DIPARTIMENTO
DI FISICA
E ASTRONOMIA
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Strongly Coupled Theories and their use
Exercise I

Fermionic Currents

Consider the Lagrangian

$$\mathcal{L} = \overline{\Psi}_i \, i\gamma^\mu \partial_\mu \Psi^i - V\left[\overline{\Psi}_i \Psi^j\right] \,,$$

and assume that the potential V respects a generic Lie group of symmetry G acting as $\Psi \to g\Psi$ on the Dirac fermions Ψ^i.

1. Applying the Noether theorem, compute the currents associated with G, and show explicitly that they are conserved on the equations of motion.
   Hint: Treat the fermion bilinear as a single field $X_i^j = \overline{\Psi}_i \Psi^j$, and express the condition of invariance of the potential in terms of the variable X.

2. Show that the conserved charges Q^a obey the canonical commutation relations.

3. Consider the case $V = 0$, what is the maximal global group of symmetry of the theory and the associated currents? Write down at least one interaction term that respects this symmetry and show that the currents are conserved in the presence of this interaction.

Exercise II

Pion decay constant

1. Consider the Gell-Mann-Levy Σ-model, and compute through the Noether theorem the conserved currents of the chiral groups $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Show that the axial current has, at tree-level, the following matrix element among the vacuum and the single pion state

$$\langle 0 | J_A^{\mu,a} | \pi^b(p) \rangle = i F_\pi \delta^{ab} p^\mu \, .$$

2. With the above result, compute the width of the decay process $\pi^+ \to \mu^+ \nu_\mu$ and, by matching with the experimental result, extract the value of F_π.

3. By assuming exact Gell-Mann $\mathrm{SU}(3)$ symmetry, compute the semileptonic decay width of the charged kaon, $K^+ \to \mu^+ \nu_\mu$ and compare the result with observations. Are the deviations compatible with the expected accuracy of the Gell-Mann symmetry?

Exercise III

Minimal Composite Higgs

Present a self-contained derivation of the Composite-Higgs Lagrangian discussed in the lectures and fill the missing steps. In particular

1. Compute explicitly the Goldstone matrix U and the d_μ symbol. The simplest way to proceed is to first derive U and d_μ, by explicit calculation, when the Goldstone fourplet has only one non-vanishing component, and then to extend the result making use of the SO(4) unbroken symmetry.

2. Introduce the SM gauge fields by gauging the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ subgroup of SO(4), and show how the d_μ symbol gets modified.

3. Identify the two complex combinations of the four real Goldstone fields that form the Higgs doublet. The requirement is that these combinations should transform like the two SM Higgs doublet components under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

4. Compute the Lagrangian in the unitary gauge, derive the expression of the vector boson's masses and of their couplings with the physical Higgs boson.

5. We saw how the SM quarks can be mixed with fermionic operators in the fiveplet of SO(5), with given assignments of an extra $\mathrm{U}(1)_X$ charge. Could we mix them with the spinorial (the **4**) or with the adjoint (the **10**) of SO(5)? Which $\mathrm{U}(1)_X$ charge should we assign in these cases?

Some of the results can be cross-checked with the ones reported in Appendix A of Ref. [1].

References

[1] A. De Simone, O. Matsedonskyi, R. Rattazzi and A. Wulzer, JHEP **1304** (2013) 004 [arXiv:1211.5663 [hep-ph]].