# Formulario di Elettrostatica

**Legge di Coulomb**

$$\vec{F} = k\,\frac{q_1\,q_2}{r^2}\,\vec{u_r}$$

**Costanti**

$$e = 1.6 \cdot 10^{-19}\ \mathrm{C}$$

$$k = 9 \cdot 10^9\,\frac{\mathrm{N\,m^2}}{\mathrm{C^2}}$$

$$\epsilon_0 = 8.85 \cdot 10^{-12}\,\frac{\mathrm{C}}{\mathrm{mV}}$$

$$k = \frac{1}{4\pi\,\epsilon_0}$$

$1\ \mathrm{eV} = 1.6 \cdot 10^{-19}\ \mathrm{J}$

**Campo elettrico**

$$\vec{E} = \sum_i k\,\frac{q_i}{r_i^2}\,\vec{u_r}$$

$$\vec{E} = k\,\frac{Q}{r^2}\,\vec{u_r}\ ; \qquad \vec{F} = q\vec{E}$$

**Energia Potenziale**

$$U_p = k\,\frac{q_1\,q_2}{r}$$

$$U_p = \sum_{i=1}^{N}\sum_{j=i+1}^{N} k\,\frac{q_i\,q_j}{r_{ij}}$$

**Potenziale Elettrico**

$$U = q\sum_i k\,\frac{q_i}{|\vec{r_i} - \vec{r}|} = q\,V(\vec{r})$$

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

Maxwell $\oint_\Gamma \vec{E} \cdot d\vec{s}$

$$\vec{E} = -\vec{\nabla} V$$

$$V(r) = \frac{k\,q}{r}$$

**Dipolo elettrico**

$\vec{P} = q\,\vec{a}$ Momento di d.e.

$$V(P) = \frac{q}{4\pi\epsilon_0} \cdot \frac{r_2 - r_1}{r_1 r_2}$$

$r \gg a \qquad r_2 - r_1 \approx a\,\cos\vartheta$

$$V = \frac{q}{4\pi\,\epsilon_0} \cdot \frac{a\,\cos\vartheta}{r^2} = \frac{\vec{P} \cdot \vec{u_r}}{4\pi\,\epsilon_0 r^2}$$

$$E_r = -\frac{\partial V}{\partial r} = \frac{2P\cos\vartheta}{4\pi\,\epsilon_0 r^3}$$

$$E_\vartheta = -\frac{1}{r}\frac{\partial V}{\partial \vartheta} = \frac{P\sin\vartheta}{4\pi\,\epsilon_0 r^3}$$

$$\Longrightarrow \vec{E} = \frac{P}{4\pi\,\epsilon_0 r^3}\,(2\cos\vartheta\,\vec{u_r} + \sin\vartheta\,\vec{u_\vartheta})$$

$$\vartheta = \pi: \quad E_r = -\frac{2P}{4\pi\epsilon_0 r^3}\ ,\ E_\vartheta = 0$$

$$\vartheta = \frac{\pi}{2}: \quad E_r = 0\ ,\ E_\vartheta = \frac{P}{4\pi\,\epsilon_0 r^2}$$

$$U = -q\,E\,a\,\cos\vartheta = -\vec{P} \cdot \vec{E}$$

**Momento torcente di un d.e.**

$$\vec{M} = (\vec{r_2} - \vec{r_1}) \times \vec{F_2} = \vec{a} \times q\vec{E} = \vec{P} \times \vec{E}$$

**Anello uniformemente carico**

$$\vec{E} = E_x\,\vec{u_x} = \frac{\lambda R x}{2\epsilon_0\,(x^2 + R^2)^{3/2}}\,\vec{u_x}$$

con $\lambda = \dfrac{Q_{tot}}{2\pi R}$

**Asta uniformemente carica**

$$\lambda = q/L\,, \quad E_x = \frac{k\,q}{x\sqrt{\frac{l^2}{4} + x^2}}$$

**Teorema di Gauss**

$$\Phi_\Sigma(\vec{E}) = \frac{q_{\,\mathrm{interna}}}{\epsilon_0}$$

**Sfera carica in modo omogeneo**

$$E(r) = \frac{1}{4\pi\,\epsilon_0}\frac{Q}{r^2}$$

$$V(r) = \int_r^\infty E(r)\,dr = \frac{Q}{4\pi\epsilon_0 r}$$

**Sfera piena carica**

$$\rho(\vec{x}) = cost. \qquad \int_{sfera} \rho\, d\vec{x} = Q$$

Per $r > R$ $\qquad E(r) = \frac{Q}{4\pi\,\epsilon_0\,r^2}$

Per $r < R$ $\qquad E(r) = \frac{\rho\, r}{3\epsilon_0}$

**Cilindro unif. carico**

$$Q = \lambda L \qquad E(r) = \frac{\lambda}{2\pi\,\epsilon_0\,r}$$

**Piano uniformemente carico**

$$E = \frac{\sigma}{2\epsilon_0}\,, \qquad \int_A \sigma\, dA = Q$$

**Parete carica**

$$q_{\text{int}} = \tau\rho = \Sigma\, d\,\rho \qquad \sigma = \rho\, d$$

- Per $|x| > d/2\,,$ $\qquad E(x) = \frac{d\,\rho}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0}$

$$V(x) = -\frac{\rho\, d}{2\epsilon_0}\,x + \frac{\rho\, d^2}{8\epsilon_0}$$

- Per $|x| < d/2\,,$ $\qquad E(x) = \frac{\rho}{\epsilon_0}\,x$

$$V(x) = -\frac{\rho}{\epsilon_0}\,\frac{1}{2}x^2$$

**Condensatore piano**

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0\,\Sigma}$$

$$|\Delta V| = E\,h = \frac{q}{\epsilon_0\,\Sigma}\,h$$

$$C = \frac{q}{\Delta V} = \frac{\epsilon_0\,\Sigma}{h}$$

$$F = \frac{1}{2}\,q\,\frac{dV}{dx} = \frac{1}{2}\,q\,E$$

$$W = -W_{\text{c}} = U_p = \frac{1}{2}\,\frac{Q^2}{C} = \frac{1}{2}\,CV^2 = \frac{1}{2}\,QV$$

$$U_p = \frac{1}{2}\,\epsilon_0\,E^2\,\Sigma h = \frac{1}{2}\,\epsilon_0\,E^2\,\tau$$

$$u_{\text{E}} = \frac{1}{2}\,\epsilon_0\,E^2$$

**Condensatore sferico**

$$E(r) = \frac{\sigma\,R_1}{r\,\epsilon_0}$$

$$|V| = \frac{\sigma\,R_1}{\epsilon_0}\,\ln\frac{R_2}{R_1}$$

$$C = \frac{2\pi\,h\,\epsilon_0}{\ln\dfrac{R_2}{R_1}}$$

**Condensatore sferico**

$$\Delta V = \frac{q}{4\pi\,\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{q}{4\pi\,\epsilon_0}\cdot\frac{R_2 - R_1}{R_1 R_2}$$

$$C = 4\pi\,\epsilon_0\,\frac{R_1\,R_2}{R_2 - R_1}$$

**Condensatori in serie**

$$\Delta V = q\sum_i \frac{1}{C_i} \quad\Longrightarrow\quad C = \frac{1}{\sum_i \dfrac{1}{C_i}}$$

**Condensatori in parallelo**

$$\Delta V_N = \frac{q_N}{C_N} \quad\Longrightarrow\quad C = \sum_i C_i$$

**Dielettrici**

$$V < V_0 \qquad V = \frac{V_0}{k} \quad \text{con } k > 1$$

$\vec{p} = \alpha\,\vec{E}$ $\qquad \alpha :$ suscettività elettrica

$$\vec{\mathbb{P}} = \frac{N\,\vec{p}}{\tau} = \frac{N\,\alpha}{\tau}\,\vec{E} \quad \text{con } n = \frac{N}{\tau}$$

$$\vec{\mathbb{P}} = n\,\alpha\vec{E}$$

$$E = \frac{\sigma - \sigma_P}{\epsilon_0} = \frac{\sigma - \mathbb{P}}{\epsilon_0}$$

$$\sigma_P = \frac{q}{\Sigma} = \frac{N\,e}{\Sigma} = \vec{\mathbb{P}}\cdot\vec{u}_N$$

$\vec{\mathbb{P}} = \epsilon_0\,\chi\,\vec{E}$ $\qquad \chi :$ Permeabilità elettrica

$$E = \frac{\sigma}{\epsilon_0}\cdot\frac{1}{1+\chi} = \frac{\sigma}{k\,\epsilon_0}$$