DIPARTIMENTO
DI FISICA
E ASTRONOMIA
Galileo Galilei

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Theory of Fundamental Interactions
Exercises II

Exercise 1

Consider n real scalar fields ϕ_i, $i = 1, \dots n$, with a Lagrangian

$$\mathcal{L} = \frac{1}{2}\sum_i \partial_\mu \phi_i \partial^\mu \phi_i + \frac{\mu^2}{2}\sum_i \phi_i^2 - \frac{\lambda}{4}\left(\sum_i \phi_i^2\right)^2 .$$

1. Identify the full (space-time, global and discrete) symmetry group of this theory.
2. Determine the classical vacuum configuration of the system and discuss which symmetries are spontaneously broken.
3. Verify explicitly the validity of the Goldstone theorem in this example by computing the particle spectrum.

Exercise 2

Within the SU(2) gauge theory, compute the spectrum of the gauge bosons and their interactions with the physical Higgs boson in the two following cases:

1. The Higgs scalar field is a doublet

   $$\Phi = \begin{pmatrix} \phi_u \\ \phi_d \end{pmatrix},$$

   with potential

   $$V[\Phi] = -\mu^2 \Phi^\dagger \Phi + \lambda \left[\Phi^\dagger \Phi\right]^2 .$$

2. The Higgs is a triplet, transforming in the adjoint of SU(2)

   $$\vec{\Phi} = \{\phi_a\}, \quad \text{con} \quad a = 1,2,3$$

   and the potential is

   $$V[\vec{\Phi}] = -\frac{\mu^2}{2}\sum_a \phi_a^2 + \frac{\lambda}{4}\left[\sum_a \phi_a^2\right]^2 .$$

   Hint: in order to write down the covariant derivative of the triplet, use that the generators in the adjoint representation are

   $$(T^a)_{bc} = i f^{acb},$$

   where f denotes the structure constants of the group.