DIPARTIMENTO
DI FISICA
E ASTRONOMIA
Galileo Galilei

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Theory of Fundamental Interactions
Exercises I

Exercise 1

1. Write, in QED, the amplitude of the process

$$e^+e^- \to \gamma\gamma\,,$$

and verify explicitly that the amplitude vanishes if one of the photons in the final state is a non-physical particle with scalar polarization.

2. Consider the SU(2) gauge theory with fermions in the fundamental representation, discussed in the Lectures, and compute the amplitude

$$\psi_1\overline{\psi_1} \to W^3W^3\,.$$

Check that the amplitude vanishes if one of the W^3 has unphysical scalar polarization. Also check that this property holds also for the process

$$\psi_1\overline{\psi_1} \to W^+W^-\,.$$

Notice that the presence of the cubic gauge boson interaction plays an essential role in this cancellation.

Exercise 2

Introduce in QED a new state, with the same charge of the electron, the muon μ with $m_\mu = 105.7$ MeV.

1. Write down the Lagrangian employing the principle of gauge invariance and verify that it *does not* contain interactions of the form

$$\mathcal{L}_{\mu e} = \widetilde{e}\,\overline{\mu}A_\mu\gamma^\mu e + h.c.\,,$$

where "*h.c.*" denotes the Hermitian conjugate and $\widetilde{e}$ is a new hypothetical free parameter. Verify explicitly that the term $\mathcal{L}_{\mu e}$ violates gauge invariance.

2. Imagine introducing the coupling $\mathcal{L}_{\mu e}$ in the Lagrangian, thus violating the gauge symmetry. Compute the amplitude

$$e^+e^- \to \gamma\gamma\,,$$

in the presence of this new vertex and check that the scalar photon amplitude does not vanish in this case, allowing for an unphysical state being produced.

3. The coupling $\mathcal{L}_{\mu e}$ mediates the decay of the muon to electron and photon. Compute, neglecting the electron mass, the decay rate for a muon at rest

$$\mu^- \to e^- \gamma_T \,,$$

averaged on the initial muon polarization and summed over the ones of the final states. Consider only the two transverse (physical) polarizations, $T = 1, 2$ for the final-state photon. Given the observed life-time of the muon, $\tau = 1/\Gamma_{tot} \simeq 2.2 \cdot 10^{-6}$ s and the experimental bound on its branching fraction to electron and photon

$$BR(\mu \to e\gamma) \equiv \frac{\Gamma(\mu \to e\gamma)}{\Gamma_{tot}} < 2.4 \cdot 10^{-12} \,,$$

extract an experimental bound on the coupling $\widetilde{e}$.

hint: Notice that the transverse polarization vectors ϵ_μ^T of the photon are orthogonal to the electron 4-momentum because the electron and photon 3-momenta are equal and opposite.

4. Instead of $\mathcal{L}_{\mu e}$, consider the $d = 5$ operator

$$\mathcal{L}'_{\mu e} = \frac{1}{\Lambda}\, \overline{\mu} \gamma^\mu \gamma^\nu e F_{\mu\nu} + h.c.\,.$$

Check that the scalar photon is not produced by this operator. Why? Given the upper limit on $BR(\mu \to e\gamma)$, compute the lower limit on the operator scale Λ.