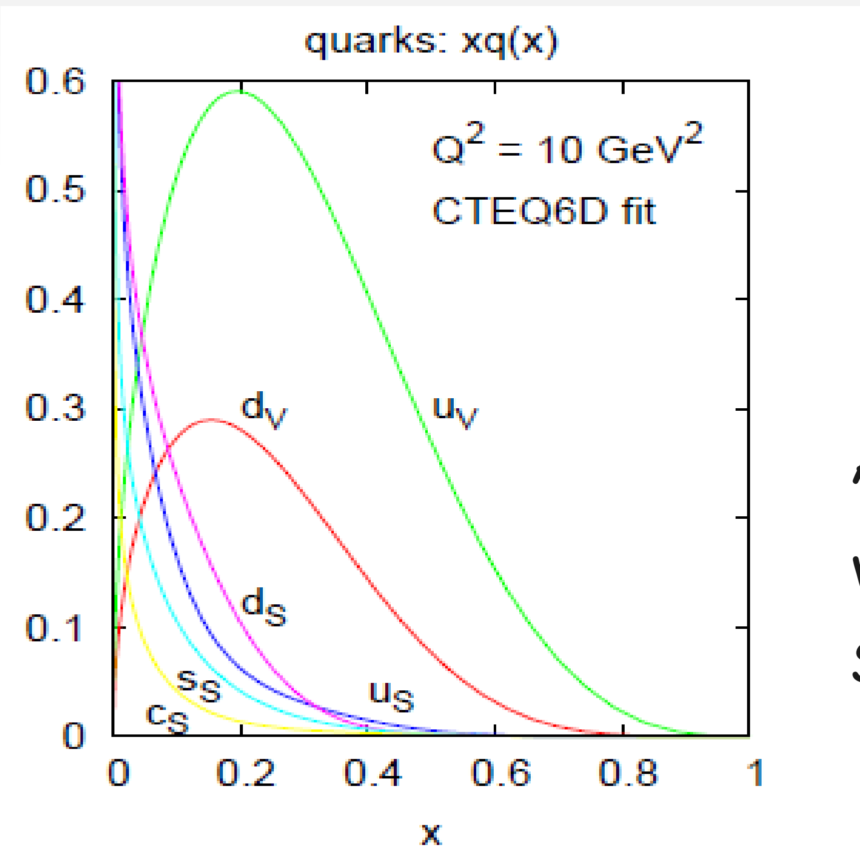
In the model we did not include the "sea" quarks but only valence quarks. In particular $\bar{u}(x)$ and $\bar{d}(x)$ are missing.:

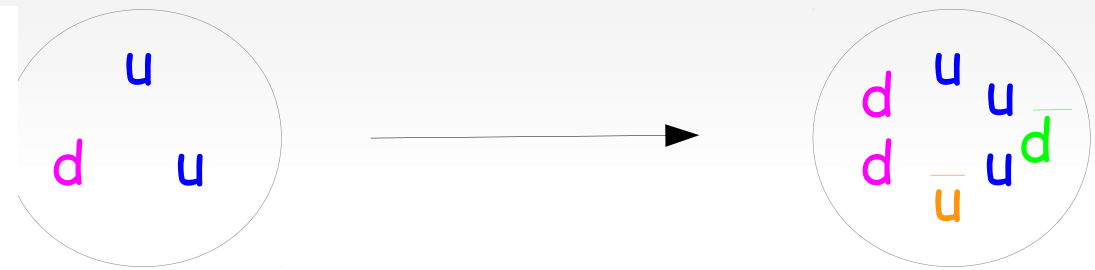
$$xu(x)+x\bar{u}(x) \qquad xd(x)+x\bar{d}(x)$$

Deep Inelastic Scattering: PDF - 3

The new proton PDF: $$F_2^{\text{proton}} = \frac{4}{9}\left(xu_p(x) + x\bar{u}_p(x)\right) + \frac{1}{9}\left(d_p(x) + \bar{d}_p(x)\right)$$

When we say p=uud → $$\int_0^1 dx\left(u(x) - \bar{u}(x)\right) = 2\,, \qquad \int_0^1 dx\left(d(x) - \bar{d}(x)\right) = 1$$





$-\bar{u}=u_v$ is valence quark distribution

Any quark can be present in the proton

Valence quark have hard distribution

Sea quark have fairly soft distribution

Deep Inelastic Scattering: PDF - 4

Check sum-rule $$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

$$\sum_q \int_0^1 dx\, x q(x) \approx 0.5$$

Where is the missing momentum?
There is one missing parton: gluon!
which indeed is very important!
Not directly probed by γ/Z or $W^{\pm}$

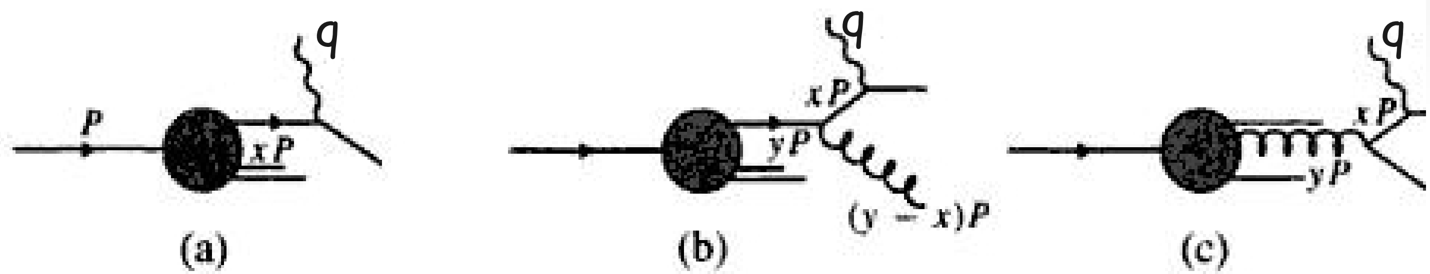
To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting

Deep Inelastic Scattering: DGLAP - 1

The PDFs depend on q^2. Let's assume $u(x, q^2)dx$ is the density of u with momentum fraction x → x+dx in a nucleon.

$$\frac{du(x, q^2)}{d \ln q^2} = \frac{\alpha_s(q^2)}{2\pi} \int_x^1 u(y, q^2) P_{QQ}\left(\frac{y}{x}\right) \frac{dy}{y}$$

(known as Altarelli-Parisi function) Let's try to understand it



Low q^2: quantum of momentum q is absorbed by quark carrying a momentum fraction x of the nucleon momentum P, case (a).
High q^2: quark can dissociated into a quark of momentum fraction x<y and a gluon (y-x). A quantum of momentum q is absorbed by the quark carrying x<y, case (b). At very small x the increasing number of gluon can generate qq, (with small x) and a quantum of momentum q can be absorbed by the quark increasing the structure function at small x, case (c).

Deep Inelastic Scattering: DGLAP - 2

The events b) + c) that happen at high q^2 are described by the AP equation.

The gluon emission probability is proportional to α_s the probability that the quark retains a fraction z=x/y of its momentum is given by the so called splitting function:

$$P_{QQ}(z)=\frac{4}{3}\frac{(1+z^2)}{(1-z)}$$

The AP equation states that the increase du in u is proportional to α_s and to the integrated number of quarks with y>x that can radiate a gluon in a such way they fall in the interval x → x+dx

$$\frac{du(x,q^2)}{d\ln q^2}=\frac{\alpha_s(q^2)}{2\pi}\int_x^1 u(y,q^2)\,P_{QQ}\left(\frac{y}{x}\right)\frac{dy}{y}$$

This for the valence quark

Deep Inelastic Scattering: DGLAP - 3

If we include the sea quarks (case c for example) and the gluon, we have a full PDF description, namely the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation:

$$\frac{d}{d\ln q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \frac{\alpha_s(q^2)}{2\pi}\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{pg} \end{pmatrix} \times \begin{pmatrix} q \\ g \end{pmatrix}$$

That for only one flavor, in general that matrix has to span over all flavor

$$P_{QQ}(z) = \frac{4}{3}\frac{(1+z^2)}{(1-z)}$$

In analogy to P_{qq} the other splitting functions are defined

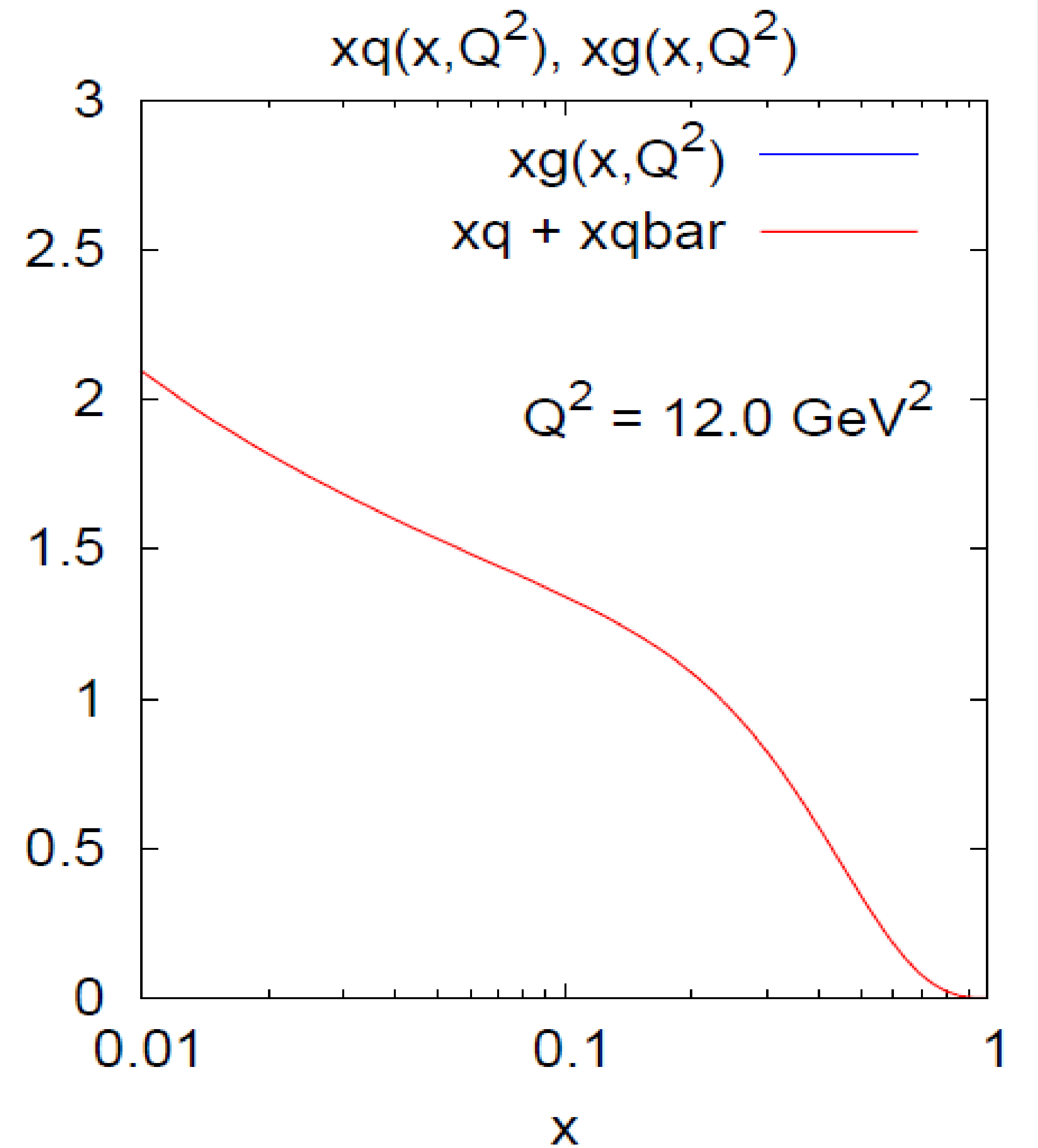
Significant properties:

P_{qg}, P_{gg}: symmetric $z \leftrightarrow 1-z$

P_{qq}, P_{gg}: diverge for $z \to 1$ Soft gluon emission

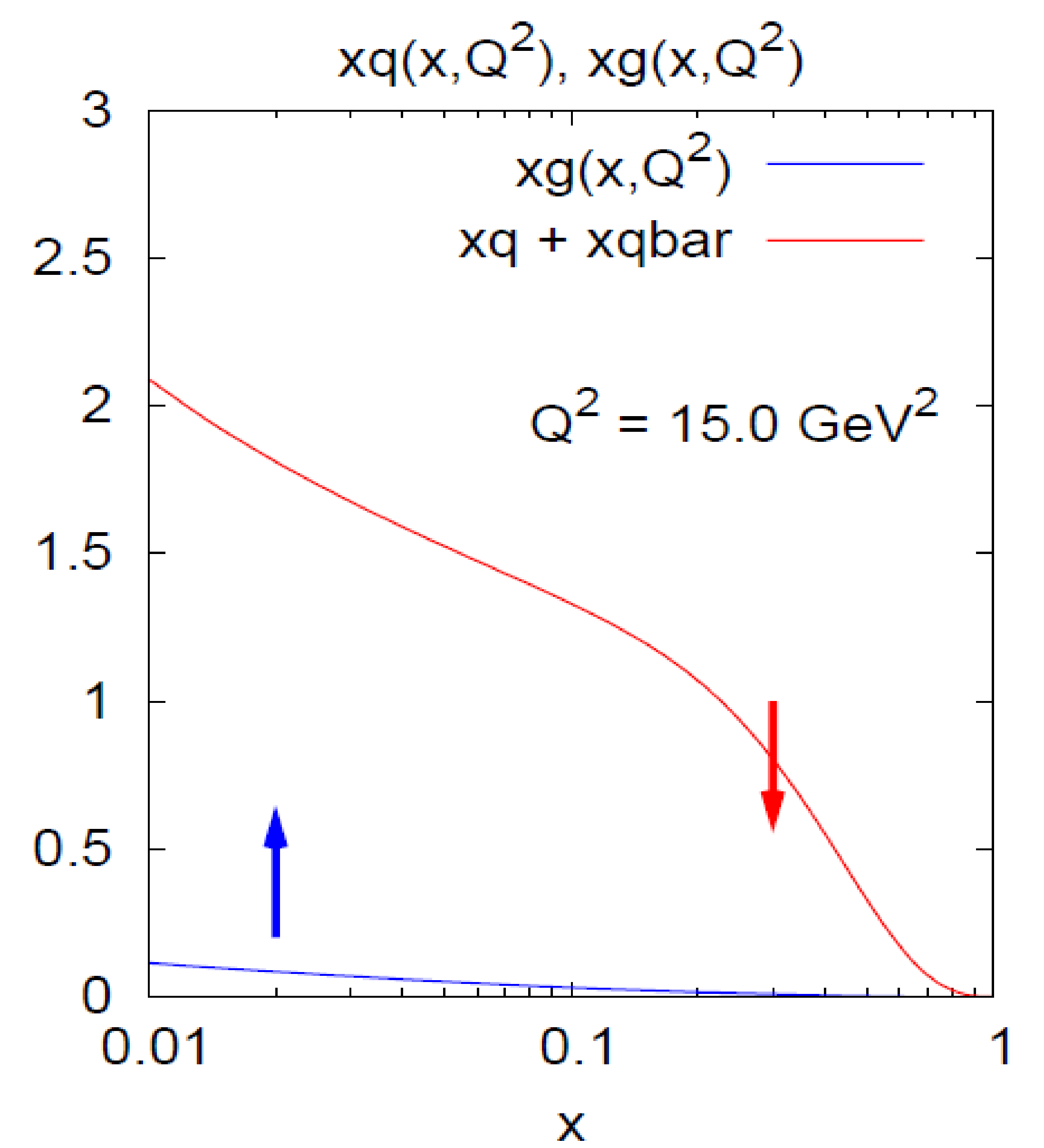
P_{gg}, P_{gq}: diverge for $z \to 0$ PDF grow at low x

Effect of DGLAP



Let's start only with quark
Quark is depleted at large x
Gluon grows at small x

Effect of DGLAP



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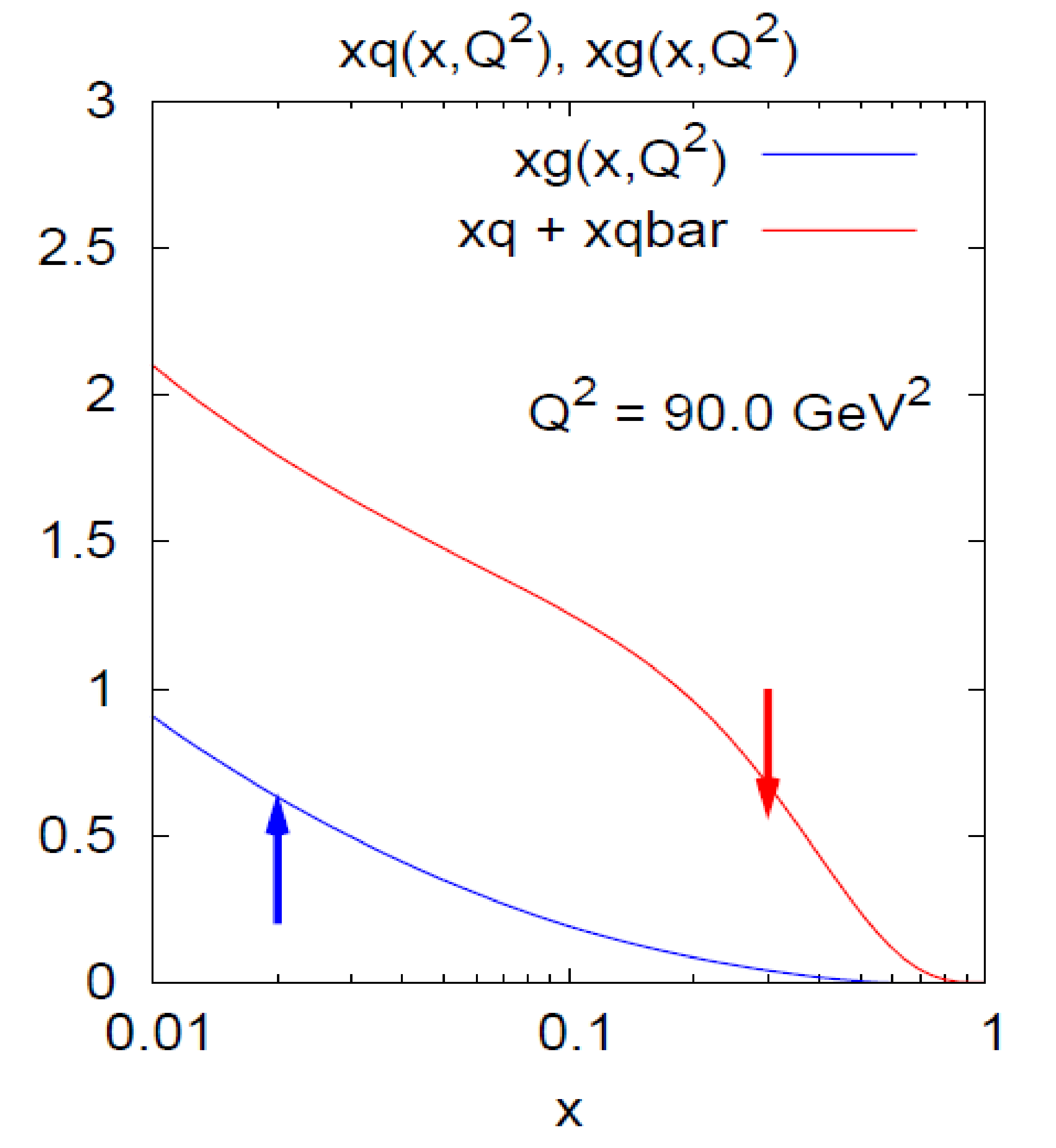

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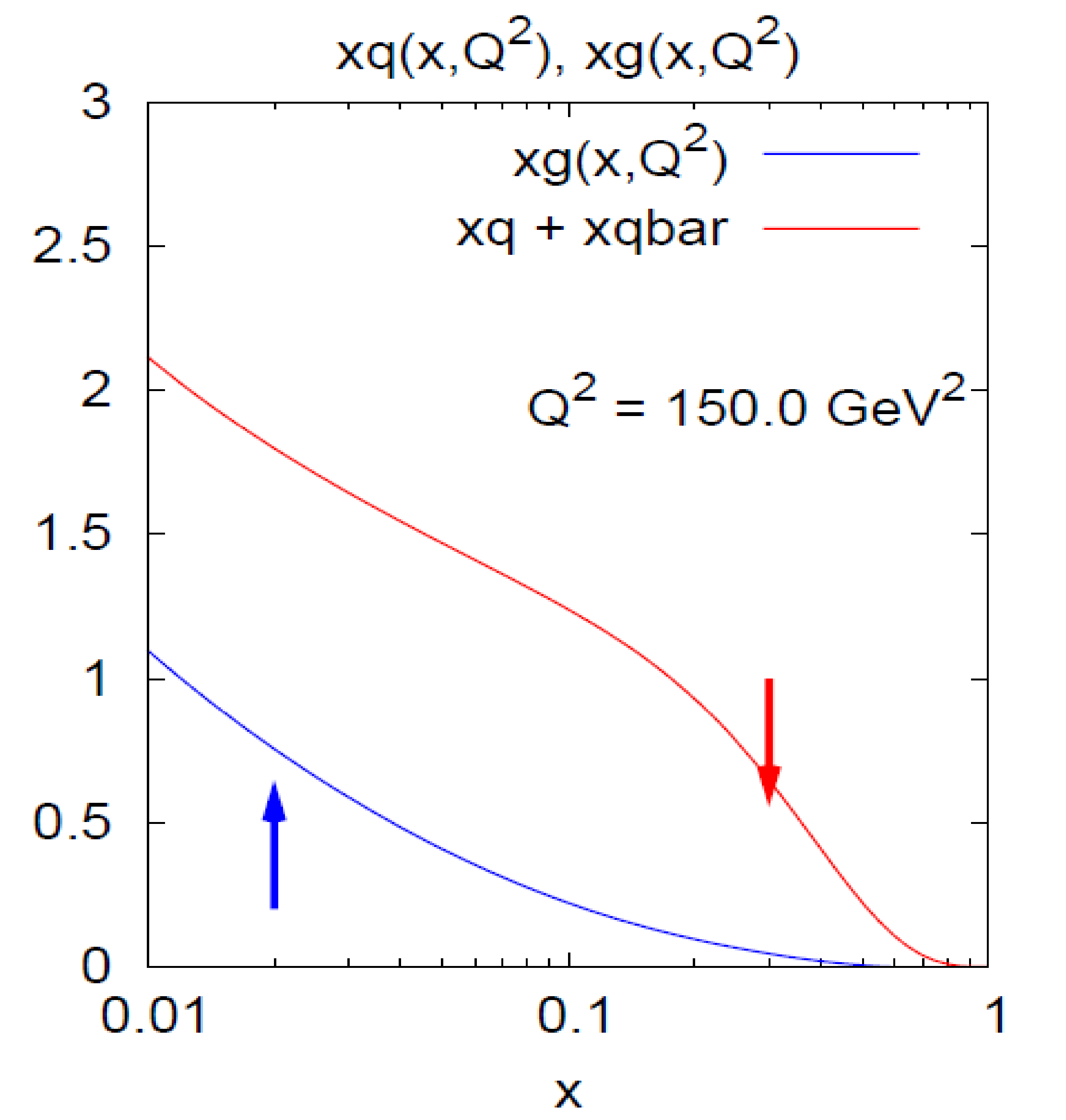

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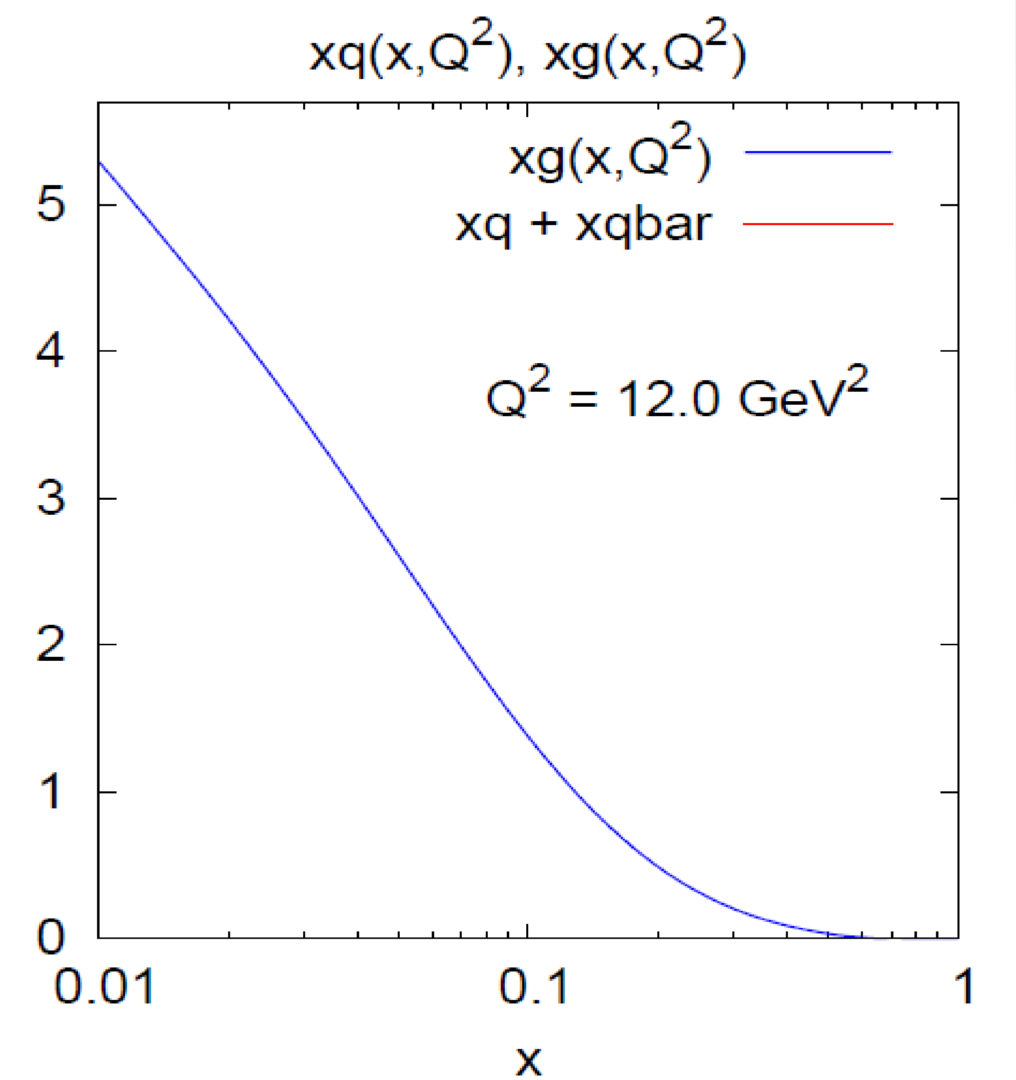
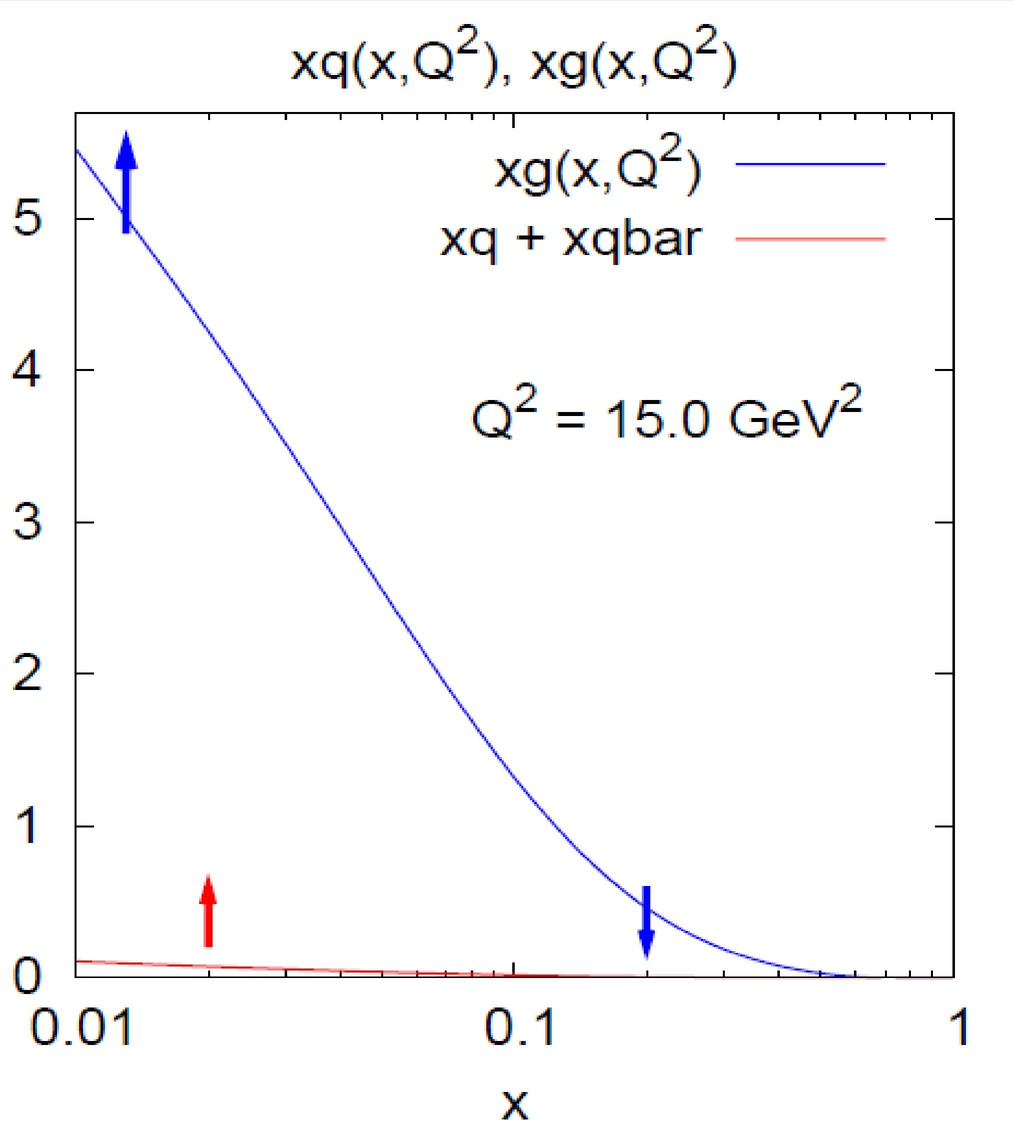


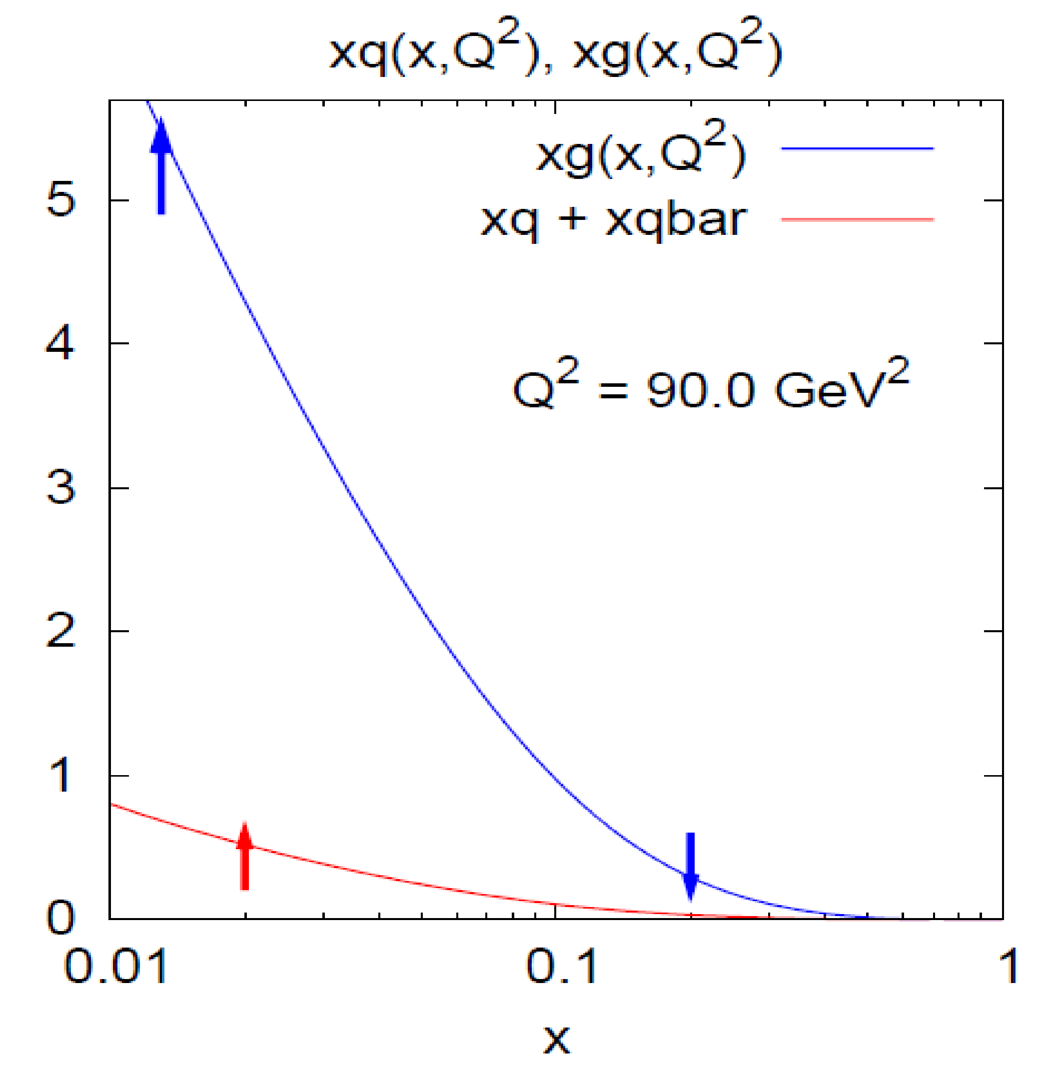
Now start with gluon.
Gluon is depleted at large x

Effect of DGLAP



Gluon decreases at large x but increase at low x as the quark

Effect of DGLAP



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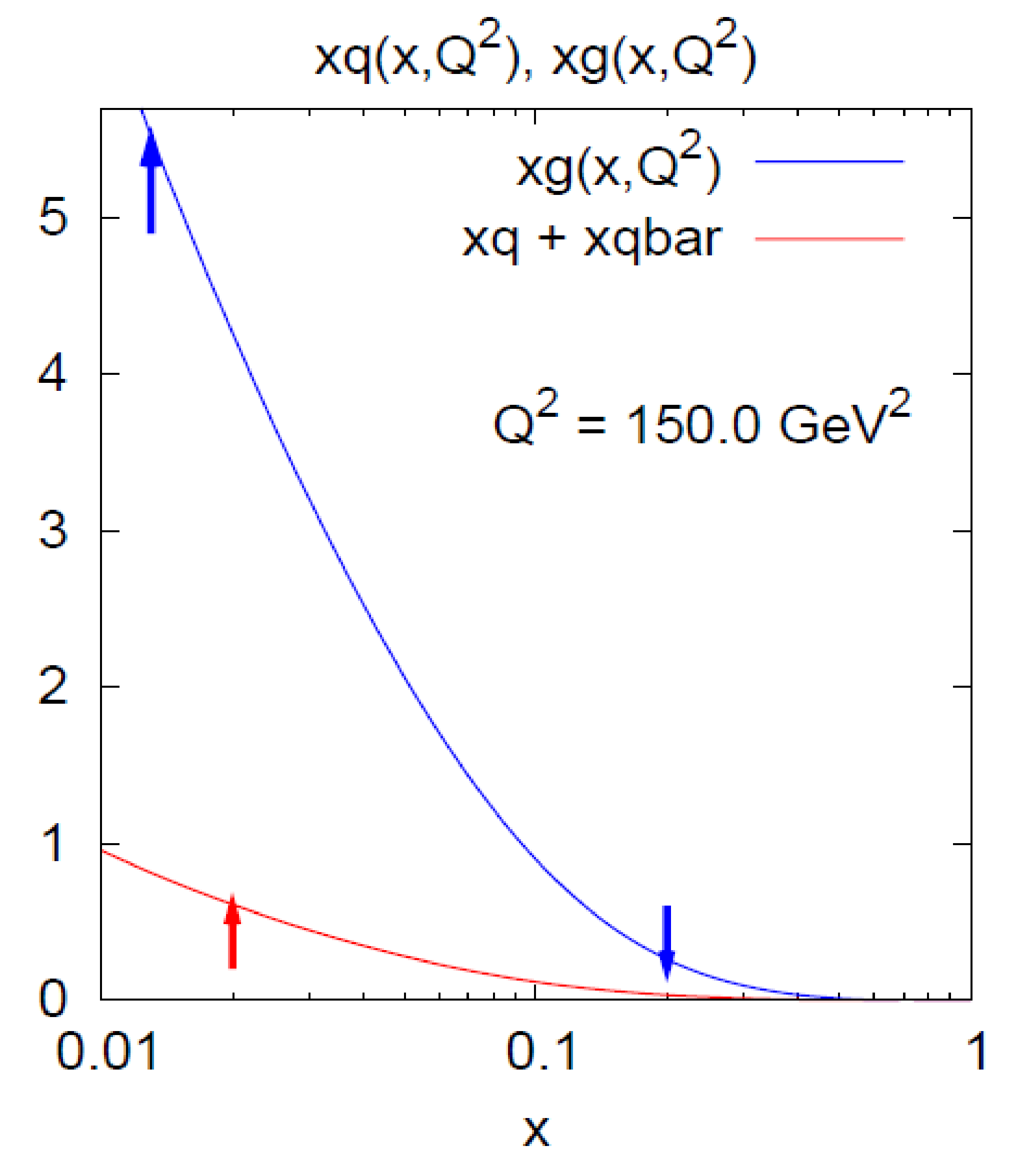
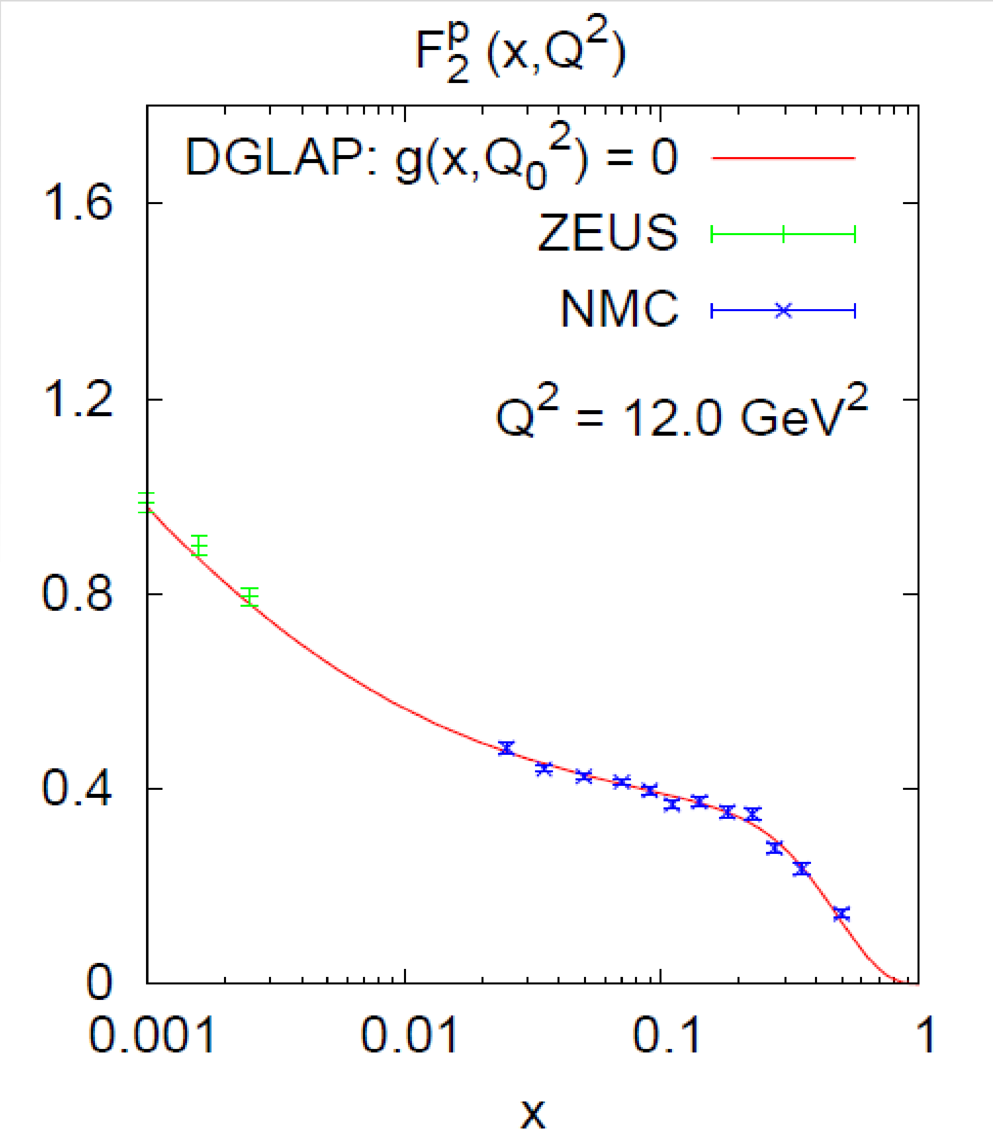
Effect of DGLAP



Gluon decreases at large x but increase at low x as the quark

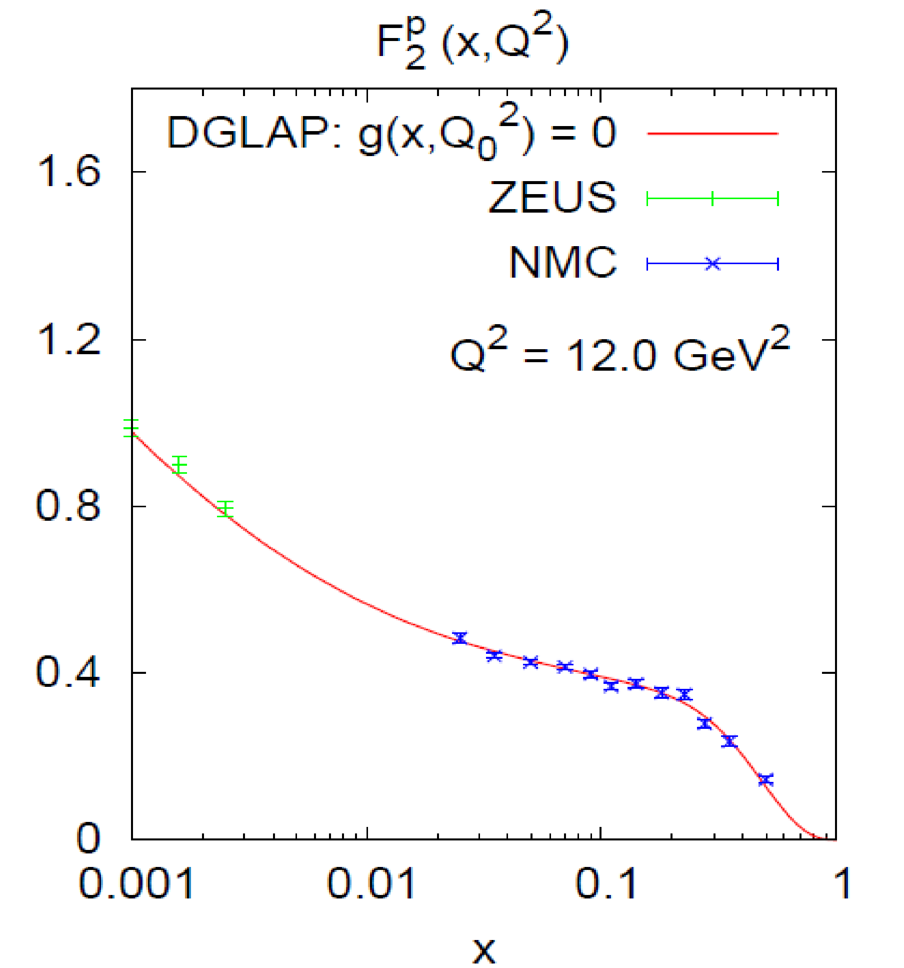
DGLAP on data

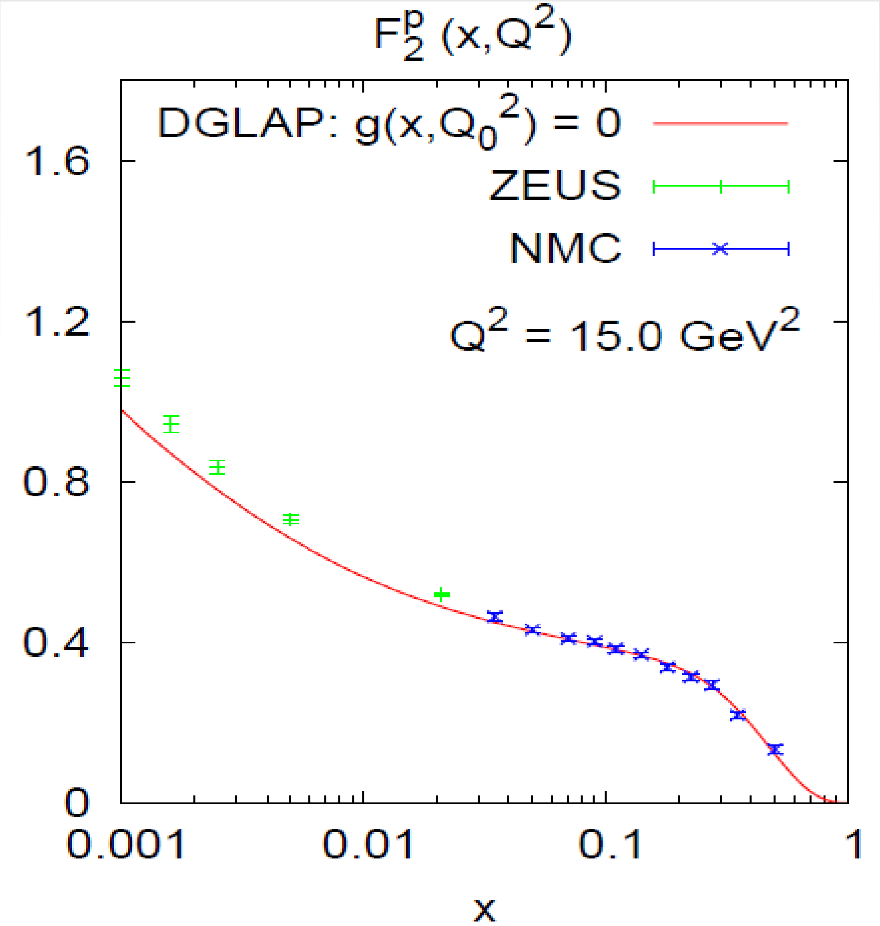


Fit data from ZEUS and New Muon Collaboration (NMC).
Fit F_2 at low q^2 assuming the gluon = 0
Then evolve F_2 to high q^2 using DGLAP equation starting from the fit on date where gluon was assumed=0

DGLAP on data

The F_2 found from evolution to high q^2 of the fit using DGLAP is compared to data at that q^2





DGLAP on data

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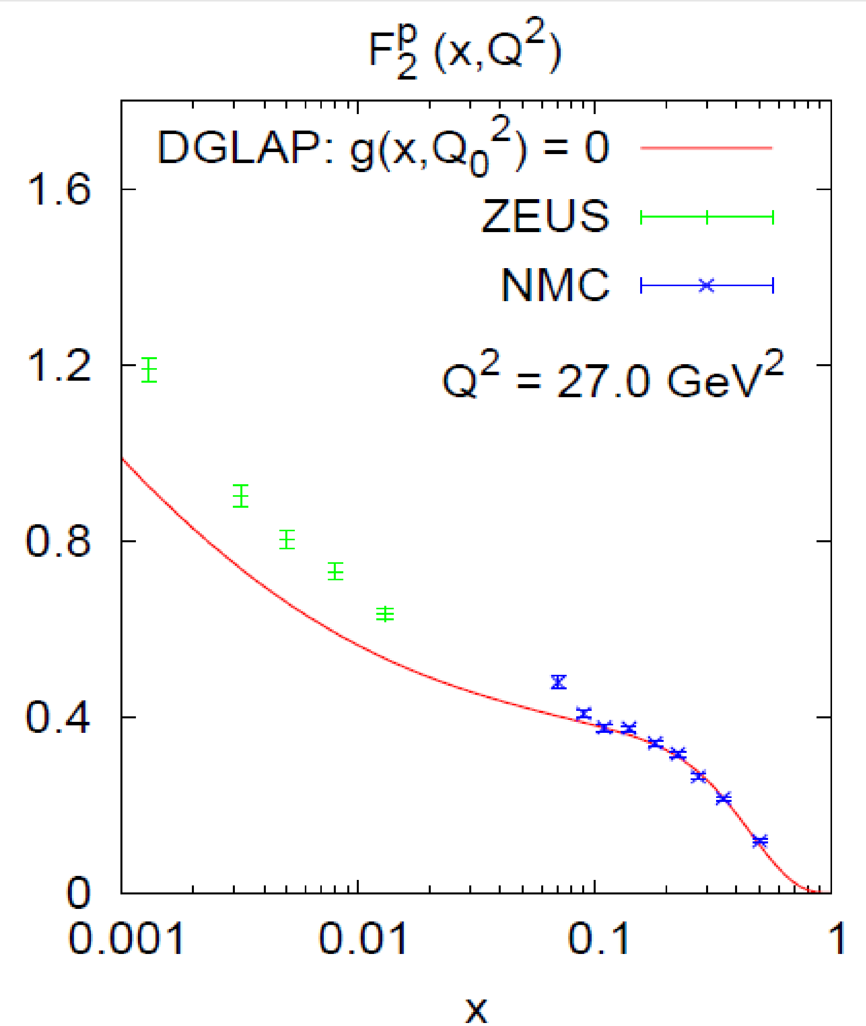
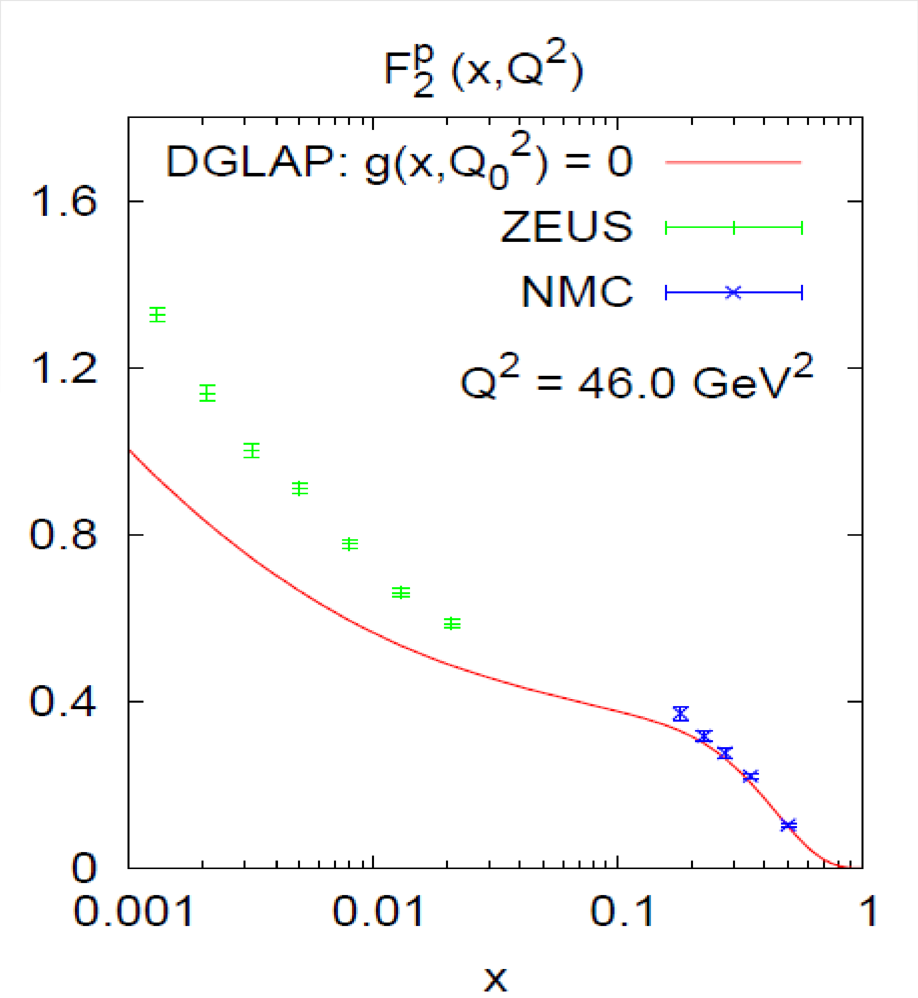
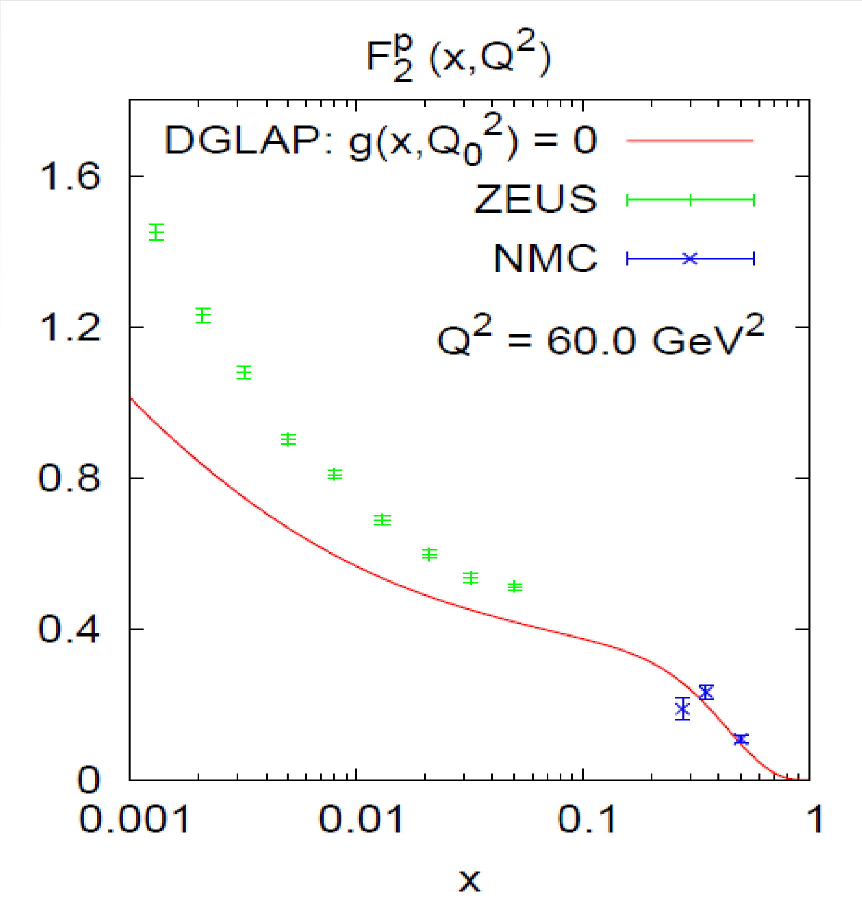
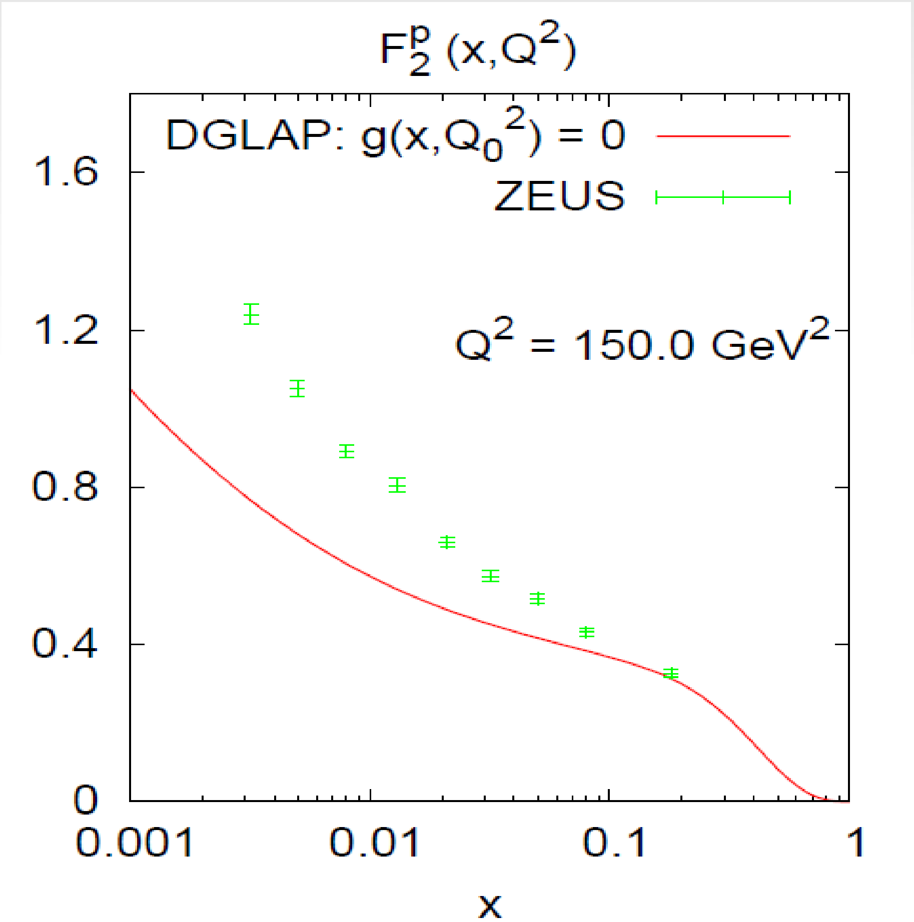




DGLAP on data no gluon

The F_2 found from evolution to high q^2 of the fit using DGLAP is compared to data at that q^2

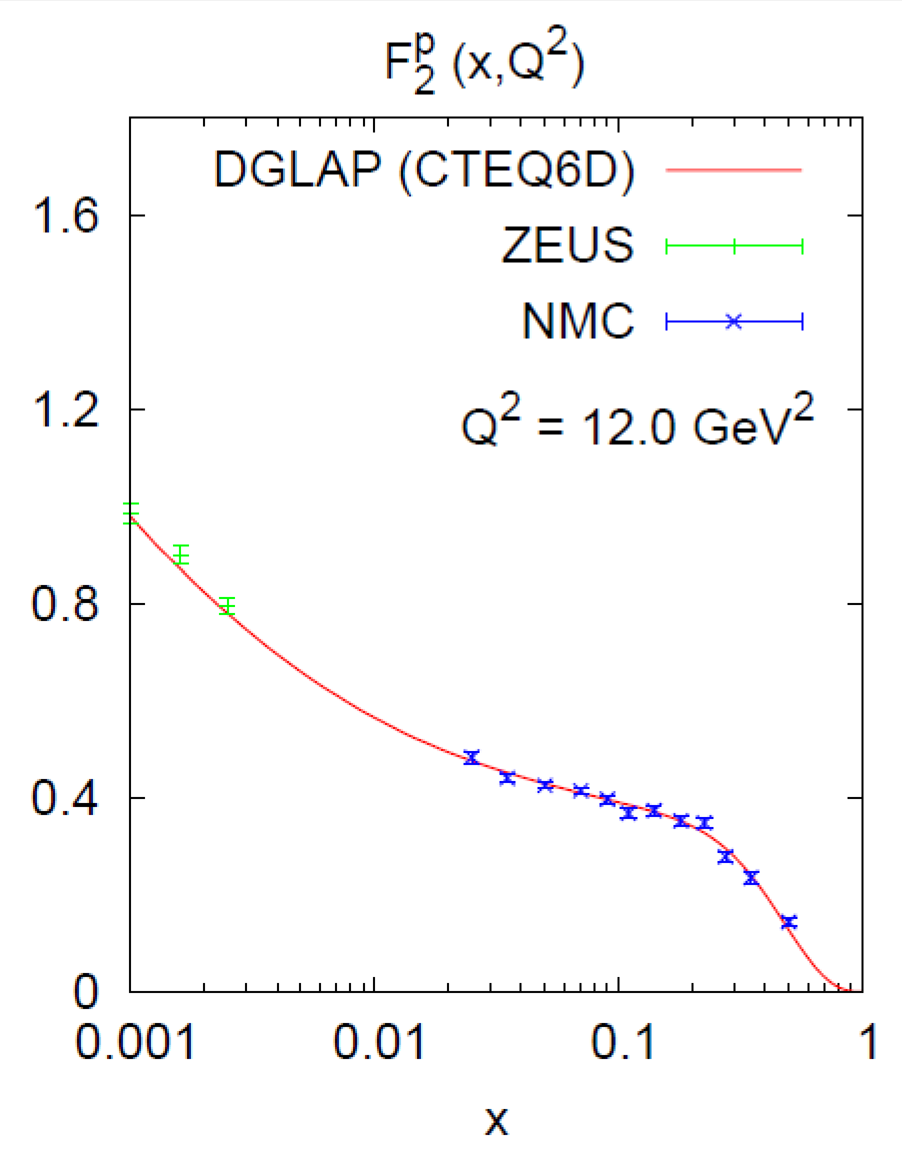


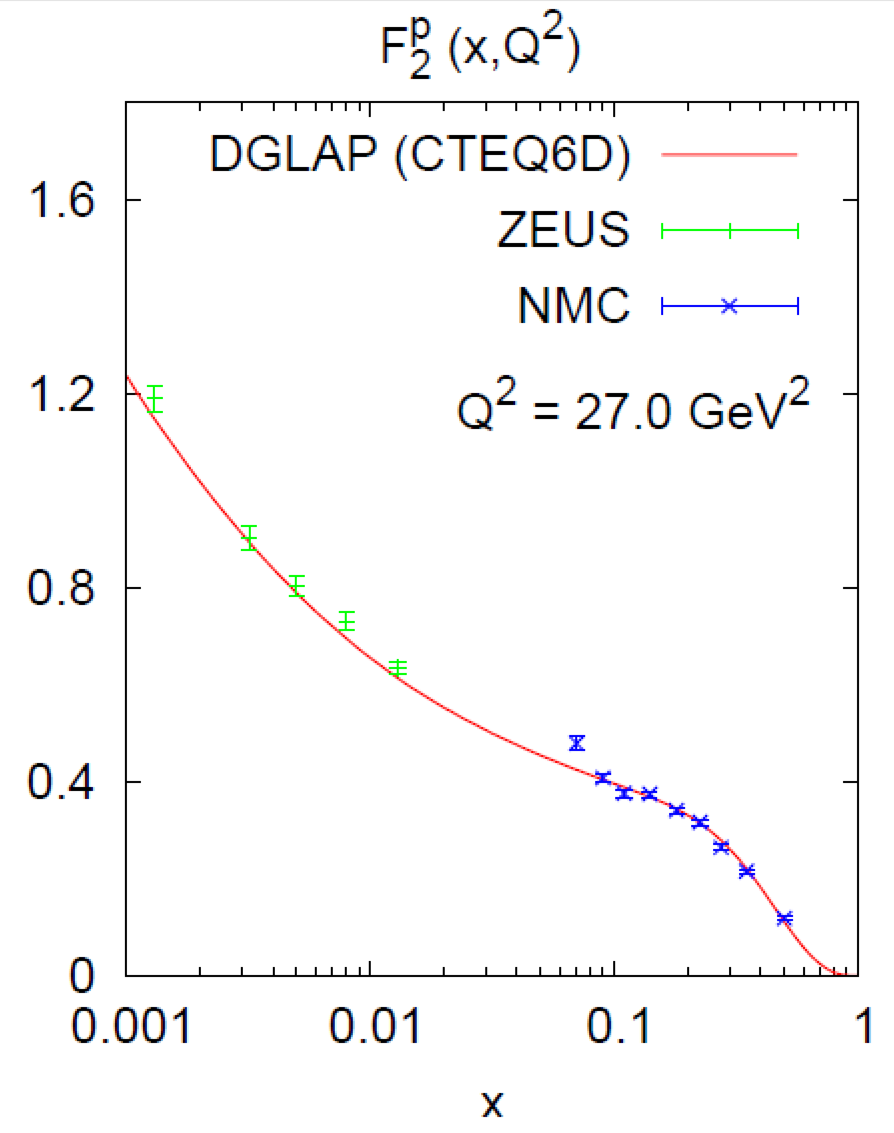


It does not work!

DGLAP on data with gluon

If F_2 is found fitting data at low q^2 with gluon $\rightarrow$ Evolve F_2 to high q^2 using DGLAP $g \rightarrow q\bar{q}$ generate extra quark at large q^2 $\rightarrow$ faster rise of F_2





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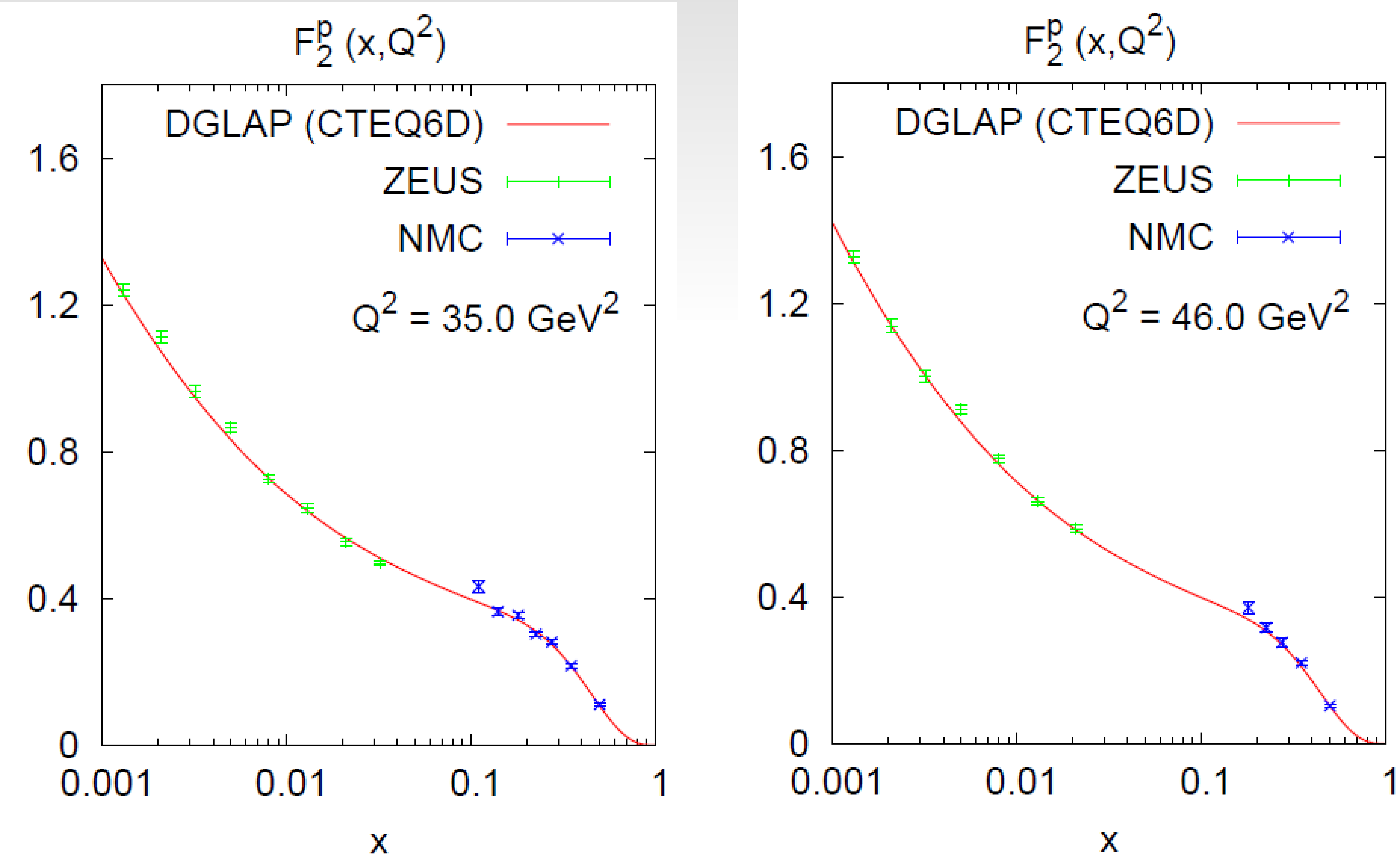

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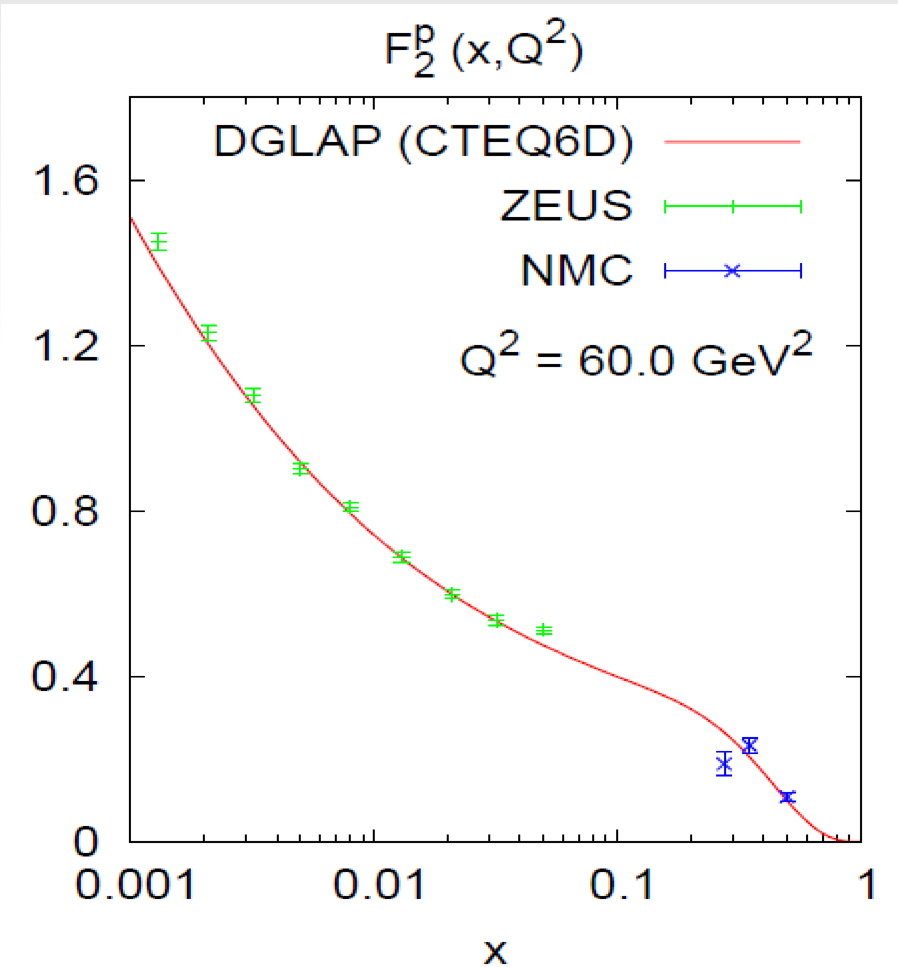
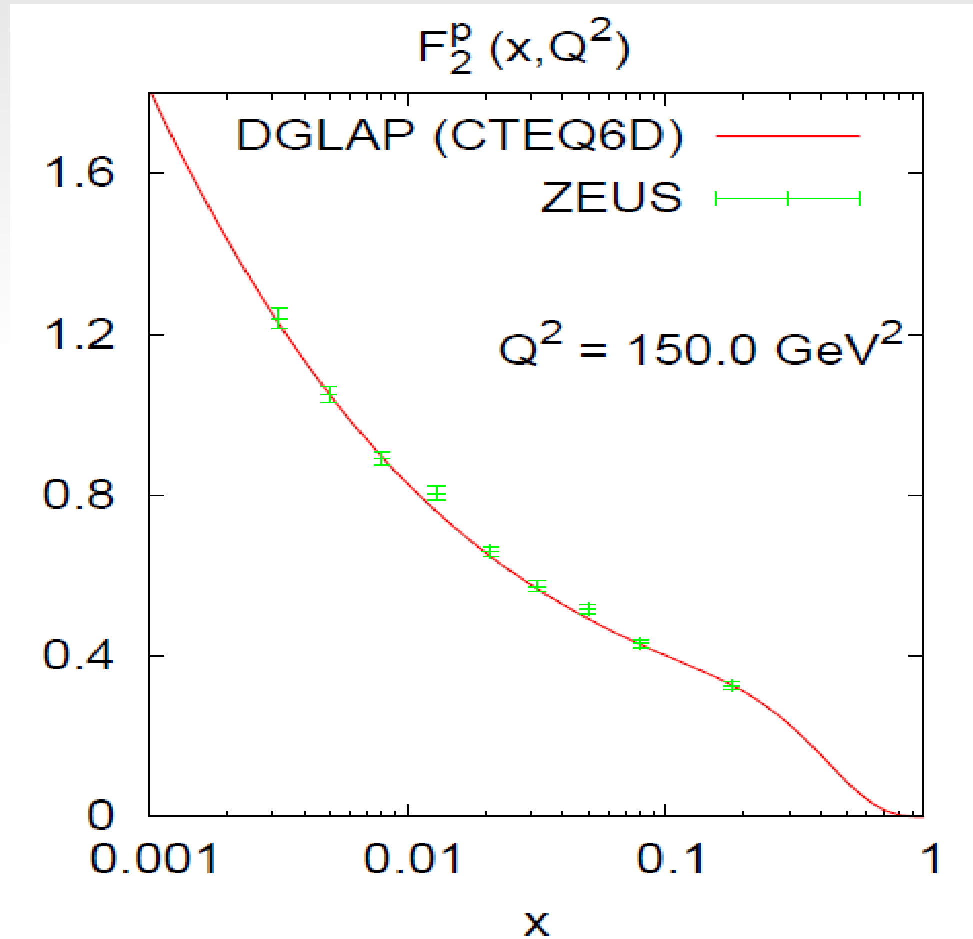
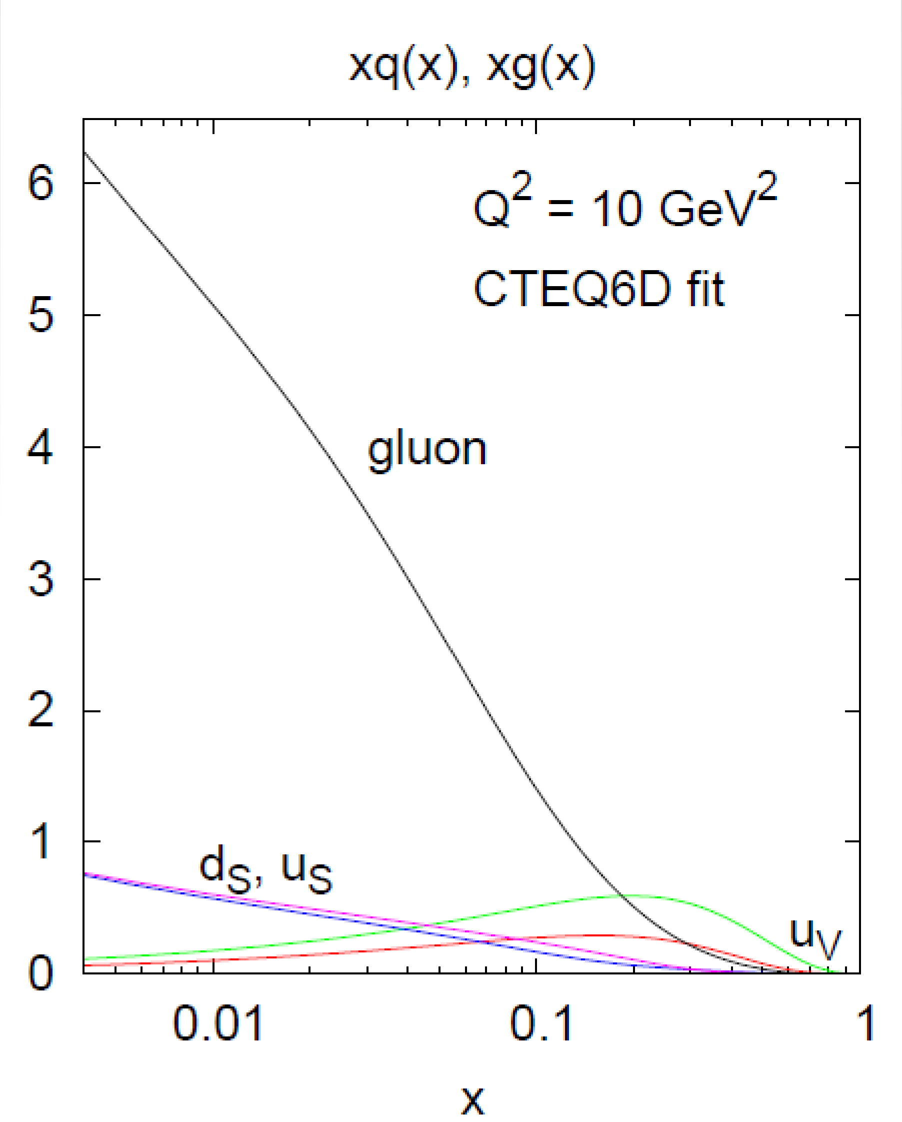




Success!

Gluon distribution



Gluon distribution is huge.

Is it real?

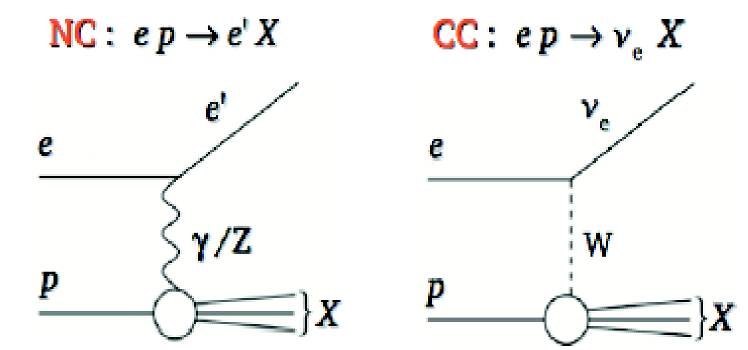
- Consistency sum rule is satisfied
- Agree with a lot of data

Success!

PDF Measurements

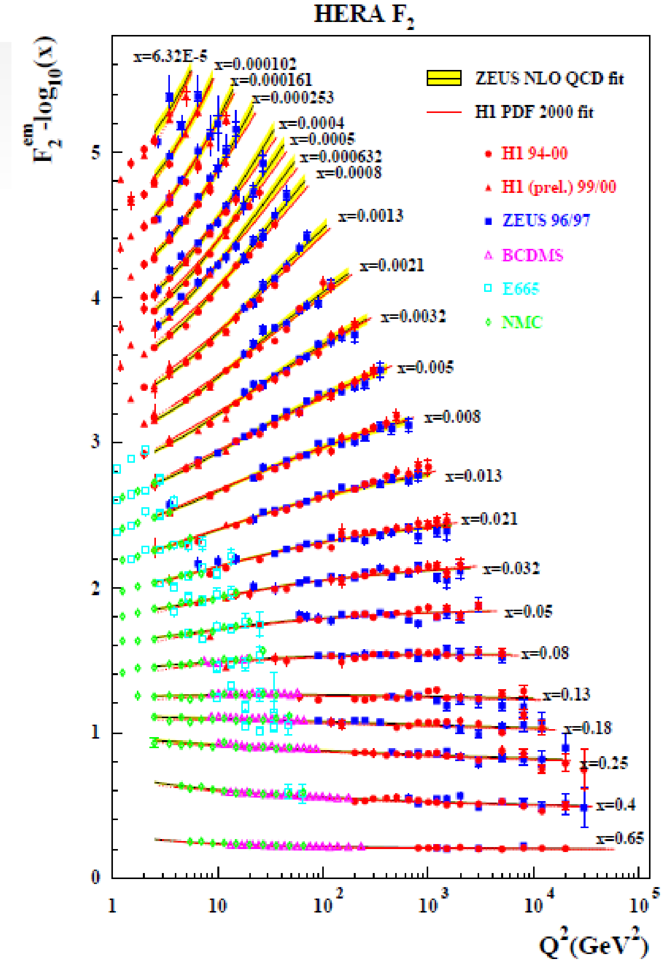
At HERA exploit these interactions. By selecting the final states it is measured the cross section:

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$



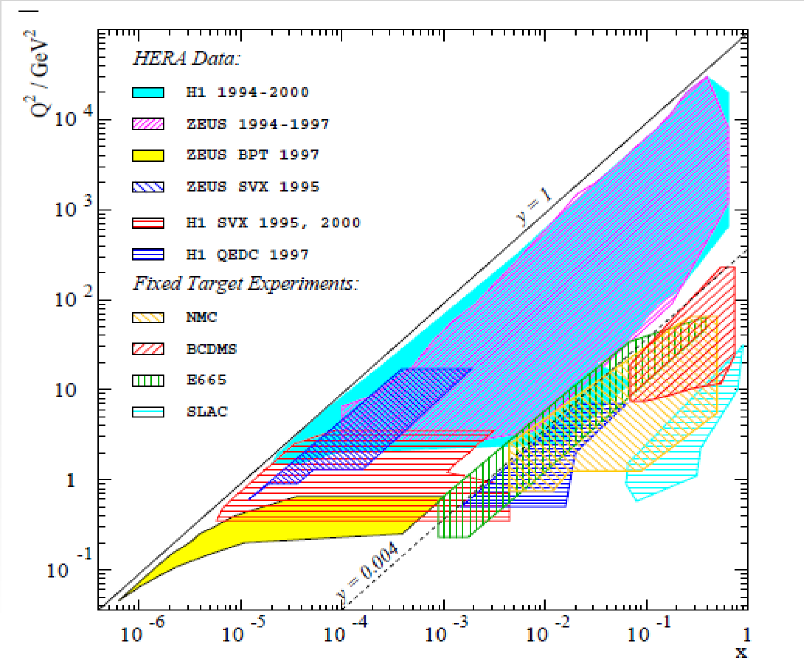
Different final states give access to different PDF

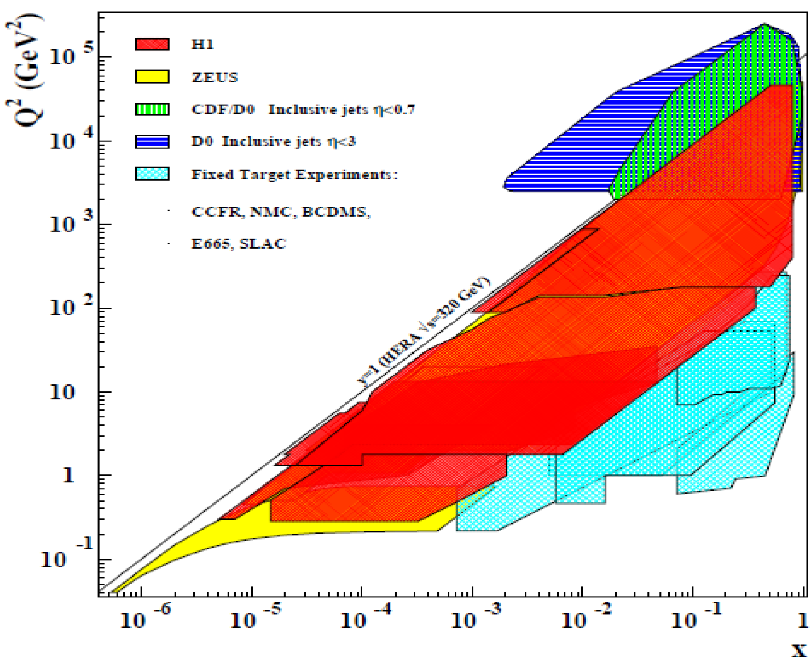
$$\sigma^+_{CC} \sim x(\bar{u}+\bar{c}) + x(1-y)^2(d+s)$$
$$\sigma^-_{CC} \sim x(u+c) + x(1-y)^2(\bar{d}+\bar{s})$$

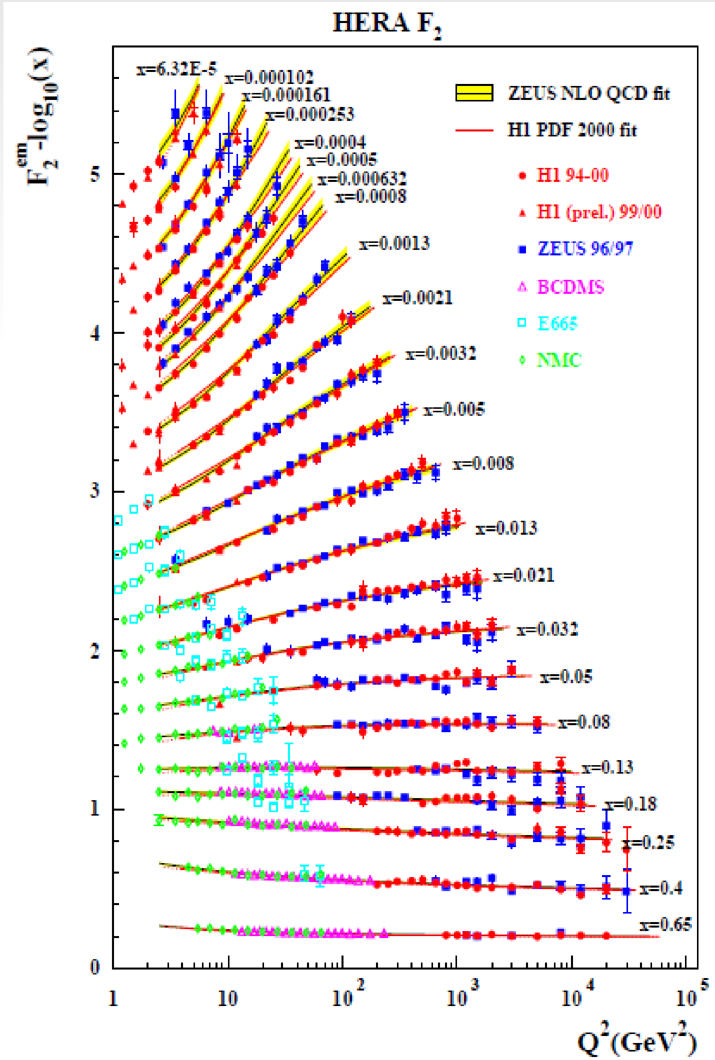


PDF Measurements

Kinematic regions of data of different experiments used for the fits and fit results.



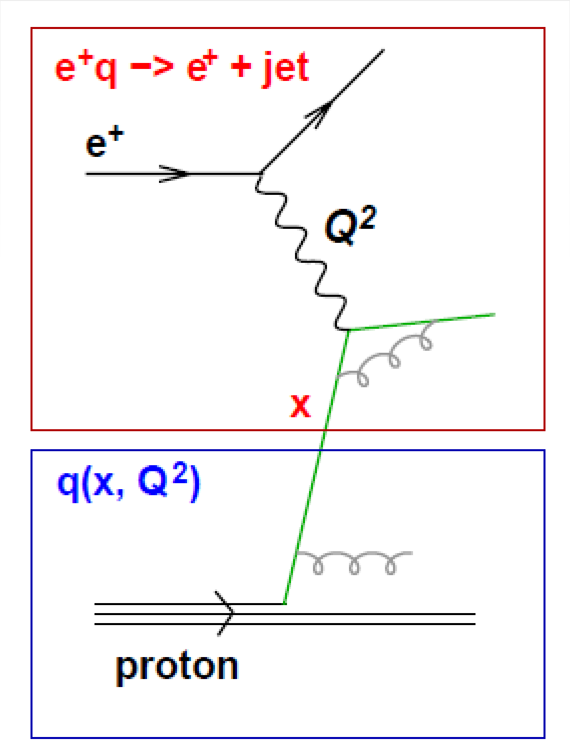




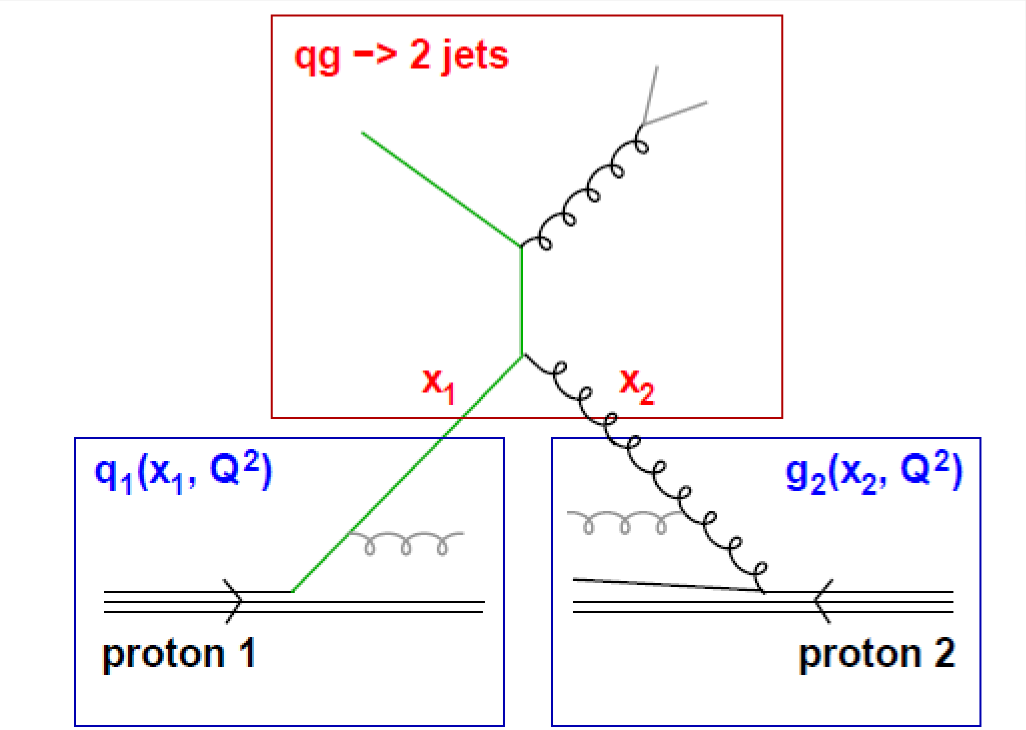
PDF Measurements checks: factorization

The interactions can be seen sum of :

- hard process dependent partonic subprocess
- non perturbative, process independent parton distribution functions



$$\sigma_{ep} = \sigma_{eq} \otimes q$$

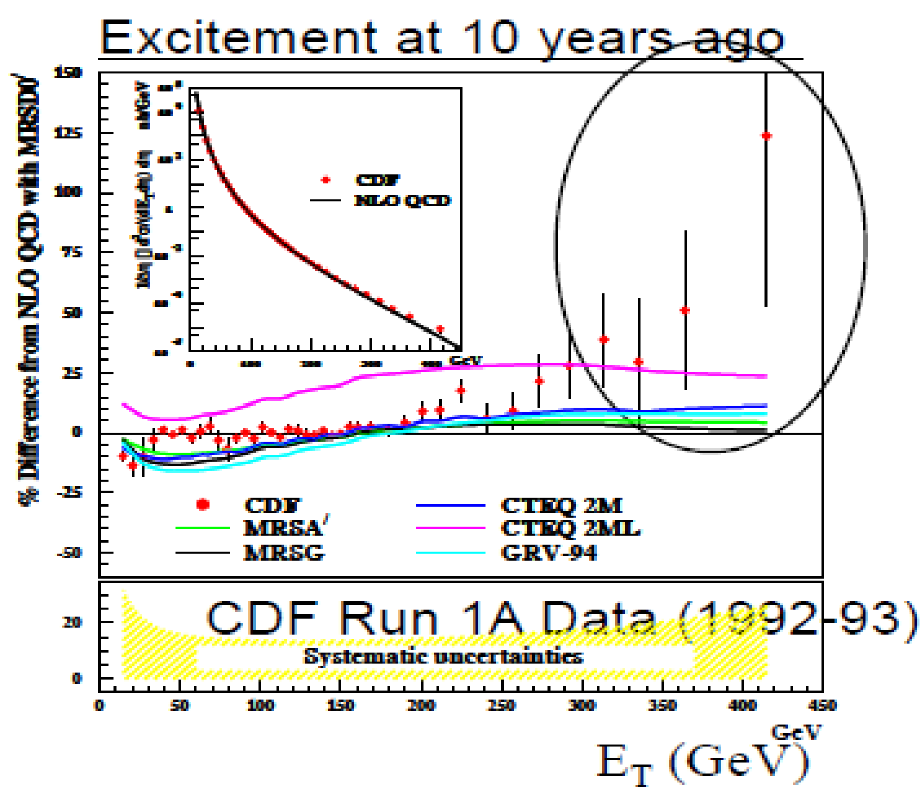


$$\sigma_{pp \to 2\,jets} = \sigma_{qg \to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

PDF Measurements checks: Jet cross section

Jet production in proton-antiproton collisions has been an important test of large gluon distribution, since there are large direct contributions from $gg \rightarrow gg$ $qg \rightarrow qg$

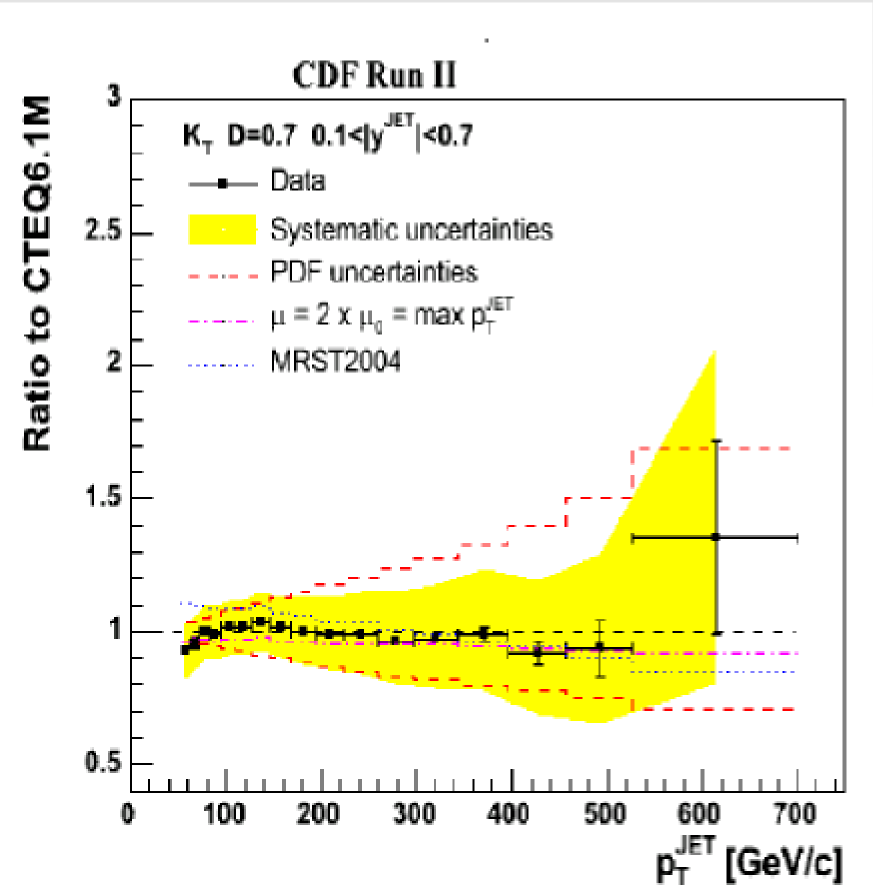
For example: in Run 1A at CDF
a discrepancy was observed in the jet cross section

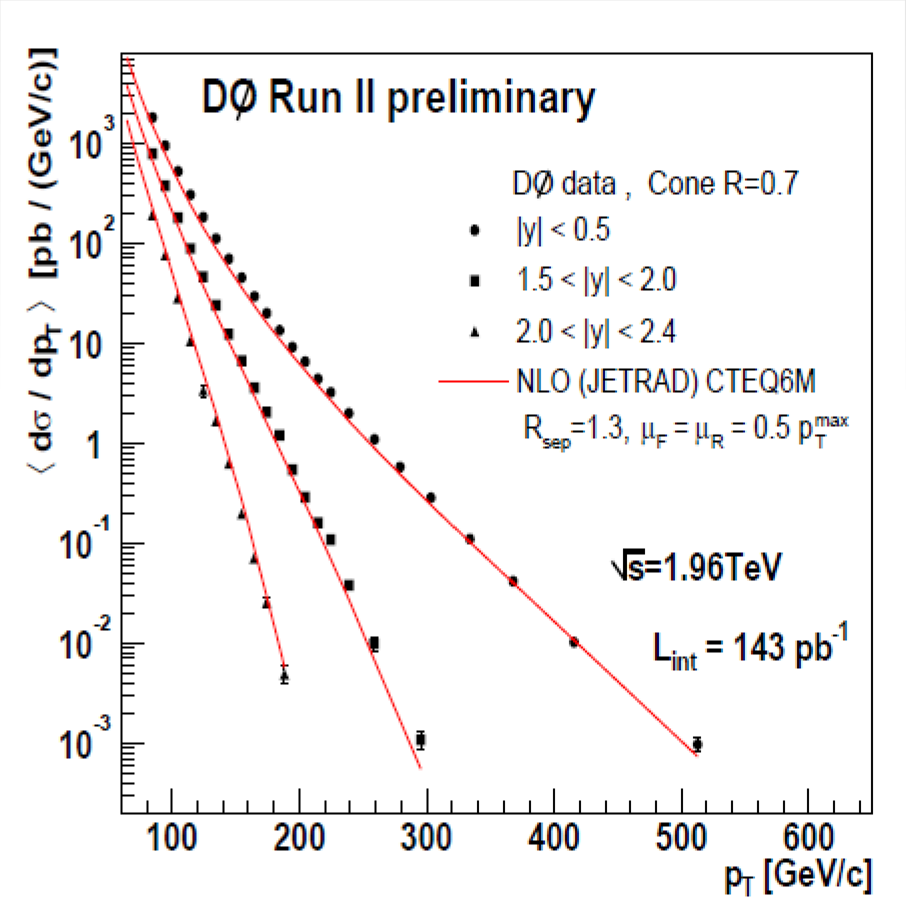


Phys. Rev. Lett. 77, 438 (1996)

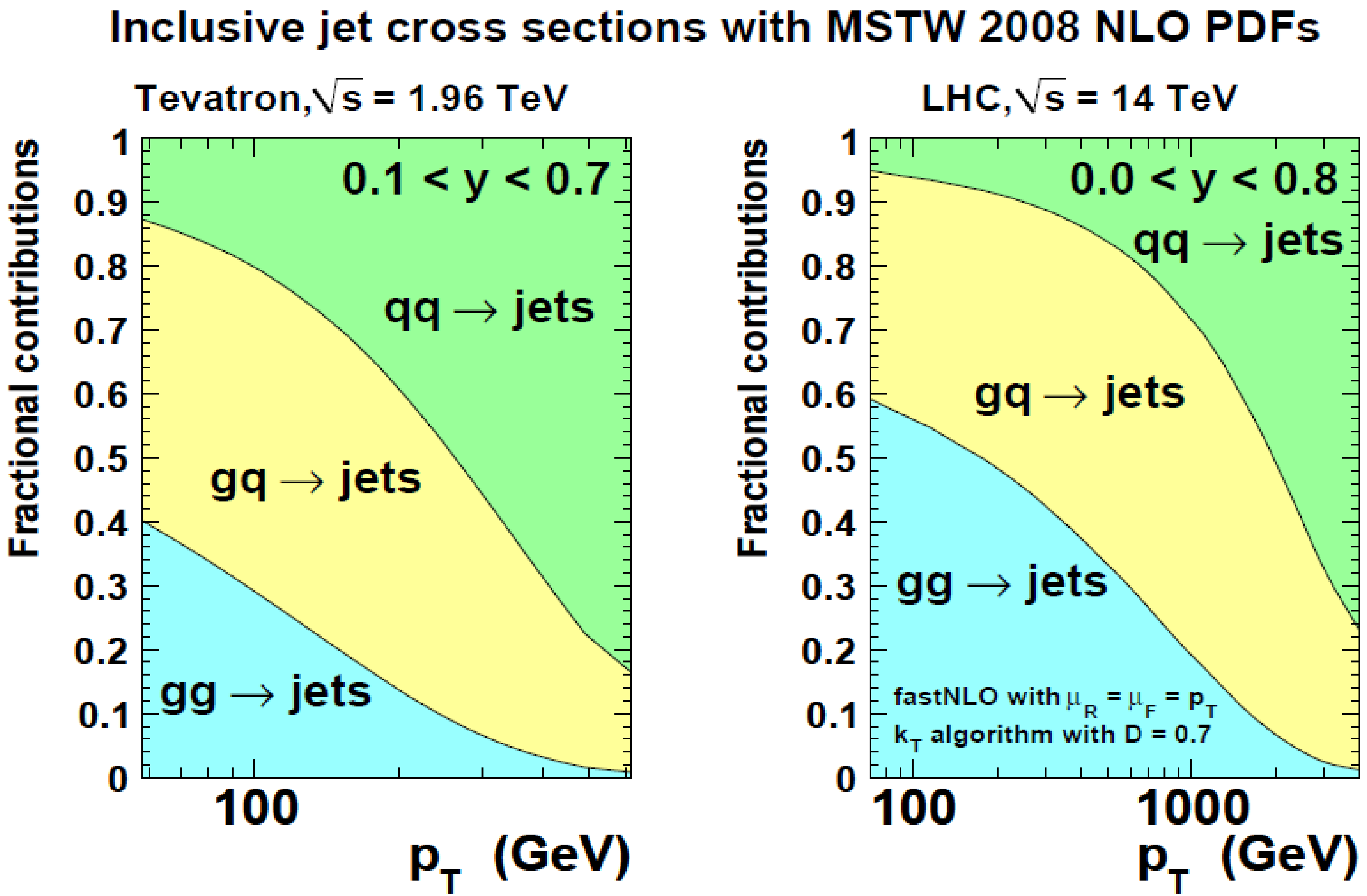
PDF Measurements checks: Jet cross section

When the gluon PDF was included

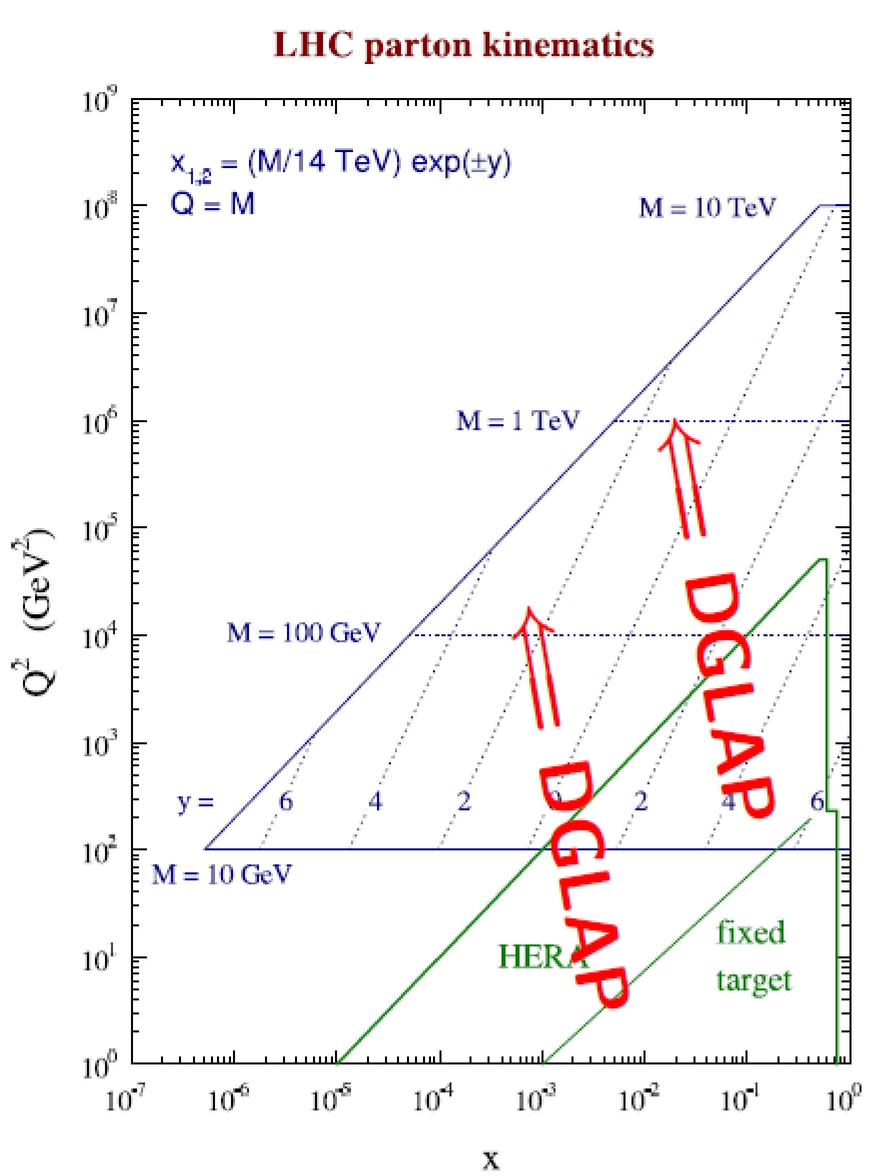




PDF Channels Contribution



PDF from HERA to LHC

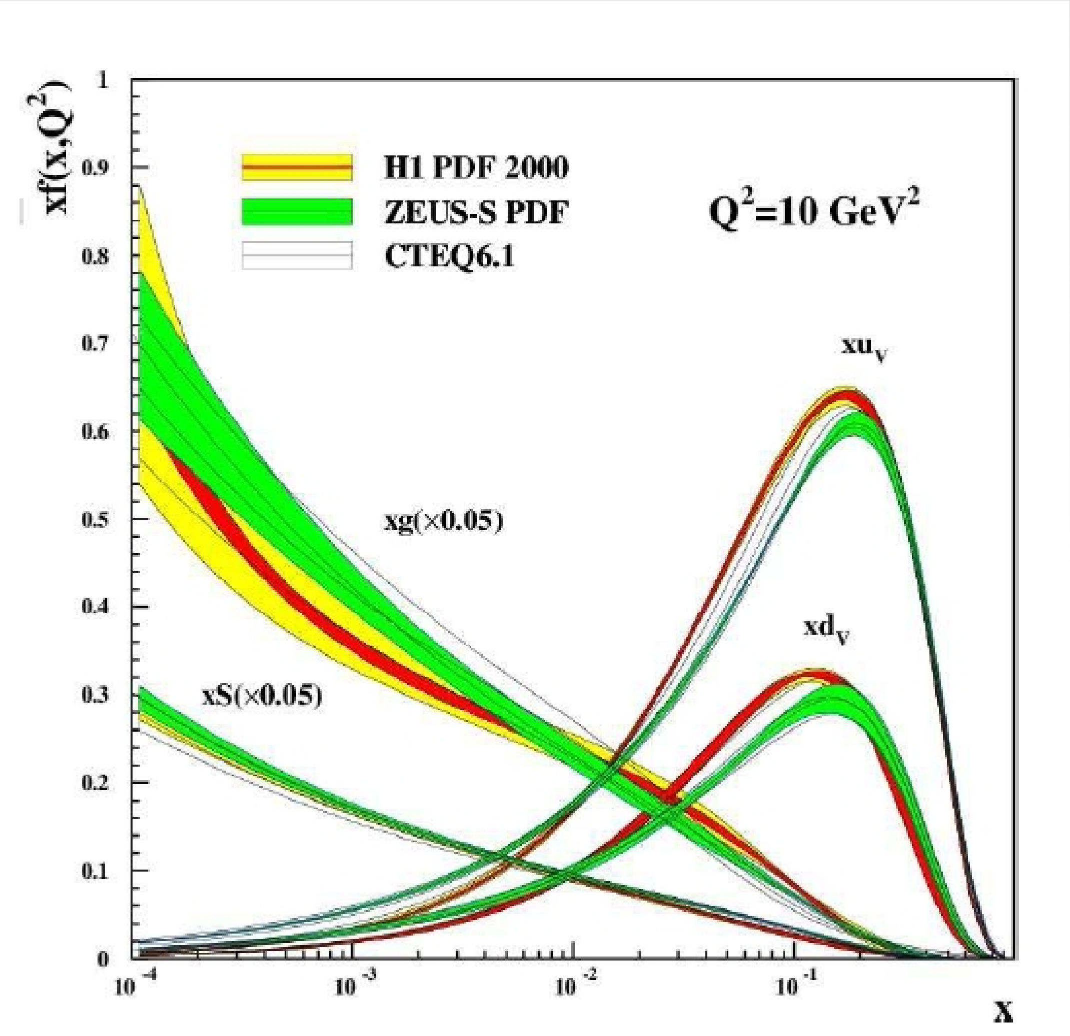


Are PDFs being used in region where measured?

Only partial kinematic overlap

DGLAP evolution is essential for the prediction of PDFs in the LHC domain.

PDF Precision



Translate the experimental errors and theoretical uncertainties into uncertainty band on extracted PDF.

Use these bands to evaluate the reliability of the Monte Carlo predictions include or should include these uncertainties