Soluzione dell'esercizio 8

$$\left(\gamma^0\right)^2 = \frac{\{\gamma^0, \gamma^0\}}{2} = \eta^{00} = +1\,,$$

$$\left(\gamma^i\right)^2 = \frac{\{\gamma^i, \gamma^i\}}{2} = \eta^{ii} = -1\,, \quad (i = 1, 2, 3)\,.$$

Scegliendo per comodità un indice spaziale $i \neq \mu$, e sfruttando l'algebra di Dirac e le proprietà cicliche della traccia:

$$tr\,\gamma^\mu = -tr\,[\gamma^\mu(\gamma^i)^2] = -tr\,[\gamma^\mu\gamma^i\gamma^i] = +tr\,[\gamma^i\gamma^\mu\gamma^i] = +tr\,[(\gamma^i)^2\gamma^\mu] = -tr\,\gamma^\mu \Rightarrow tr\,\gamma^\mu = 0\,,$$

$$\left(\gamma^5\right)^2 = (-i)\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,(-i)\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3 =$$

$$-\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3 = \gamma^1\,\gamma^2\,\gamma^3\,\gamma^1\,\gamma^2\,\gamma^3 = -\gamma^1\,\gamma^2\,\gamma^1\,\gamma^2 = -(\gamma^1)^2 = 1\,,$$

$$\left\{\gamma^5\,,\,\gamma^\mu\right\} = (-i)\ \left\{\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,,\,\gamma^\mu\right\} = (-i)(\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,\gamma^\mu + \gamma^\mu\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3) =$$

$$(-i)(\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,\gamma^\mu - \gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,\gamma^\mu) = 0\,.$$